1.8 Represent Functions as Graphs

Before You represented functions as rules and tables.

Now You will represent functions as graphs.

Why? So you can describe sales trends, as in Example 4.

Key Vocabulary
- **function**
- **domain**
- **range**

REVIEW THE COORDINATE PLANE
For help with the coordinate plane, see p. SR11.

COMMON CORE

CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*

You can use a graph to represent a function. Given a table that represents a function, each corresponding pair of input and output values forms an ordered pair of numbers that can be plotted as a point. The x-coordinate is the input. The y-coordinate is the output.

Table

Input, x	Output, y
1	2
2	3
4	5

Ordered Pairs

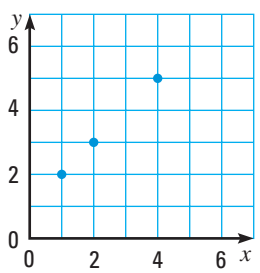
(input, output)

(1, 2)

(2, 3)

(4, 5)

Graph



The horizontal axis of the graph is labeled with the input variable. The vertical axis is labeled with the output variable.

EXAMPLE 1 Graph a function

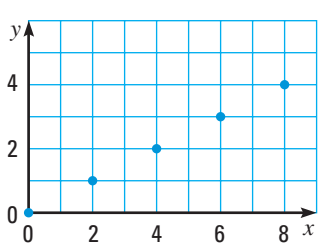
Graph the function $y = \frac{1}{2}x$ with domain 0, 2, 4, 6, and 8.

Solution

STEP 1 **Make** an input-output table.

x	0	2	4	6	8
y	0	1	2	3	4

STEP 2 **Plot** a point for each ordered pair (x, y).



GUIDED PRACTICE for Example 1

1. Graph the function $y = 2x - 1$ with domain 1, 2, 3, 4, and 5.

EXAMPLE 2 Graph a function

SAT SCORES The table shows the average score s on the mathematics section of the Scholastic Aptitude Test (SAT) in the United States from 1997 to 2003 as a function of the time t in years since 1997. In the table, 0 corresponds to the year 1997, 1 corresponds to 1998, and so on. Graph the function.

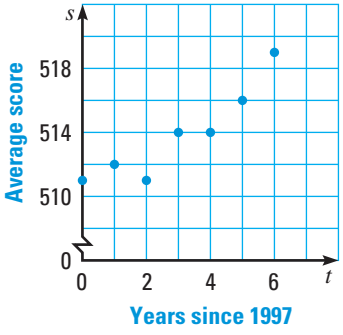
Years since 1997, t	0	1	2	3	4	5	6
Average score, s	511	512	511	514	514	516	519

READING
The symbol ≀ on the vertical number line represents a break in the axis.

Solution

STEP 1 **Choose** a scale. The scale should allow you to plot all the points on a graph that is a reasonable size.

- The t-values range from 0 to 6, so label the t-axis from 0 to 6 in increments of 1 unit.
- The s-values range from 511 to 519, so label the s-axis from 510 to 520 in increments of 2 units.



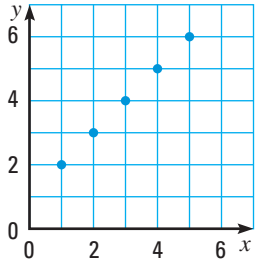
STEP 2 **Plot** the points.

GUIDED PRACTICE for Example 2

2. WHAT IF? In Example 2, suppose that you use a scale on the s-axis from 0 to 520 in increments of 1 unit. *Describe* the appearance of the graph.

EXAMPLE 3 Write a function rule for a graph

Write a rule for the function represented by the graph. Identify the domain and the range of the function.



Solution

STEP 1 **Make** a table for the graph.

x	1	2	3	4	5
y	2	3	4	5	6

STEP 2 **Find** a relationship between the inputs and the outputs. Notice from the table that each output value is 1 more than the corresponding input value.

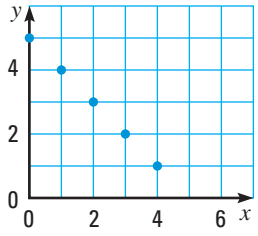
STEP 3 **Write** a function rule that describes the relationship: $y = x + 1$.

▶ A rule for the function is $y = x + 1$. The domain of the function is 1, 2, 3, 4, and 5. The range is 2, 3, 4, 5, and 6.

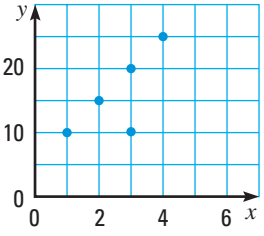
GUIDED PRACTICE for Example 3

Write a rule for the function represented by the graph. Identify the domain and the range of the function.

3.

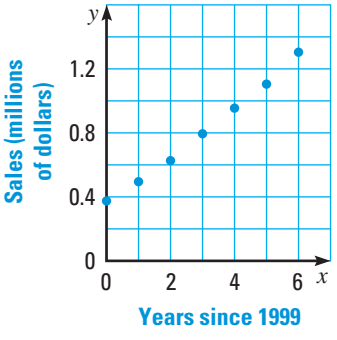


4.



EXAMPLE 4 Analyze a graph

GUITAR SALES The graph shows guitar sales (in millions of dollars) for a chain of music stores for the period 1999–2005. Identify the independent variable and the dependent variable. Describe how sales changed over the period and how you would expect sales in 2006 to compare to sales in 2005.



Solution

The independent variable is the number of years since 1999. The dependent variable is the sales (in millions of dollars). The graph shows that sales were increasing. If the trend continued, sales would be greater in 2006 than in 2005.

GUIDED PRACTICE for Example 4

5. REASONING Based on the graph in Example 4, is \$1.4 million a reasonable prediction of the chain's sales for 2006? *Explain*.

CONCEPT SUMMARY *For Your Notebook*

Ways to Represent a Function

You can use a verbal rule, an equation, a table, or a graph to represent a function.

Verbal Rule

The output is 1 less than twice the input.

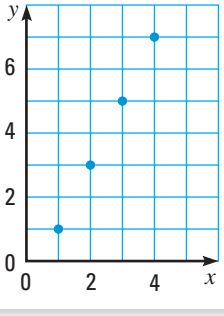
Equation

$y = 2x - 1$

Table

x	y
1	1
2	3
3	5
4	7

Graph



1.8 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 3 and 17

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 18, 19, and 20

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Each point on the graph of a function corresponds to an ordered pair (x, y) where x is in the _?_ of the function and y is in the _?_ of the function.

2. ★ **WRITING** Given the graph of a function, describe how to write a rule for the function.

EXAMPLE 1 for Exs. 3–9

GRAPHING FUNCTIONS **Graph the function.**

3. $y = x + 3$; domain: 0, 1, 2, 3, 4, and 5

4. $y = \frac{1}{2}x + 1$; domain: 0, 1, 2, 3, 4, and 5

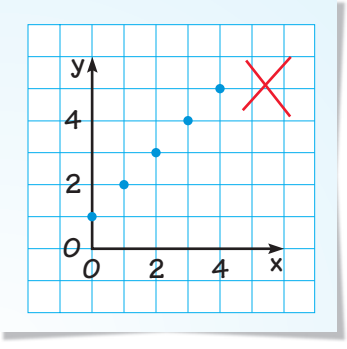
5. $y = 2x + 2$; domain: 0, 2, 5, 7, and 10

6. $y = 3x - 1$; domain: 1, 2, 3, 4, and 5

7. $y = x + 5$; domain: 0, 2, 4, 6, 8, and 10

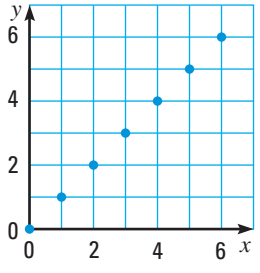
8. $y = 2.5x$; domain: 0, 1, 2, 3, and 4

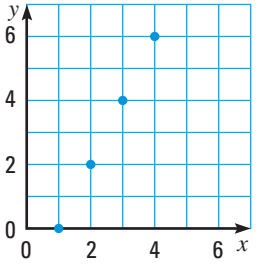
9. **ERROR ANALYSIS** *Describe* and correct the error in graphing the function $y = x - 1$ with domain 1, 2, 3, 4, and 5.

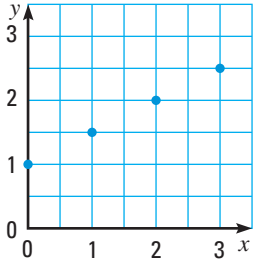


EXAMPLE 3 for Exs. 10–12

WRITING FUNCTION RULES **Write a rule for the function represented by the graph. Identify the domain and the range of the function.**

10.


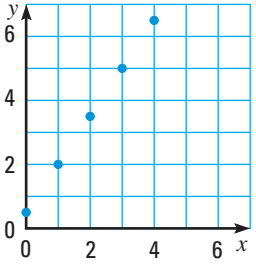
11.


12.


13. ★ **MULTIPLE CHOICE** The graph of which function is shown?

Ⓐ $y = \frac{1}{2}x + \frac{1}{2}$ Ⓑ $y = x + \frac{1}{2}$

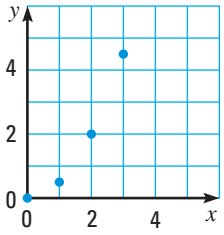
Ⓒ $y = \frac{3}{2}x + \frac{1}{2}$ Ⓓ $y = 2x + \frac{1}{2}$



14. CHALLENGE The graph represents a function.

a. Write a rule for the function.

b. Find the value of y so that $(1.5, y)$ is on the graph of the function.



PROBLEM SOLVING

EXAMPLE 2 for Exs. 15–17

15. ADVERTISING The table shows the cost C (in millions of dollars) of a 30 second Super Bowl ad on TV as a function of the time t (in years) since 1997. Graph the function.

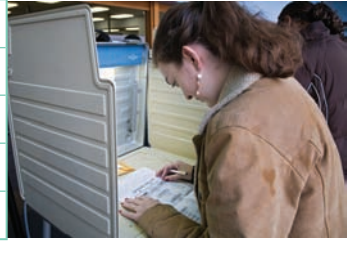
Years since 1997, t	0	1	2	3	4	5	6	7
Cost (millions of dollars), C	1.2	1.3	1.6	2.1	2.1	1.9	2.1	2.3

16. CONGRESS The table shows the number r of U.S. representatives for Texas as a function of the time t (in years) since 1930. Graph the function.

Years since 1930, t	0	10	20	30	40	50	60	70
Number of representatives, r	21	21	22	23	24	27	30	32

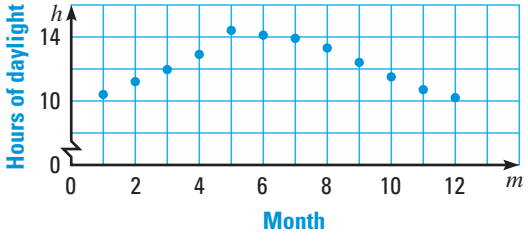
17. ELECTIONS The table shows the number v of voters in U.S. presidential elections as a function of the time t (in years) since 1984. First copy and complete the table. Round to the nearest million. Then graph the function represented by the first and third columns.

Years since 1984	Voters	Voters (millions)
0	92,652,680	?
4	91,594,693	?
8	104,405,155	?
12	96,456,345	?
16	105,586,274	?

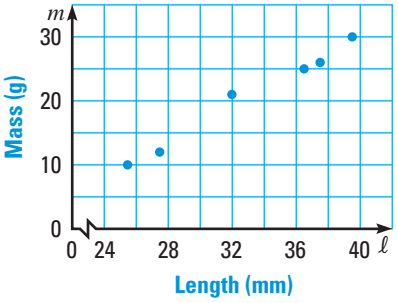


EXAMPLE 4 for Exs. 18–19

18. ★ WRITING The graph shows the number of hours of daylight in Houston, Texas, on the fifteenth day of the month, with 1 representing January, and so on. Identify the independent variable and the dependent variable. *Describe* how the number of hours of daylight changes over a year.



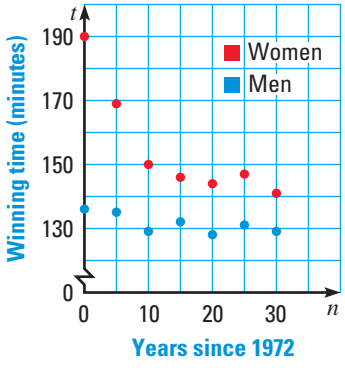
19. ★ **SHORT RESPONSE** A field biologist collected and measured alligator snapping turtle eggs. The graph shows the mass m (in grams) of an egg as a function of its length ℓ (in millimeters).



a. Describe As the lengths of the eggs increase, what happens to the masses of the eggs?

b. Estimate Is 27.5 g a reasonable estimate for the mass of an egg that is 38 mm long? *Explain*.

20. ★ **SHORT RESPONSE** Women first officially ran in the Boston Marathon in 1972. The graph shows the winning time t (in minutes) for both men and women as a function of the number n of years since 1972 for that year and every five years thereafter.



a. CHALLENGE *Explain* how you can estimate the difference in the men's and women's winning time for any year shown.

b. CHALLENGE *Compare* any trends you see in the graphs.

QUIZ

Choose the more precise measurement.

1. 1.5 ft; 18 in. **2.** 25 mm or 1.2 m **3.** 1.7 cm or 2.45 cm

4. SIGNIFICANT DIGITS Identify the number of significant digits in the measurement **(a)** 16.002 m and **(b)** 10.05.

5. The domain of the function $y = 12 - 2x$ is 0, 2, 3, 4, and 5. Make a table for the function, then identify the range of the function.

Tell whether the pairing is a function.

6.

x	5	6	7	11
y	1	2	3	7

7.

x	4	6	9	15
y	1	3	6	3

Graph the function.

8. $y = 2x - 5$; domain: 5, 6, 7, 8, and 9

9. $y = 7 - x$; domain: 1, 2, 3, 4, and 5

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

Extension

Determine Whether a Relation Is a Function

GOAL Determine whether a relation is a function when the relation is represented by a table or a graph.

Key Vocabulary
- **relation**

A **relation** is any pairing of a set of inputs with a set of outputs. Every function is a relation, but not every relation is a function. A relation is a function if for every input there is exactly one output.

COMMON CORE

CC.9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

EXAMPLE 1 Determine whether a relation is a function

Determine whether the relation is a function.

a.

Input	4	4	5	6	7
Output	0	1	2	3	4

b.

Input	3	5	7	9
Output	1	2	3	2

Solution

a. The input 4 has two different outputs, 0 and 1. So, the relation is *not* a function.

b. Every input has exactly one output, so the relation is a function.

USING THE GRAPH OF A RELATION You can use the *vertical line test* to determine whether a relation represented by a graph is a function. When a relation is *not* a function, its graph contains at least two points with the same x-coordinate and different y-coordinates. Those points lie on a vertical line.

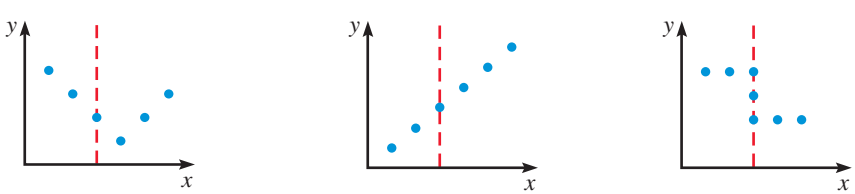
KEY CONCEPT *For Your Notebook*

Vertical Line Test

Words

A relation represented by a graph is a function provided that no vertical line passes through more than one point on the graph.

Graphs

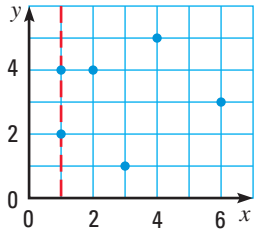


Function **Function** ***Not* a function**

EXAMPLE 2 Use the vertical line test

Determine whether the graph represents a function.

a.



You can draw a vertical line through the points (1, 2) and (1, 4). The graph does *not* represent a function.

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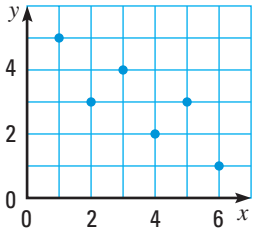
b.



No vertical line can be drawn through more than one point. The graph represents a function.

PRACTICE

EXAMPLE 1 for Exs. 1–3

IDENTIFYING FUNCTIONS **Determine whether the relation is a function.**

1.

Input	Output
0	1
2	6
5	12
7	5
8	4

2.

Input	Output
3	7
4	8
4	9
5	10
6	11

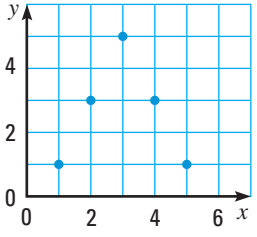
3.

Input	Output
0.7	1.9
1.2	2.4
3.5	4.7
7.5	8.7
7.5	9.7

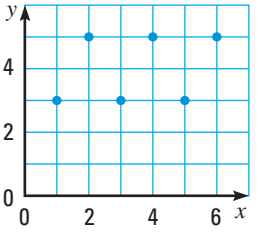
EXAMPLE 2 for Exs. 4–6

IDENTIFYING FUNCTIONS **Determine whether the graph represents a function.**

4.



5.



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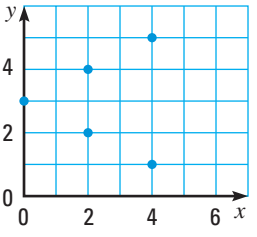

REASONING **Tell whether the pairing of *x*-values and *y*-values is necessarily a function. *Explain* your reasoning.**

7. A teacher makes a table that lists the number x of letters in the first name and the number y of letters in the last name of each student in the class.

8. Your doctor records your height x (in inches) and your weight y (in pounds) each time you have a medical exam.

9. You have a record of your age x (in years) and your height y (in inches) on each of your birthdays since you were born.

MIXED REVIEW of Problem Solving

MATHEMATICAL PRACTICES Make sense of problems and persevere in solving them.

1. **MULTI-STEP PROBLEM** A pizza shop charges $7 for a large cheese pizza plus $.95 for each topping.
   - **a.** Use a verbal model to write an equation for the total cost C (in dollars) of a pizza with n toppings.
   - **b.** The pizza shop offers 10 toppings. Write an input-output table for the total cost (in dollars) of a pizza as a function of the number n of toppings. *Explain* why the table represents a function and describe the domain and range of the function.
   - **c.** You have $15 to spend on a large pizza. What is the greatest number of toppings you can afford?

2. **SHORT RESPONSE** Your class is planning a car wash. You need $75 worth of materials.

   - **a.** Use a verbal model to write an equation that relates your profit to the number of cars you wash. Find your profit if you wash 120 cars.
   - **b.** Does doubling the number of cars you wash double your profit? *Explain*.

3. **MULTI-STEP PROBLEM** You are painting a room in a community center. The room has four walls that are each 9 feet high and 25 feet long. There are two rectangular windows and two rectangular doors that do not need to be painted. Each window is 3.5 feet wide and 4 feet high. Each door is 3.5 feet wide and 7 feet high.
   - **a.** Find the combined area of the windows and doors.
   - **b.** Find the combined area of all four walls, excluding the windows and the doors.
   - **c.** A gallon of paint covers about 400 square feet. How many one-gallon cans of paint will you need in order to give the room one coat of paint?
   - **d.** The paint costs $24.95 per gallon. How much will it cost for one coat of paint?

4. **GRIDDED ANSWER** You consider 68°F to be a comfortable room temperature. The temperature in a room is 18°C. How many degrees Celsius should you raise the temperature so that it will be 68°F?

5. **SHORT RESPONSE** Your family is driving from Charleston, South Carolina, to Jacksonville, Florida, a total distance of about 250 miles. You leave Charleston at 1:00 P.M. You travel at an average speed of 55 miles per hour without stopping. Will you get to Jacksonville before the 5:00 P.M. rush hour? *Explain*.

6. **GRIDDED ANSWER** A person invests $1200 in an account earning 3% simple annual interest. How much will be in the account after 2 years?

7. **OPEN-ENDED** Write a problem that involves a real-world situation and that can be solved using the formula for distance traveled. Solve the problem and explain what the solution means in the situation.

8. **EXTENDED RESPONSE** You pay $40 per hour for windsurfing lessons and rent equipment for $20 per hour. The cost (in dollars) of lessons and the cost (in dollars) of rentals are both functions of the time (in hours).
   - **a.** Write a rule for each function.
   - **b.** Let the domains of the functions be the whole numbers from 0 to 6. Graph each function.
   - **c.** You rent equipment for every lesson you take. What function gives your total cost? How would the graph of this function compare with the graphs in part (b)?