Name _____ Date _____

# CHAPTER 1 Extended Problem Solving

## The Division Algorithm

Given any two integers $a$ and $b$ with $a > 0$, there are unique integers $q$ and $r$ such that $b = qa + r$, where $0 \leq r < a$. The integer $a$ is the divisor, $b$ is the dividend, $q$ is the quotient, and $r$ is the remainder. The remainder is less than the divisor.

For $a = 5$ and $b = 13$ in the division algorithm, we have $13 = 2 \cdot 5 + 3$, so that $q = 2$ and $r = 3$. Note the value of $r$, 3, is less than the value of $a$, 5.

If an integer is divided by 5, the possible remainders are 0, 1, 2, 3, and 4.

## EXAMPLE 1 Find a divisor

The head of a political action committee has several volunteers who will call the offices of the 535 members of the U.S. Congress. When she tries to divide the 100 Senators evenly among the volunteers, she has four Senators left over. When she tries to divide the 435 House members evenly among the volunteers, she has three left over. What are the possible numbers of volunteers?

**Solution:**

**Step 1 Read and Understand**

The problem involves division with remainders. When the numbers 100 and 435 are both divided by the same unknown divisor, the remainders are 4 and 3, respectively. We want to find all possible values for the unknown divisor.

**Step 2 Make a Plan**

We will list all the divisors of 100 that result in a remainder of 4. Then we will check to see which of those divisors result in a remainder of 3 when divided into 435.

**Step 3 Solve the Problem**

We begin by listing all the divisors of 100 that result in a remainder of 4. Since the remainder, 4, is less than the unknown divisor, we can conclude that the divisor is at least 5. We can reason further that since the remainder is 4, the divisor divides evenly into $100 - 4 = 96$. So, we are looking for the divisors of 96 that are greater than or equal to 5. Those numbers are shown below.

6, 8, 12, 16, 24, 32, 48, 96

We could reason similarly about the divisors of 435 and find the common numbers in the two lists, but instead we will check these eight numbers by performing the divisions.

| Divisor | | Remainder |
|---|---|---|
| 6 | $435 = 72 \cdot 6 + 3$ | 3 |
| 8 | $435 = 54 \cdot 8 + 3$ | 3 |
| 12 | $435 = 36 \cdot 12 + 3$ | 3 |
| 16 | $435 = 27 \cdot 16 + 3$ | 3 |
| 24 | $435 = 18 \cdot 24 + 3$ | 3 |
| 32 | $435 = 13 \cdot 32 + 19$ | 19 |
| 48 | $435 = 9 \cdot 48 + 3$ | 3 |
| 96 | $435 = 4 \cdot 96 + 51$ | 51 |

The divisors 6, 8, 12, 16, 24, and 48 of 435 result in a remainder of 3, so the political action committee has 6, 8, 12, 16, 24, or 48 volunteers.

**Step 4 Look Back**

None of the possible numbers of volunteers seems to be unreasonable. You can check the numbers by dividing them into both 100 and 435 and verifying that you get the correct remainders. ■

## EXAMPLE 2 Find a dividend

When Coach Allen attempted to divide the Lettermen's Club into three equally sized groups, he had two athletes left over. When he attempted to divide the club into five equally sized groups, he also had two athletes left over. Assuming that the Letterman's Club has at least 20 members, what is the smallest number of members it could have?

**Solution:**

**Step 1 Read and Understand**

The problem involves division with remainders. When an unknown dividend is divided by both 3 and by 5, the remainder is 2. We want to find the smallest possible value of the dividend that is greater than or equal to 20.

**Step 2 Make a Plan**

We will use the division algorithm to list the possible dividends that result in a remainder of 2 for each divisor, 3 and 5. Then we will choose the least common dividend from the two lists that is greater than or equal to 20.

**Step 3 Solve the Problem**

We will list the dividends in tables.

**Divisor: 3**

| Quotient | | Dividend |
|---|---|---|
| 1 | $1 \cdot 3 + 2 = 5$ | 5 |
| 2 | $2 \cdot 3 + 2 = 8$ | 8 |
| 3 | $3 \cdot 3 + 2 = 11$ | 11 |
| 4 | $4 \cdot 3 + 2 = 14$ | 14 |
| 5 | $5 \cdot 3 + 2 = 17$ | 17 |
| 6 | $6 \cdot 3 + 2 = 20$ | 20 |
| 7 | $7 \cdot 3 + 2 = 23$ | 23 |
| 8 | $8 \cdot 3 + 2 = 26$ | 26 |
| 9 | $9 \cdot 3 + 2 = 29$ | 29 |
| 10 | $10 \cdot 3 + 2 = 32$ | 32 |

**Divisor: 5**

| Quotient | | Dividend |
|---|---|---|
| 1 | $1 \cdot 5 + 2 = 7$ | 7 |
| 2 | $2 \cdot 5 + 2 = 12$ | 12 |
| 3 | $3 \cdot 5 + 2 = 17$ | 17 |
| 4 | $4 \cdot 5 + 2 = 22$ | 22 |
| 5 | $5 \cdot 5 + 2 = 27$ | 27 |
| 6 | $6 \cdot 5 + 2 = 32$ | 32 |
| 7 | $7 \cdot 5 + 2 = 37$ | 37 |
| 8 | $8 \cdot 5 + 2 = 42$ | 42 |
| 9 | $9 \cdot 5 + 2 = 47$ | 47 |
| 10 | $10 \cdot 5 + 2 = 52$ | 52 |

The smallest dividend common to both lists is 17, but we know that the club has at least 20 members. The next common dividend is 32, which is greater than 20. The smallest number of members that the Lettermen's Club could have is 32.

**Step 4 Look Back**

The answer seems to be reasonable. You can check the answer by dividing 32 by both 3 and 5 and verifying that you get a remainder of 2 both times. You can also verify that this will not happen for the numbers between 20 and 31, inclusive. ■

## Practice

1. Which letter represents the dividend in the equation $b = qa + r$?
2. What are the possible remainders when an integer is divided by 8?
3. If, in a division problem, the divisor is greater than the dividend, then what is the quotient? Assume that both the dividend and the divisor are positive.
4. When a number is divided by 9, the quotient is 7 and the remainder is 8. What is the number?
5. When one integer is divided by another, the remainder is 15. What is the smallest possible value of the divisor?
6. When 75 is divided by a number, the remainder is 3. What is the greatest possible value of the divisor?

# Extended Problem Solving *continued*

7. When a number is divided by 18, the remainder is 5. What is the smallest possible value of the number?

8. When 52 is divided by a number, the remainder is 1. What are the possible values of the divisor?

### Problem Solving

9. Halley has 15 CDs and 25 DVDs. When she divides the CDs evenly among her friends, she has three left over. When she divides the DVDs evenly among her friends, she has one left over. List the numbers of friends that Halley could have.

10. Jerome deals the cards in a game evenly to all the players. When there are 3 players, he has one card left over. When there are four players, he has two cards left over. If there are at least 50 cards in the game, what is the smallest possible number of cards?

11. Bella chooses a number between 1 and 100. If she divides her number by 10, the remainder is 8. If she divides her number by 11, the remainder is 1. What is her number?

12. Michael is given a box of baseball cards. If he divides the cards evenly among himself and seven of his friends, he will have six cards left over. If he divides the cards only among his seven friends, he will have five cards left over. What is the least number of baseball cards that could be in the box?

13. The owner of an auto shop has 23 cars waiting for an oil change and 17 cars waiting for a tire rotation. If he divides the oil changes evenly among his employees, he has three left over. If he divides the tire rotation evenly among his employees, he has two left over. How many employees does the auto shop have?

14. Vincente has 1000 paper clips, 500 rubber bands, and several small boxes. When he divides the paper clips evenly among the boxes, he has 16 left over. When he divides the rubber bands evenly among the boxes, he has 20 left over. What is the minimum number of boxes that Vincente could have?

15. **Challenge** Lance has a box of 100 candy bars. If he divides the candy bars evenly among the students in his math class, he will have four left over. If he divides them evenly among the students in his Spanish class, he will have 8 left over. If there is one more student in his math class than in his Spanish class, how many students are in each class?

16. **Challenge** Tana chooses a number between 1 and 200. If she divides her number by 10, the remainder is 9. If she divides her number by 8, the remainder is 7. If she divides her number by 6, the remainder is 5. What is her number?