

**Scoring Rubric****Full Credit**

- solution is complete and correct

**Partial Credit**

- solution is complete but has errors, or
- solution is without error but incomplete

**No Credit**

- no solution is given, or
- solution makes no sense

**SHORT RESPONSE QUESTIONS****PROBLEM**

A survey of 887 households found that 270 households have a dog, 327 have a cat, and 82 have both. One of the households surveyed is chosen at random. Which is greater, the probability that the household has either a dog *or* a cat, or that the household has neither?

Below are sample solutions to the problem. Read each solution and the comments on the left to see why the sample represents full credit, partial credit, or no credit.

**SAMPLE 1: Full credit solution**

Because some households have both a cat and a dog, having a dog and having a cat are overlapping events.

So,  $P(\text{dog or cat}) = P(\text{dog}) + P(\text{cat}) - P(\text{dog and cat})$ .

$$P(\text{dog}) = \frac{271}{887} \approx 0.306 \quad \text{and} \quad P(\text{cat}) = \frac{327}{887} \approx 0.369$$

$$P(\text{dog and cat}) = \frac{82}{887} \approx 0.092$$

$$P(\text{dog or cat}) \approx 0.306 + 0.369 - 0.092 = 0.583$$

$$P(\text{neither}) = \frac{887 - ((270 + 327) - 82)}{887} = \frac{372}{887} \approx 0.419$$

The probability that the household has a dog or a cat, about 0.583, is greater than the probability that the household has neither, 0.419.

**SAMPLE 2: Partial credit solution**

The events “having a dog” and “having a cat” are overlapping.

$P(\text{dog or cat}) = P(\text{dog}) + P(\text{cat}) - P(\text{dog and cat})$ .

$$P(\text{dog}) = \frac{271}{877} \approx 0.309 \quad \text{and} \quad P(\text{cat}) = \frac{326,591}{877,403} \approx 0.373$$

$$P(\text{dog and cat}) = \frac{82}{877} \approx 0.094$$

$$P(\text{dog or cat}) \approx 0.309 + 0.373 - 0.094 = 0.588$$

$$P(\text{neither}) = \frac{877 - ((270 + 327) - 82)}{877} \approx 0.413$$

$$P(\text{dog or cat}) > P(\text{neither})$$

**SAMPLE 3: Partial credit solution**

$$P(\text{dog or cat}) = \approx 0.306 + 0.369 - 0.092 = 0.583$$

$$P(\text{dog or cat}) > P(\text{neither})$$

.....→  
The student correctly identified and used language and formulas.

.....→  
The answer is correct and clearly stated in a sentence.

.....→  
The reasoning is complete but has errors

.....→  
The solution is correct, but incomplete. No explanation is provided.

#### SAMPLE 4: No credit solution

.....→  
The reasoning and the answer are both incorrect.

The probability that a household has a dog or a cat is the sum of the probability that the household has a dog and the probability that it has a cat, which is about  $0.369 + 0.306$ , or  $0.675$ . The probability that the household has neither is  $\frac{887 - 82}{887} \approx 0.908$ , so the probability that the household has neither is greater.

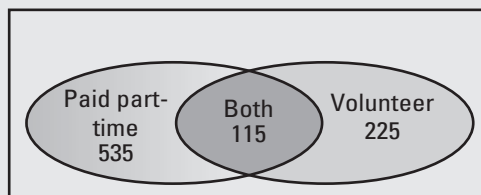
### PRACTICE

#### Apply the Scoring Rubric

Score the solution to the problem below as *full credit*, *partial credit*, or *no credit*. Explain your reasoning.

**PROBLEM** A survey of 1200 high school students found that 650 of the students have a paid part-time job, 340 have a volunteer job, and 115 have both. One of the students surveyed is chosen at random. Which is greater, the probability that the student has either a paid part-time job or a volunteer job, or the probability that the student has neither?

1. The number of students who have either a paid part-time job, a volunteer job, or both is 1105, so the probability the student chosen at random has either is  $\frac{1105}{1200} \approx 0.921$ . To check, notice that 1105 is close to 1200, so  $\frac{1105}{1200} \approx 1$ .  
The number of students who have neither is  $1200 - (650 + 340 + 115) = 95$ , so the probability the student has neither is  $\frac{95}{1200} \approx 0.079$ . To check, notice that  $\frac{95}{1000} \approx \frac{100}{1000} = 0.1$ .  
Then the probability that the student has either a paid part-time job or a volunteer job is greater than the probability that the student has neither.
2. I made a Venn diagram to display the data.



Because there are 115 students with both a paid part-time job and a volunteer job, there are  $650 - 115$  students with a paid part-time job only and  $340 - 115 = 225$  students with a volunteer job only. So, the number of students with either is  $535 + 115 + 225 = 875$ . The probability that the student has either a paid part-time job or a volunteer job is  $\frac{875}{1200} \approx 0.729$ .  
The number of students with neither is  $1200 - 875 = 325$ . The probability that the student has neither is  $\frac{325}{1200} \approx 0.271$ .  
The probability that the student has either a paid part-time job or a volunteer job is greater than the probability that the student has neither a paid part-time job nor a volunteer job.

## SHORT RESPONSE

1. Your English teacher gives you a list of 5 books that you are required to read over summer vacation. You read the books in a random order.
  - a. In how many different ways can you read the 5 books?
  - b. What is the probability that you read the longest book first or second? *Explain* how you found this probability.
2. You are ordering a pizza with 3 toppings. There are 8 toppings available.
  - a. How many possible pizzas with 3 toppings can you order?
  - b. Did you answer the question in part (a) using combinations or permutations? *Explain* your choice.
3. You must choose a password for an online account. The password must have between 4 and 6 characters, consisting of letters or digits. The letters or digits may repeat. How many passwords are possible? *Explain* how you found your answer.
4. A meteorologist claims that there is a 70% chance of rain. Brett knows that if it rains, there is a 75% chance that his softball game will be rescheduled. Is it *more likely* or *less likely* that the game will be rescheduled? *Explain* your reasoning.
5. In one high school, 40% of the students are involved in sports, 25% are involved in community service clubs, and 15% are involved in both. Suppose a student is selected at random. *Compare* the following probabilities: the probability that a student is on a sports team, given that the student is in a service club, and the probability that a student is in a service club given that the student is on a sports team.
6. A stock market analyst predicts that the probability that a company's stock will rise next week is about 20%. The analyst also predicts that if the stock *does* rise next week, the probability that it will rise the following week is 50%. How likely is it that the stock will rise both weeks? *Explain*.
7. Of the 8 members of a math club, 2 are being chosen to compete as a team for a statewide competition. To choose the team, the team adviser plans to observe each possible pair of club members working together for 20 minutes to solve problems, then choose the pair that works best together. How long will the observations take altogether?
8. In a game, player A thinks of a number from 1 to 10, and player B (knowing the number is from 1 to 10) guesses what it is. Player A then tells player B whether the guess was correct, or whether or not the actual number is higher or lower than the guess. Player B gets one more chance to guess the number.
  - a. Find the probability that player B guesses correctly on the first try.
  - b. Given that player B guesses "5" on the first try and is wrong, find the probability that player B wins the game.

## MULTIPLE CHOICE

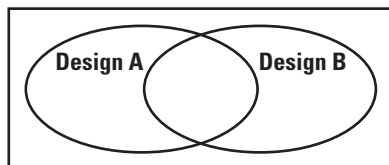
9. The odds in favor of an event are 3 : 4. What is the probability of the event?
- (A)  $\frac{1}{4}$  (B)  $\frac{3}{7}$   
(C) 75% (D)  $\frac{3}{4}$
10. A bag contains 4 red marbles, 3 green marbles, and 5 blue marbles. You randomly choose a marble from the bag. What is the probability that you choose a blue marble?
- (A)  $\frac{1}{5}$  (B)  $\frac{5}{12}$   
(C)  $\frac{5}{11}$  (D)  $\frac{5}{7}$
11. You roll a number cube. What is the probability that you roll a multiple of 2 or a multiple of 3?
- (A)  $\frac{1}{6}$  (B)  $\frac{1}{3}$   
(C)  $\frac{2}{3}$  (D)  $\frac{5}{6}$

## GRIDDED ANSWER

12. What is the value of  ${}_4P_3$ ?
13. In how many ways can you arrange the letters in the word BEACH?
14. In how many ways can a president, vice president, and treasurer be chosen from among the 10 members of a club?
15. Given that  $A$  and  $B$  are dependent events, and that  $P(A \text{ and } B) = 0.4$  and  $P(B|A) = 0.8$ , what is  $P(A)$ ?
16. There are 13 girls and 12 boys in a class. What is the probability that a student selected at random is a girl?
17. You draw a marble from a bag containing 6 green marbles and 4 red marbles, replace it, and draw a second marble. What is the probability that both marbles you drew are green?

## EXTENDED RESPONSE

18. A computer software company is performing a market test on two designs, A and B, for its new software program. Out of 250 people who view the designs, 85 like design A, 135 like design B, and 45 like both designs.
- a. Copy and complete the Venn diagram.



- b. What is the probability that a person likes design A or design B?
- c. What is the probability that a person does not like either design?
- d. *Explain* how you can calculate the probability from part (c) if you know the probability from part (b).