

11.1 Find Probabilities and Odds



Before

You made organized lists and tree diagrams.

Now

You will find sample spaces and probabilities.

Why?

So you can find the likelihood of an event, as in Example 2.

Key Vocabulary

- outcome
- event
- sample space
- probability
- odds

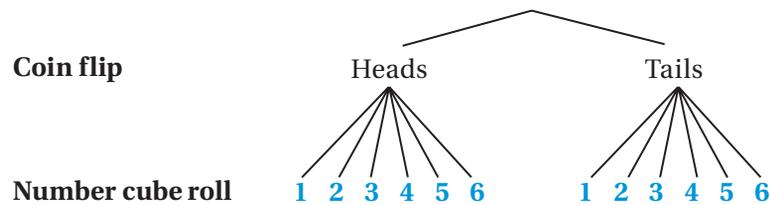
A possible result of an experiment is an **outcome**. For instance, when you roll a number cube there are 6 possible outcomes: a 1, 2, 3, 4, 5, or 6. An **event** is an outcome or a collection of outcomes, such as rolling an odd number. The set of all possible outcomes is called a **sample space**.

EXAMPLE 1 Find a sample space

You flip a coin and roll a number cube. How many possible outcomes are in the sample space? List the possible outcomes.

Solution

Use a tree diagram to find the outcomes in the sample space.



The sample space has 12 possible outcomes. They are listed below.

Heads, 1 Heads, 2 Heads, 3 Heads, 4 Heads, 5 Heads, 6
Tails, 1 Tails, 2 Tails, 3 Tails, 4 Tails, 5 Tails, 6

REVIEW TREE DIAGRAMS

For help with tree diagrams, see p. SR22.



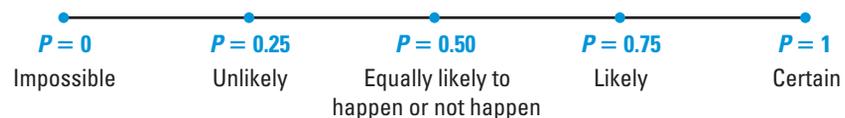
CC.9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*



GUIDED PRACTICE for Example 1

1. You flip 2 coins and roll a number cube. How many possible outcomes are in the sample space? List the possible outcomes.

PROBABILITY The **probability of an event** is a measure of the likelihood, or chance, that the event will occur. Probability is a number from 0 to 1 and can be expressed as a decimal, fraction, or percent.



THEORETICAL PROBABILITY The outcomes for a specified event are called *favorable outcomes*. When all outcomes are equally likely, the **theoretical probability** of the event can be found using the following:

$$\text{Theoretical probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The probability of event A is written as $P(A)$.

EXAMPLE 2 Find a theoretical probability

T-SHIRTS You and your friends designed T-shirts with silk screened emblems, and you are selling the T-shirts to raise money. The table below shows the number of T-shirts you have in each design. A student chooses a T-shirt at random. What is the probability that the student chooses a red T-shirt?

| | Gold emblem | Silver emblem |
|---------------|-------------|---------------|
| Green T-shirt | 10 | 8 |
| Red T-shirt | 6 | 6 |



Solution

You and your friends have a total of $10 + 6 + 8 + 6 = 30$ T-shirts. So, there are 30 possible outcomes. Of all the T-shirts, 12 T-shirts are red. There are 12 favorable outcomes.

$$\begin{aligned} P(\text{red T-shirt}) &= \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} \\ &= \frac{\text{Number of red T-shirts}}{\text{Total number of T-shirts}} \\ &= \frac{12}{30} \\ &= \frac{2}{5} \end{aligned}$$



GUIDED PRACTICE for Example 2

- T-SHIRTS** In Example 2, what is the probability that the student chooses a T-shirt with a gold emblem?
- You toss a coin and roll a number cube. What is the probability that the coin shows tails and the number cube shows 4?

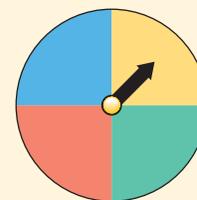
EXPERIMENTAL PROBABILITY An **experimental probability** is based on repeated *trials* of an experiment. The number of trials is the number of times the experiment is performed. Each trial in which a favorable outcome occurs is called a *success*.

$$\text{Experimental probability} = \frac{\text{Number of successes}}{\text{Number of trials}}$$



EXAMPLE 3 Standardized Test Practice

Each section of the spinner shown has the same area. The spinner was spun 20 times. The table shows the results. For which color is the experimental probability of stopping on the color the same as the theoretical probability?



| Spinner Results | | | |
|-----------------|-------|------|--------|
| Red | Green | Blue | Yellow |
| 5 | 9 | 3 | 3 |

- (A) Red (B) Green (C) Blue (D) Yellow

Solution

The theoretical probability of stopping on each of the four colors is $\frac{1}{4}$. Use the outcomes in the table to find the experimental probabilities.

$$P(\text{red}) = \frac{5}{20} = \frac{1}{4} \qquad P(\text{green}) = \frac{9}{20} \qquad P(\text{blue}) = \frac{3}{20} \qquad P(\text{yellow}) = \frac{3}{20}$$

▶ The correct answer is A. (A) (B) (C) (D)



ODDS The odds of an event compare the number of favorable and unfavorable outcomes when all outcomes are equally likely.

$$\text{Odds in favor} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}}$$

$$\text{Odds against} = \frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}}$$

EXAMPLE 4 Find the odds

READING

Odds are read as the ratio of one number to another. For instance, the odds $\frac{3}{1}$ are read as “three to one.” Odds are usually written as $a : b$.

SPINNER In Example 3, find the odds against stopping on green.

Solution

The 4 possible outcomes are all equally likely. Green is the 1 favorable outcome. The other 3 colors are unfavorable outcomes.

$$\text{Odds against green} = \frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}} = \frac{3}{1} \text{ or } 3 : 1.$$

GUIDED PRACTICE for Examples 3 and 4

- In Example 3, for which color is the experimental probability of stopping on the color greater than the theoretical probability?
- In Example 3, what are the odds in favor of stopping on blue?

11.1 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**
Exs. 3 and 21

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 14–16, 21, and 22

SKILL PRACTICE

- VOCABULARY** Copy and complete: A number that describes the likelihood of an event is the ? of the event.
- ★ **WRITING** Explain how the probability of an event differs from the odds in favor of the event when all outcomes are equally likely.

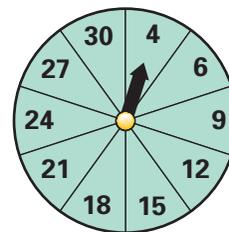
EXAMPLE 1
for Exs. 3–6

SAMPLE SPACE In Exercises 3–6, find the number of possible outcomes in the sample space. Then list the possible outcomes.

- A bag contains 4 red cards numbered 1–4, 4 white cards numbered 1–4, and 4 black cards numbered 1–4. You choose a card at random.
- You toss two coins.
- You roll a number cube and toss three coins.
- You roll two number cubes.

EXAMPLE 2
for Exs. 7–8

PROBABILITY AND ODDS In Exercises 7–13, refer to the spinner shown. The spinner is divided into sections with the same area.



- What is the probability that the spinner stops on a multiple of 3?
- ERROR ANALYSIS** Describe and correct the error in finding the probability of stopping on a multiple of 9.

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{2}{10} = \frac{1}{5} \quad \times$$

EXAMPLE 3
for Exs. 9–10

- You spin the spinner 30 times. It stops on 12 three times. What is the experimental probability of stopping on 12?
- You spin the spinner 10 times. It stops on an even number 6 times. What is the experimental probability of stopping on an even number?

EXAMPLE 4
for Exs. 11–14

- What are the odds in favor of stopping on a multiple of 4?
- What are the odds against stopping on a number less than 12?
- ERROR ANALYSIS** Describe and correct the error in finding the odds in favor of stopping on a multiple of 3.

$$\text{Odds in favor of a multiple of 3} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{9}{10} \text{ or } 9 : 10 \quad \times$$

- ★ **MULTIPLE CHOICE** The odds in favor of an event are 5 : 8. What are the odds against the event?

(A) 3 : 8 (B) 8 : 3 (C) 5 : 8 (D) 8 : 5

- ★ **WRITING** A manufacturer tests 1200 computers and finds that 1191 of them have no defects. Find the probability that a computer chosen at random has no defects. Predict the number of computers without defects in a shipment of 15,000 computers. Explain your reasoning.

16. ★ **MULTIPLE CHOICE** According to a meteorologist, there is a 40% chance that it will rain today. What are the odds in favor of rain?
- (A) 2 : 5 (B) 2 : 3 (C) 3 : 2 (D) 4 : 1
17. **DECISION MAKING** A driver wants to determine which of two possible routes to work he should choose. For 60 work days he recorded which route he took and whether or not he encountered heavy traffic. On 28 days he took route A, and on 7 of those days he encountered heavy traffic. On 32 days he took route B, and on 12 of those days he encountered heavy traffic. Which route would you suggest he choose? *Explain* your answer using experimental probabilities.
18. **CHALLENGE** You randomly draw a marble from a bag containing white, red, and blue marbles. The odds against drawing a white marble are 47 : 3.
- There are fewer than 100 marbles in the bag. How many marbles are in the bag? *Justify* your answer.
 - The probability of drawing a red marble is 0.5. What is the probability of drawing a blue marble? *Explain* how you found your answer.

PROBLEM SOLVING

EXAMPLE 2
for Exs. 19–20

19. **MUSIC PROGRAM** You have created a playlist of 7 songs on your MP3 player. You play these songs in a random shuffle, where each song has an equally likely chance of being played. What is the probability that the second song on the list will be played first?

20. **SURVEY** A survey asked a total of 600 students (100 male students and 100 female students who were 11, 13, and 15 years old) about their exercise habits. The table shows the numbers of students who said they exercise 2 hours or more each week.

| | 11 years | 13 years | 15 years |
|--------|----------|----------|----------|
| Female | 53 | 57 | 51 |
| Male | 65 | 68 | 67 |



- What is the probability that a randomly selected female student who participated in this survey exercises 2 hours or more each week?
- What is the probability that a randomly selected 15-year-old student who participated in this survey exercises 2 hours or more each week?
- What is the probability that a randomly selected student who participated in this survey exercises 2 hours or more each week?

EXAMPLES 2 and 4
for Ex. 21

21. ★ **SHORT RESPONSE** Suppose there are 15 girls and 12 boys in your homeroom. The teacher chooses one student representative at random. What is the probability that a boy is chosen? What are the odds in favor of choosing a boy? *Explain* how the probability and odds are related.

22. ★ **EXTENDED RESPONSE** The table shows the 2003 regular season field goal statistics for kicker Adam Vinatieri.

| | Point difference at end of game | | |
|-----------------------|---------------------------------|-------------|-------------|
| | 0–7 points | 8–14 points | ≥ 15 points |
| Field goals attempted | 20 | 11 | 3 |
| Field goals made | 16 | 7 | 2 |



- During the 2003 regular season, what was the probability that Adam Vinatieri would make an attempted field goal, regardless of the point difference?
- Find the probabilities that Vinatieri made an attempted field goal when the point difference at the end of the game was 0–7 points, 8–14 points, and at least 15 points.
- During what kinds of games was Adam Vinatieri most likely to make attempted field goals? *Justify* your answer.

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23. **CHALLENGE** The table shows the results of Congressional elections that involved incumbent candidates (representatives or senators who ran for re-election) during the period 1980–2000.

| | Incumbent representatives | | Incumbent senators | |
|----------------------------|---------------------------|------------|--------------------|------------|
| | Ran | Re-elected | Ran | Re-elected |
| Presidential election year | 2373 | 2235 | 163 | 130 |
| Midterm election year | 1984 | 1873 | 145 | 130 |

- Did a representative or a senator have a better chance of being re-elected? *Justify* your answer using the data in the table.
- Did a member of Congress have a better chance of being re-elected during a presidential election year than during a midterm election year? *Justify* your answer.