1.3 Write Expressions

Before	You evaluated expressions.
Now	You will translate verbal phrases into expressions.
Why?	So you can find a bicycling distance, as in Ex. 36.

Key Vocabulary
• verbal model
• rate
• unit rate

To translate verbal phrases into expressions, look for words that indicate mathematical operations.

KEY CONCEPT *For Your Notebook*

Translating Verbal Phrases

Operation	Verbal Phrase	Expression
Addition: sum, plus, total, more than, increased by	The sum of 2 and a number x	$2 + x$
	A number n plus 7	$n + 7$
Subtraction: difference, less than, minus, decreased by	The difference of a number n and 6	$n - 6$
	A number y minus 5	$y - 5$
Multiplication: times, product, multiplied by, of	12 times a number y	$12y$
	$\frac{1}{3}$ of a number x	$\frac{1}{3}x$
Division: quotient, divided by, divided into	The quotient of a number k and 2	$\frac{k}{2}$

Order is important when writing subtraction and division expressions. For instance, "the difference of a number n and 6" is written $n - 6$, *not* $6 - n$, and "the quotient of a number k and 2" is written $\frac{k}{2}$, *not* $\frac{2}{k}$.

EXAMPLE 1 Translate verbal phrases into expressions

AVOID ERRORS
When you translate verbal phrases, the words "the quantity" tell you what to group. In part (a), you write $6n - 4$, *not* $(6 - 4)n$.

Verbal Phrase	Expression
a. 4 less than the quantity 6 times a number n	$6n - 4$
b. 3 times the sum of 7 and a number y	$3(7 + y)$
c. The difference of 22 and the square of a number m	$22 - m^2$

✓ **GUIDED PRACTICE** for Example 1

1. Translate the phrase "the quotient when the quantity 10 plus a number x is divided by 2" into an expression.
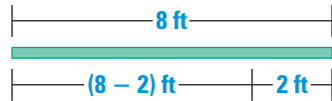
EXAMPLE 2 Write an expression

CUTTING A RIBBON A piece of ribbon ℓ feet long is cut from a ribbon 8 feet long. Write an expression for the length (in feet) of the remaining piece.
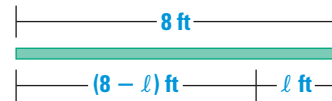
Solution

Draw a diagram and use a specific case to help you write the expression.

Suppose the piece cut is 2 feet long.

|←——— 8 ft ———→|

|←—(8 − 2) ft—→|—2 ft—|

The remaining piece is $(8 - 2)$ feet long.

Suppose the piece cut is ℓ feet long.

|←——— 8 ft ———→|

|←—(8 − ℓ) ft—→|—ℓ ft—|

The remaining piece is $(8 - \ell)$ feet long.

▶ The expression $8 - \ell$ represents the length (in feet) of the remaining piece.

VERBAL MODEL A **verbal model** describes a real-world situation using words as labels and using math symbols to relate the words. You can replace the words with numbers and variables to create a *mathematical model*, such as an expression, for the real-world situation.

EXAMPLE 3 Use a verbal model to write an expression

TIPS You work with 5 other people at an ice cream stand. All the workers put their tips into a jar and share the amount in the jar equally at the end of the day. Write an expression for each person's share (in dollars) of the tips.

Solution

STEP 1 **Write** a verbal model.

STEP 2 **Translate** the verbal model into an algebraic expression. Let a represent the amount (in dollars) in the jar.

Amount in jar	÷	Number of people
a	÷	6
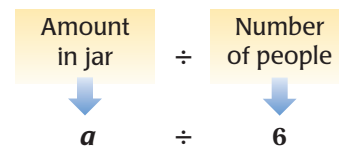
▶ An expression that represents each person's share (in dollars) is $\frac{a}{6}$.

✓ **GUIDED PRACTICE**   for Examples 2 and 3

2. **WHAT IF?** In Example 2, suppose that you cut the original ribbon into p pieces of equal length. Write an expression that represents the length (in feet) of each piece.

3. **WHAT IF?** In Example 3, suppose that each of the 6 workers contributes an equal amount for an after-work celebration. Write an expression that represents the total amount (in dollars) contributed.

RATES A **rate** is a fraction that compares two quantities measured in different units. If the denominator of the fraction is 1 unit, the rate is called a **unit rate**.

EXAMPLE 4 **Find a unit rate**

A car travels 120 miles in 2 hours. Find the unit rate in feet per second.

$$\frac{120 \text{ miles}}{2 \text{ hours}} = \frac{120 \text{ miles}}{2 \text{ hours}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{88 \text{ feet}}{1 \text{ second}}$$

▶ The unit rate is 88 feet per second.

EXAMPLE 5 **Solve a multi-step problem**

TRAINING For a training program, each day you run a given distance and then walk to cool down. One day you run 2 miles and then walk for 20 minutes at a rate of 0.1 mile per 100 seconds. What total distance do you cover?
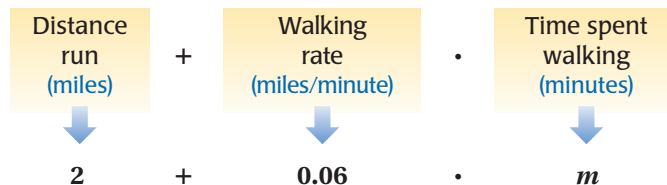
Solution

STEP 1 **Convert** your walking rate to miles per minute.

$$\frac{0.1 \text{ mile}}{100 \text{ seconds}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{6 \text{ miles}}{100 \text{ minutes}} = \frac{0.06 \text{ mile}}{1 \text{ minute}}$$

STEP 2 **Write** a verbal model and then an expression. Let m be the number of minutes you walk.

Distance run (miles)	+	Walking rate (miles/minute)	·	Time spent walking (minutes)
2	+	0.06	·	m

Use *unit analysis* to check that the expression $2 + 0.06m$ is reasonable.

$$\text{miles} + \frac{\text{miles}}{\text{minute}} \cdot \text{minutes} = \text{miles} + \text{miles} = \text{miles}$$

Because the units are miles, the expression is reasonable.

STEP 3 **Evaluate** the expression when $m = 20$.

$$2 + 0.06(20) = 3.2$$

▶ You cover a total distance of 3.2 miles.

✓ **GUIDED PRACTICE** **for Examples 4 and 5**

4. WHAT IF? In Example 5, suppose tomorrow you run 3 miles and then walk for 15 minutes at a rate of 0.1 mile per 90 seconds. What total distance will you cover?

1.3 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS**
Exs. 11, 21, and 33

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 13, 14, 34, and 37

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A(n) __?__ is a fraction that compares two quantities measured in different units.

2. ★ **WRITING** *Explain* how to write $\dfrac{20 \text{ miles}}{4 \text{ hours}}$ as a unit rate.

EXAMPLE 1
for Exs. 3–14

TRANSLATING PHRASES Translate the verbal phrase into an expression.

3. 8 more than a number x

4. The product of 6 and a number y

5. $\frac{1}{2}$ of a number m

6. 50 divided by a number h

7. The difference of 7 and a number n

8. The sum of 15 and a number x

9. The quotient of twice a number t and 12

10. 3 less than the square of a number p

11. 7 less than twice a number k

12. 5 more than 3 times a number w

13. ★ **MULTIPLE CHOICE** Which expression represents the phrase "the product of 15 and the quantity 12 more than a number x"?

Ⓐ $15 + 12 \cdot x$ Ⓑ $(15 + 12)x$ Ⓒ $15(x + 12)$ Ⓓ $15 \cdot 12 + x$

14. ★ **MULTIPLE CHOICE** Which expression represents the phrase "twice the quotient of 50 and the sum of a number y and 8"?

Ⓐ $\dfrac{2 \cdot 50}{y} + 8$ Ⓑ $2\left(\dfrac{50 + y}{8}\right)$ Ⓒ $2\left(\dfrac{50}{y + 8}\right)$ Ⓓ $\dfrac{2}{50} + (y + 8)$

EXAMPLES 2 and 3
for Exs. 15–21

WRITING EXPRESSIONS Write an expression for the situation.

15. Number of tokens needed for v video games if each game takes 4 tokens

16. Number of pages of a 5 page article left to read if you've read p pages

17. Each person's share if p people share 16 slices of pizza equally

18. Amount you spend if you buy a shirt for $20 and jeans for j dollars

19. Number of days left in the week if d days have passed so far

20. Number of hours in m minutes

21. Number of months in y years

EXAMPLE 4
for Exs. 22–27

UNIT RATES Find the unit rate in feet per second.

22. $\dfrac{300 \text{ yards}}{1 \text{ minute}}$

23. $\dfrac{240 \text{ yards}}{1 \text{ hour}}$

24. $\dfrac{180 \text{ miles}}{2 \text{ hours}}$

25. $\dfrac{171 \text{ miles}}{3 \text{ hours}}$

ERROR ANALYSIS *Describe* and correct the error in the units.

26.
$$\dfrac{\$2}{\text{foot}} \cdot 24 \text{ feet} = \dfrac{\$48}{\text{ft}^2}$$ ✗

27.
$$9 \text{ yards} \cdot \dfrac{3 \text{ feet}}{1 \text{ yard}} \cdot \dfrac{\$2}{\text{foot}} = \dfrac{\$54}{\text{ft}}$$ ✗

COMPARING RATES In Exercises 28 and 29, tell which rate is greater.

28. $1\frac{1}{4}$ miles in 2 minutes and 4 seconds, or $1\frac{3}{16}$ miles in 1 minute and 55 seconds

29. $1.60 for 5 minutes, or $19.50 for 1 hour

30. **CHALLENGE** Look for a pattern in the expressions shown below. Use the pattern to write an expression for the sum of the whole numbers from 1 to n. Then find the sum of the whole numbers from 1 to 50.

$$1 + 2 = \frac{2 \cdot 3}{2} \qquad\qquad 1 + 2 + 3 = \frac{3 \cdot 4}{2} \qquad 1 + 2 + 3 + 4 = \frac{4 \cdot 5}{2}$$

## PROBLEM SOLVING

31. **TICKET PRICES** Tickets to a science museum cost $19.95 each. There is a $3 charge for each order no matter how many tickets are ordered. Write an expression for the cost (in dollars) of ordering tickets. Then find the total cost if you order 5 tickets.

32. **FOSSIL FUELS** Fossil fuels are produced by the decay of organic material over millions of years. To make one gallon of gas, it takes about 98 tons of organic material, roughly the amount of wheat that could be harvested in a 40 acre field. Write an expression for the amount (in tons) of organic material it takes to make g gallons of gas. How many tons would it take to make enough gas to fill a car's 20 gallon gas tank?

33. **MULTI-STEP PROBLEM** A 48 ounce container of juice costs $2.64. A 64 ounce container of the same juice costs $3.84.

   a. Find the cost per ounce of each container.

   b. Which size container costs less per ounce?

   c. You want to buy 192 ounces of juice. How much do you save using the container size from your answer to part (b)?

34. ★ **OPEN–ENDED** *Describe* a real-world situation that can be modeled by the rate $\frac{30}{x}$ where x is a period of time (in hours). Identify the units for 30. Choose a value for x and find the unit rate.

35. **WILDLIFE EDUCATION** A wildlife center presents a program about birds of prey. The center charges a basic fee of $325 and an additional fee for each bird exhibited. If 5 birds are exhibited, the additional fee is $125. What is the total cost if 7 birds are exhibited?

36. **CYCLING** To prepare for a bicycling event, you warm up each day at a moderate pace and then you ride hard for a number of miles. One day, you warm up for 15 minutes at a rate of 0.1 mile per 25 seconds and then you ride hard for 12 miles. What total distance do you cover?

37. ★ **EXTENDED RESPONSE** A national survey determines the champion tree in a species. The champion is the tree with the greatest score, based on the tree's girth, its height, and its crown spread as shown.

A tree's score is the sum of the girth in inches, the height in feet, and $\frac{1}{4}$ the crown spread in feet. The data for three champion trees are given. Note that the girth is given in feet.

Crown spread
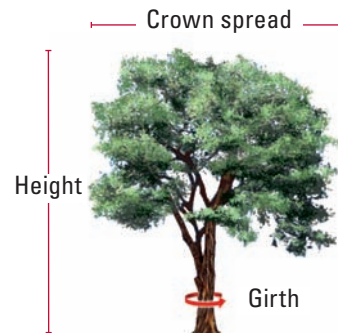
Height

Girth

Species	Girth (ft)	Height (ft)	Crown spread (ft)
Narrowleaf cottonwood	12	97	24
Green ash	21.5	95	95
Green buttonwood	14.5	51	68

a. Write Write an expression for a tree's score.

b. Evaluate Find the score for each tree in the table.

c. CHALLENGE Let *n* be any number greater than 0. Which change would have the greatest effect on a tree's score, an increase of *n* feet in the girth, in the height, or in the crown spread? *Explain* your reasoning.

QUIZ

Evaluate the expression.

1. $y + 10$ when $y = 43$

2. $15 - b$ when $b = 9$

3. t^2 when $t = 20$

4. $3n - 5$ when $n = 8$

5. $2y^2 - 1$ when $y = 5$

6. $\dfrac{3x - 6}{8}$ when $x = 8$

Translate the verbal phrase into an expression.

7. 7 less than a number *y*

8. 5 more than a number *t*

9. Twice a number *k*

10. CAMPING The rental cost for a campsite is $25 plus $2 per person. Write an expression for the total cost. Then find the total cost for 5 people.

See **EXTRA PRACTICE** in Student Resources 🧭 **ONLINE QUIZ** at my.hrw.com