LESSON 10

Independent and Dependent Events

GOAL Find the probability of independent and dependent events, and find conditional probabilities.

Two events are **independent events** if the occurrence of one event does not affect the occurrence of the other. Two events are **dependent events** if the occurrence of one event *does* affect the occurrence of the other.

EXAMPLE 1 Identifying Independent and Dependent Events

A jar contains red and blue marbles. You randomly choose a marble from the jar, and you do not replace it. Then you randomly choose another marble. Tell whether the events are *independent* or *dependent*.

Event A: **The first marble you choose is red.**

Event B: **The second marble you choose is blue.**

SOLUTION

After you choose a red marble, there are fewer marbles left in the jar. This affects the probability that the second marble you choose is blue. So, the events are dependent.

The probability that event B occurs given that event A has occurred is called the **conditional probability** of B given A and is written $P(B|A)$. Note that A and B are independent events if and only if $P(B) = P(B|A)$ since the probability of event B does not depend on the occurrence of event A.

Use the formulas below to find the probabilities of independent and dependent events.

Probabilities of Independent and Dependent Events

Independent Events

For two independent events A and B, the probability that both events occur is the product of the probabilities of the events.

$P(A \text{ and } B) = P(A) \cdot P(B)$ *Events A and B are independent.*

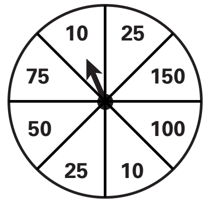
Dependent Events

For two dependent events A and B, the probability that both occur is the product of the probability of the first event and the conditional probability of the second event given the first event.

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ *Events A and B are dependent.*

EXAMPLE 2 Probability of Independent Events

As part of a board game, you need to spin the spinner at the right, which is divided into equal parts. Find the probability that you get 25 on your first spin and 50 on your second spin.



SOLUTION

Let event A be "get 25 on first spin" and let event B be "get 50 on second spin." Find the probability of each event. Then multiply the probabilities.

$P(A) = \frac{2}{8}$ *"25" appears twice.*

$P(B) = \frac{1}{8}$ *"50" appears once.*

$$P(A \text{ and } B) = \frac{2}{8} \cdot \frac{1}{8} = \frac{2}{64} \approx 0.031$$

The probability that you get 25 on your first spin and 50 on your second spin is about 3.1%.

EXAMPLE 3 Probability of Dependent Events

A bowl contains 36 green grapes and 14 purple grapes. You randomly choose a grape, eat it, and randomly choose another grape. Find the probability that both events A and B will occur.

Event A: The first grape is green.

Event B: The second grape is green.

SOLUTION

Find $P(A)$ and $P(B|A)$. Then multiply the probabilities.

$P(A) = \frac{36}{50}$ *Of the 50 grapes, 36 are green.*

$P(B|A) = \frac{35}{49}$ *Of the 49 remaining grapes, 35 are green.*

$$P(A \text{ and } B) = \frac{36}{50} \cdot \frac{35}{49} = \frac{1260}{2450} \approx 0.514$$

The probability that both of the grapes are green is about 51.4%.

CHECK Examples 1, 2, and 3

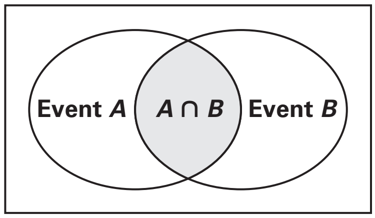
Tell whether the situation describes *independent* or *dependent events*. Then answer the question.

1. A drawer contains 12 white socks and 8 black socks. You randomly choose one sock, and you do not replace it. Then you randomly choose another sock. What is the probability that both socks chosen are white?
2. Suppose you flip a coin twice. What is the probability that you get tails on the first flip and tails on the second flip?

Conditional Probability The formula for dependent events on page 41 can be rewritten to give a rule for finding conditional probabilities. Dividing both sides of the formula by $P(A)$ gives the following.

$$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$$

A Venn diagram shows why this formula for $P(B|A)$ makes sense. Event A is known to have occurred, so it becomes the sample space for $P(B|A)$. The shaded region represents the intersection of A and B, written $A \cap B$. This intersection consists of the outcomes in B that are also in A. So, the probability of B given A is the number of outcomes in B that are also in A, divided by the total number of outcomes in A.



$$P(B|A) = \frac{\text{Number of outcomes in } A \cap B}{\text{Number of outcomes in } A}$$

Dividing both the numerator and denominator by the number of outcomes in the original sample space gives the rule $P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$.

EXAMPLE 4 Conditional Probability

The sedans, hatchbacks, and convertibles at a rental company are available with automatic or manual transmissions, as shown in the two-way table.

	Sedan (S)	Hatchback (H)	Convertible (C)	Total
Automatic (A)	8	4	6	18
Manual (M)	6	2	6	14
Total	14	6	12	32

a. Find the probability that a randomly chosen car is a sedan.

b. Find the probability that a randomly chosen car is a sedan given that the car has an automatic transmission.

c. Are the events "car is a sedan" and "car has an automatic transmission" independent? Why or why not?

SOLUTION

a. Since 14 of the 32 cars are sedans, $P(S) = \frac{14}{32} = \frac{7}{16}$.

b. $P(S|A) = \dfrac{P(S \text{ and } A)}{P(A)} = \dfrac{\frac{8}{32}}{\frac{18}{32}} = \dfrac{8}{18} = \dfrac{4}{9}$

c. By parts (a) and (b), $P(S) \neq P(S|A)$, so the events "car is a sedan" and "car has an automatic transmission" are *not* independent.

CHECK Example 4

Use the data in the table from Example 4.

3. Find the probability that a randomly-chosen car has a manual transmission.

4. Find the probability that a randomly-chosen car has a manual transmission given that the car is a hatchback.

5. Are the events "car has a manual transmission" and "car is a hatchback" independent? Why or why not?

EXERCISES

Tell whether the events are *independent* or *dependent*.

1. A box of energy bars contains an assortment of flavors. You randomly choose an energy bar and eat it. Then you randomly choose another bar.

Event *A*: You choose a honey-peanut bar first.

Event *B*: You choose a chocolate chip bar second.

2. You roll a number cube and flip a coin.

Event *A*: You get a 4 when rolling the number cube.

Event *B*: You get tails when flipping the coin.

3. Your CD collection contains hip-hop and rock CDs. You randomly choose a CD, then choose another without replacing the first CD.

Event *A*: You choose a hip-hop CD first.

Event *B*: You choose a rock CD second.

4. There are 22 volumes of an encyclopedia on a shelf. You randomly choose a volume and put it back. Then you randomly choose another volume.

Event *A*: You choose volume 7 first.

Event *B*: You choose volume 5 second.

Events *A* and *B* are independent. Find the missing probability.

5. $P(A) = 0.7$
$P(B) = 0.3$
$P(A \text{ and } B) = \underline{\ ?\ }$

6. $P(\text{A}) = 0.22$
$P(B) = \underline{\ ?\ }$
$P(A \text{ and } B) = 0.11$

7. $P(A) = \underline{\ ?\ }$
$P(B) = 0.4$
$P(A \text{ and } B) = 0.13$

Events *A* and *B* are dependent. Find the missing probability.

8. $P(A) = 0.5$
$P(B|A) = 0.4$
$P(A \text{ and } B) = \underline{\ ?\ }$

9. $P(A) = 0.9$
$P(B|A) = \underline{\ ?\ }$
$P(A \text{ and } B) = 0.72$

10. $P(A) = \underline{\ ?\ }$
$P(B|A) = 0.6$
$P(A \text{ and } B) = 0.15$

In Exercises 11–13, tell whether the situation describes *independent* or *dependent events*. Then answer the question.

11. You roll a number cube two times in a row. What is the probability that you get a 5 on the first roll and an even number on the second roll?

12. A jar contains 18 square and 27 round buttons. You randomly choose one button, and you do not replace it. Then you randomly choose another button. What is the probability that both buttons are square?

13. So far this season, Ryan has made 20 free throws in 28 attempts and Jay has made 11 free throws in 15 attempts. What is the probability that Ryan and Jay both make their next free throws?

14. Write the letters of your first and last name on slips of paper and place them in a cup. Include all the letters, even repeating ones. Design and carry out an experiment to find the probability of drawing two consonants in a row, assuming you do not replace the first letter after you draw it. Describe your experiment and your results. Compare the experimental probability to the theoretical probability.

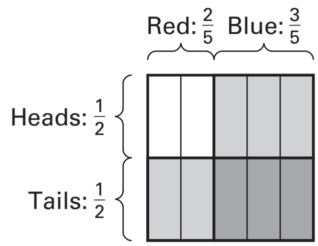
In Exercises 15 and 16, use the following information.

The formula for finding the probability of independent events can be extended to three or more events. This is also true for dependent events.

15. You are given a 4 digit password, which is randomly chosen by a computer, to access your new voice mail account. Each of the digits is a whole number from 0 through 9, and the digits can be repeated. What is the probability that your password is 2468?

16. In Exercise 15, suppose that the digits cannot be repeated. What is the probability that your password is 2468?

17. The area model at the right represents the probabilities when flipping a coin and randomly choosing a marble from a jar containing red and blue marbles. Use the area model to find the probability of getting tails and choosing a blue marble.



In Exercises 18–21, use the following information.

A standard deck of playing cards has 52 cards, with 13 cards in each of four *suits*: clubs, spades, diamonds, and hearts. *Face cards* are jacks, queens, and kings.

Clubs ♣: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king
Spades ♠: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king
Diamonds ♦: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king
Hearts ♥: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king

18. You randomly choose one card. What is the probability that the card is an ace or a king?

19. You randomly choose one card. What is the probability that the card is a face card or a spade?

20. You randomly choose one card and replace it. Then you randomly choose another card. What is the probability that the first card is a club and the second card is a king?

21. You randomly choose one card and do not replace it. Then you randomly choose another card. What is the probability that the first card is a 9 and the second card is a 6?

In Exercises 22 and 23, use the following information.

A basket contains bottles of apple juice and orange juice in two sizes. The number of each type of bottle is shown in the table.

22. What is the probability that a randomly-chosen bottle of juice is a 6-ounce bottle given that the bottle contains apple juice?

23. What is the probability that a randomly-chosen 6-ounce bottle contains orange juice?

	6 oz	**8 oz**
Apple	4	8
Orange	6	9

In Exercises 24–26, use the following information.

The students at Allen High School were surveyed to find out how they get to school each day. The table shows the number of students at each grade level who walk, bike, or take the bus to school.

	Walk	Bike	Bus	Total
Freshman	52	16	72	140
Sophomore	41	18	61	120
Junior	43	35	72	150
Senior	28	40	52	120
Total	164	109	257	530

24. a. What is the probability that a randomly-chosen student is a senior?

b. What is the probability that a randomly-chosen student is a senior given that the student bikes to school?

c. Are the events "student is a senior" and "student bikes to school" independent events? Why or why not?

25. What is the probability that a randomly-chosen student takes the bus given that the student is a junior?

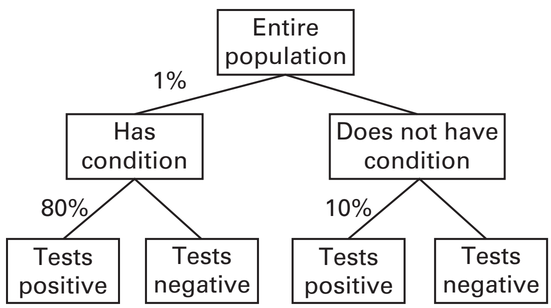
26. What is the probability that a randomly-chosen sophomore does not take the bus?

In Exercises 27 and 28, use the following information.

According to Bayes' Theorem, the conditional probability of event A given event B can be calculated using the following formula, assuming $P(B) \neq 0$.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

27. Suppose 1% of the population is known to have a medical condition. There is a test for the condition, and 80% of people with the condition test positive for it. Also, 10% of people without the condition test positive for it. Follow these steps to find the probability that a person actually has the condition given that he or she tests positive.



a. Let A be the event that a person has the condition and let B be the event that the person tests positive. What are $P(B|A)$ and $P(A)$?

b. Find $P(B)$. (*Hint*: Consider the portion of the population with the condition that tests positive and the portion of the population without the condition that tests positive.)

c. Use your results from parts (a) and (b) and Bayes' Theorem to find the probability that a person has the condition given that he or she tests positive.

28. Sara plans a picnic for May 18. The probability of rain on any day in May in Sara's town is 2%. However, the forecast predicts rain for May 18. When it rains, the forecast correctly predicts rain 85% of the time. When it does not rain, the forecast incorrectly predicts rain 3% of the time. What is the probability that it will rain during the picnic given that the forecast predicts rain?