1.4 Write Equations and Inequalities

Before You translated verbal phrases into expressions.

Now You will translate verbal sentences into equations or inequalities.

Why So you can calculate team competition statistics, as in Ex. 41.

Key Vocabulary
- **equation**
- **inequality**
- **open sentence**
- **solution of an equation**
- **solution of an inequality**

COMMON CORE

CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.*

An **equation** is a mathematical sentence formed by placing the symbol = between two expressions. An **inequality** is a mathematical sentence formed by placing one of the symbols <, ≤, >, or ≥ between two expressions.

An **open sentence** is an equation or an inequality that contains an algebraic expression.

KEY CONCEPT *For Your Notebook*

Symbol	Meaning	Associated Words
=	is equal to	the same as
<	is less than	fewer than
≤	is less than or equal to	at most, no more than
>	is greater than	more than
≥	is greater than or equal to	at least, no less than

COMBINING INEQUALITIES Sometimes two inequalities are combined. For example, the inequalities $x > 4$ and $x < 9$ can be combined to form the inequality $4 < x < 9$, which is read "x is greater than 4 and less than 9."

EXAMPLE 1 Write equations and inequalities

Verbal Sentence	Equation or Inequality
a. The difference of twice a number k and 8 is 12.	$2k - 8 = 12$
b. The product of 6 and a number n is at least 24.	$6n \geq 24$
c. A number y is no less than 5 and no more than 13.	$5 \leq y \leq 13$

Animated Algebra at my.hrw.com

GUIDED PRACTICE for Example 1

1. Write an equation or an inequality: The quotient of a number p and 12 is at least 30.

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SOLUTIONS When you substitute a number for the variable in an open sentence like $x + 2 = 5$ or $2y > 6$, the resulting statement is either true or false. If the statement is true, the number is a **solution of the equation** or a **solution of the inequality**.

EXAMPLE 2 Check possible solutions

Check whether 3 is a solution of the equation or inequality.

Equation/Inequality	Substitute	Conclusion
a. $8 - 2x = 2$	$8 - 2(3) \stackrel{?}{=} 2$	$2 = 2$ ✓ 3 is a solution.
b. $4x - 5 = 6$	$4(3) - 5 \stackrel{?}{=} 6$	$7 = 6$ ✗ 3 is *not* a solution.
c. $2z + 5 > 12$	$2(3) + 5 \stackrel{?}{>} 12$	$11 > 12$ ✗ 3 is *not* a solution.
d. $5 + 3n \leq 20$	$5 + 3(3) \stackrel{?}{\leq} 20$	$14 \leq 20$ ✓ 3 is a solution.

READING
A question mark above a symbol indicates a question. For instance, $8 - 2(3) \stackrel{?}{=} 2$ means "Is $8 - 2(3)$ equal to 2?"

USING MENTAL MATH Some equations are simple enough to solve using mental math. Think of the equation as a question. Once you answer the question, check the solution.

EXAMPLE 3 Use mental math to solve an equation

Equation	Think	Solution	Check
a. $x + 4 = 10$	What number plus 4 equals 10?	6	$6 + 4 = 10$ ✓
b. $20 - y = 8$	20 minus what number equals 8?	12	$20 - 12 = 8$ ✓
c. $6n = 42$	6 times what number equals 42?	7	$6(7) = 42$ ✓
d. $\frac{a}{5} = 9$	What number divided by 5 equals 9?	45	$\frac{45}{5} = 9$ ✓

✓ GUIDED PRACTICE for Examples 2 and 3

Check whether the given number is a solution of the equation or inequality.

2. $9 - x = 4;\ 5$ **3.** $b + 5 < 15;\ 7$ **4.** $2n + 3 \geq 21;\ 9$

Solve the equation using mental math.

5. $m + 6 = 11$ **6.** $5x = 40$ **7.** $\frac{r}{4} = 10$

EXAMPLE 4 Solve a multi-step problem

MOUNTAIN BIKING The last time you and 3 friends went to a mountain bike park, you had a coupon for $10 off and paid $17 for 4 tickets. What is the regular price of 4 tickets? If you pay the regular price this time and share it equally, how much does each person pay?

Solution

STEP 1 **Write** a verbal model. Let p be the regular price of 4 tickets. Write an equation.

Regular price	−	Amount of coupon	=	Amount paid
p	−	10	=	17

STEP 2 **Use** mental math to solve the equation $p - 10 = 17$. Think: 10 less than what number is 17? Because $27 - 10 = 17$, the solution is 27.

▶ The regular price for 4 tickets is $27.

STEP 3 **Find** the cost per person: $\frac{\$27}{4 \text{ people}} = \$6.75$ per person

▶ Each person pays $6.75.

EXAMPLE 5 Write and check a solution of an inequality

BASKETBALL A basketball player scored 351 points last year. If the player plays 18 games this year, will an average of 20 points per game be enough to beat last year's total?

Solution

STEP 1 **Write** a verbal model. Let p be the average number of points per game. Write an inequality.

Number of games	·	Points per game	>	Total points last year
18	·	p	>	351

USE UNIT ANALYSIS
Unit analysis shows that games $\cdot \frac{\text{points}}{\text{games}} = \text{points}$, so the inequality is reasonable.

STEP 2 **Check** that 20 is a solution of the inequality $18p > 351$. Because $18(20) = 360$ and $360 > 351$, 20 is a solution. ✓

▶ An average of 20 points per game will be enough.

GUIDED PRACTICE for Examples 4 and 5

8. **WHAT IF?** In Example 4, suppose that the price of 4 tickets with a half-off coupon is $15. What is each person's share if you pay full price?

9. **WHAT IF?** In Example 5, suppose that the player plays 16 games. Would an average of 22 points per game be enough to beat last year's total?

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1.4 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 7 and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 37, 44, 45, and 46

SKILL PRACTICE

1. **VOCABULARY** Give an example of an open sentence.

2. ★ **WRITING** *Describe* the difference between an expression and an equation.

EXAMPLE 1 for Exs. 3–16

WRITING OPEN SENTENCES **Write an equation or an inequality.**

3. The sum of 42 and a number n is equal to 51.
4. The difference of a number z and 11 is equal to 35.
5. The difference of 9 and the quotient of a number t and 6 is 5.
6. The sum of 12 and the quantity 8 times a number k is equal to 48.
7. The product of 9 and the quantity 5 more than a number t is less than 6.
8. The product of 4 and a number w is at most 51.
9. The sum of a number b and 3 is greater than 8 and less than 12.
10. The product of 8 and a number k is greater than 4 and no more than 16.
11. The difference of a number t and 7 is greater than 10 and less than 20.

STORE SALES **Write an inequality for the price p (in dollars) described.**

12. 

13. 

ERROR ANALYSIS ***Describe*** **and correct the error in writing the verbal sentence as an equation or an inequality.**

14. The sum of a number n and 4 is no more than 13.

$n + 4 < 13$ ✗

15. The quotient of a number t and 4.2 is at most 15.

$\frac{t}{4.2} > 15$ ✗

16. ★ **MULTIPLE CHOICE** Which inequality corresponds to the sentence "The product of a number b and 3 is no less than 12"?

Ⓐ $3b < 12$ Ⓑ $3b \leq 12$ Ⓒ $3b > 12$ Ⓓ $3b \geq 12$

EXAMPLE 2 for Exs. 17–28

CHECK POSSIBLE SOLUTIONS **Check whether the given number is a solution of the equation or inequality.**

17. $x + 9 = 17$; 8
18. $9 + 4y = 17$; 1
19. $6f - 7 = 29$; 5
20. $\frac{k}{5} + 9 = 11$; 10
21. $\frac{r}{3} - 4 = 4$; 12
22. $\frac{x - 5}{3} \geq 2.8$; 11
23. $15 - 4y > 6$; 2
24. $y - 3.5 < 6$; 9
25. $2 + 3x \leq 8$; 2
26. $2p - 1 \geq 7$; 3
27. $4z - 5 < 3$; 2
28. $3z + 7 > 20$; 4

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EXAMPLE 3
for Exs. 29–34

MENTAL MATH **Solve the equation using mental math.**

29. $x + 8 = 13$ **30.** $y + 16 = 25$ **31.** $z - 11 = 1$

32. $5w = 20$ **33.** $8b = 72$ **34.** $\frac{f}{6} = 4$

EQUATIONS AND INEQUALITIES **In Exercises 35 and 36, write an open sentence. Then check whether $3\frac{1}{2}$ is a solution of the open sentence.**

35. 2 less than the product of 3 and a number x is equal to the sum of x and 5.

36. 4 more than twice a number k is no greater than the sum of k and 11.

37. ★ **MULTIPLE CHOICE** Which equation has the same solution as $z - 9 = 3$?

Ⓐ $z - 4 = 16$ Ⓑ $\frac{1}{2}z = 7$ Ⓒ $z + 15 = 27$ Ⓓ $5z = 45$

38. **CHALLENGE** Use mental math to solve the equation $3x + 4 = 19$. *Explain* your thinking.

PROBLEM SOLVING

EXAMPLES 4 and 5
for Exs. 39–43

39. **CHARITY WALK** You are taking part in a charity walk, and you have walked 12.5 miles so far. Your goal is to walk 20 miles. How many more miles do you need to walk to meet your goal?

40. **COMPACT DISCS** You buy a storage rack that holds 40 CDs. You have 27 CDs. Write an inequality that describes how many more CDs you can buy and still have no more CDs than the rack can hold. You buy 15 CDs. Will they all still fit?

41. **ECO-CHALLENGE** Eco-Challenge Fiji was a competition that included jungle trekking, ocean swimming, mountain biking, and river kayaking. In 2002, the U.S. team finished second about 6 hours after the winning team from New Zealand. The U.S. team finished in about 173 hours. What was the winning team's time?

42. **BAKING MEASUREMENTS** You are baking batches of cookies for a bake sale. Each batch takes 2.5 cups of flour. You have 18 cups of flour. Can you bake 8 batches? *Explain*.

43. **EMPLOYMENT** Your friend takes a job cleaning up a neighbor's yard and mowing the grass, and asks you and two other friends to help. Your friend divides the amount the neighbor pays equally among all the members of the group. Each of you got $25. How much did the neighbor pay?

44. ★ **OPEN-ENDED** Describe a real-world situation you could model using the equation $5x = 50$. Use mental math to solve the equation. *Explain* what the solution means in this situation.

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45. ★ **SHORT RESPONSE** You have two part-time jobs. You earn $6 per hour running errands and $5 per hour walking dogs. You can work a total of 10 hours this weekend and hope to earn at least $55. Let r be the number of hours you spend running errands.

a. Write an inequality that describes the situation. Your inequality should involve only one variable, r.

b. If you spend the same amount of time at each job, will you meet your goal? *Explain*.

c. Can you meet your goal by working all 10 hours at only one job? *Explain*.

46. ★ **EXTENDED RESPONSE** Your school's service club is sponsoring a dance in the school gym to raise money for a local charity. The expenses will be $600. The club members will sell tickets for $10. They hope to raise enough money to cover the expenses and have enough left to donate $1000 to the charity.

a. How many tickets must they sell to cover their expenses?

b. How many tickets must they sell to cover their expenses and meet their goal?

c. The school allows no more than 200 students in the gymnasium for a dance. Can the club members sell enough tickets to exceed their goal? What is the greatest possible amount by which they can exceed their goal? *Explain* your reasoning.

47. **CHALLENGE** You and your friend are reading the same series of science fiction books. You tell your friend, "I've read 3 times as many books as you have." Your friend replies, "You've read only 4 more books than I have." How many books have each of you read?

48. **CHALLENGE** Each of the long sides of a rectangle has a length of x inches. Each of the other sides is 1 inch shorter than the long sides. The perimeter of the rectangle is 22 inches. Find the length and the width of the rectangle. *Justify* your answer.

MIXED REVIEW of Problem Solving

MATHEMATICAL PRACTICES Make sense of problems and persevere in solving them.

1. **MULTI-STEP PROBLEM** You are making a photo quilt by transferring photos to squares of fabric. Each square should be big enough so that you can turn over an edge $\frac{5}{8}$ inch long on each side and have a finished square with a side length of $5\frac{3}{4}$ inches.
   - **a.** What are the dimensions of each fabric square?
   - **b.** How many square inches of fabric do you need if you want to include 48 squares?
   - **c.** The fabric you buy is 36 inches wide. How long a piece of fabric do you need?
   - **d.** You buy a piece of fabric that has the length you found in part (c). Once you've cut all the squares, how many square inches of fabric are left over?

2. **MULTI-STEP PROBLEM** A rule of thumb states that the ideal weight (in ounces) of a baseball bat for a high school baseball player is 5 ounces more than one third of the player's height (in inches).

   - **a.** Write an expression that describes the ideal weight (in ounces) of a bat for a high school baseball player who is h inches tall.
   - **b.** One player was 66 inches tall last year. This year the player is 69 inches tall. How much heavier should the player's new bat be than the bat used last year?

3. **SHORT RESPONSE** You collect miniature cars and display them on shelves that hold 20 cars each.
   - **a.** Which expression would you evaluate to find the number of shelves you need for x cars: $20x$, $\frac{x}{20}$, or $\frac{20}{x}$? *Justify* your choice.
   - **b.** Find the number of shelves you need to display 120 cars.

4. **OPEN-ENDED** *Describe* a real-world situation that you could model with the inequality $3x < 15$. *Explain* what a solution of the inequality means in this situation.

5. **SHORT RESPONSE** You pay $7.50 for 3 quarts of strawberries. You realize that you need more strawberries for your recipe. You return to the store with $4.50. Will you have enough money to buy 2 more quarts of strawberries? *Explain* your reasoning.

6. **EXTENDED RESPONSE** The number of calories in one serving of any food is the sum of the calories from fat, protein, and carbohydrate. The table shows the calories in 1 gram of each of the three food components.

Component	Calories in 1 gram
Fat	9
Protein	4
Carbohydrate	4

   - **a.** Write an expression for the total number of calories in a serving of food that contains f grams of fat, p grams of protein, and c grams of carbohydrate.
   - **b.** A serving of cheddar cheese contains 14 grams of fat, 11 grams of protein, and 1 gram of carbohydrate. How many calories are in a serving of cheddar cheese?
   - **c.** A 100 pound teenager requires about 45 grams of protein per day. If the teenager tried to get all the required protein for one day from cheddar cheese, how many calories would the teenager consume? *Explain*.

7. **GRIDDED ANSWER** You are comparing two dorm-size refrigerators, both with cube-shaped interiors. One model has an interior edge length of 14 inches. Another model has an interior edge length of 16 inches. How many more cubic inches of storage space does the larger model have?

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