Name _____ Date _____

CHAPTER 11 Probability Distributions and Expected Value

Probability is defined as the number of favorable outcomes divided by the total number of outcomes. The number of favorable outcomes will always be less than or equal to the total number of outcomes, which makes the value of any probability less than or equal to 1.

KEY CONCEPT

Laws of Probability

There are two important laws to become familiar with when working with probabilities:

Law #1: The probability that a certain outcome will occur is a number greater than or equal to 0, and less than or equal to 1.

Law #2: The sum of the probabilities of all possible individual outcomes is equal to 1.

EXAMPLE 1

Demonstrate Law #1 and Law #2

A small bookshelf has 50 books: 20 fiction and 30 non-fiction. A book is randomly selected from the shelf. What is the probability that the book is non-fiction?

Solution:

The probability that the book selected is non-fiction is equal to 30 (number of favorable outcomes) divided by 50 (total number of outcomes), which is equal to 0.6. This demonstrates Law #1.

There are two outcomes in this experiment: selecting a fiction book or selecting a non-fiction book. The probability of selecting a fiction book is equal to 0.4 and the probability of selecting a non-fiction book is equal to 0.6. Notice that the sum of these probabilities is equal to 1, which demonstrates Law #2. ■

An **experiment** is a situation that involves a probability that leads to certain outcomes. Rolling a number cube is considered an experiment. The outcome of the experiment is the number that the cube lands on (1, 2, 3, 4, 5, or 6). The probability of each individual outcome is $\frac{1}{6}$, which is approximately equal to 0.17.

A **probability distribution** is a table, chart, or graph that shows the probability for each outcome.

EXAMPLE 2

Find a probability distribution with equal outcomes

Consider again the experiment of rolling a number cube. What is the probability distribution for this experiment?

Solution:

We want to make a probability distribution table for this situation. We know that there are a total of 6 outcomes, and that the probability of each outcome is $\frac{1}{6}$.

Below is a probability distribution table for this situation.

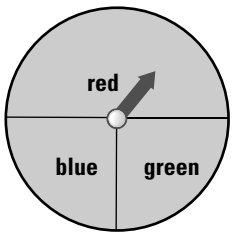
Outcome	Probability of Outcome
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$

Notice that the sum of all 6 probabilities is equal to 1 (Law #2) ■

In the last example, all of the outcomes had an equal chance of occurring. In the next example, the outcomes do not have an equal chance of happening.

EXAMPLE 3 Find a probability distribution with unequal outcomes

Consider another experiment using the spinner shown below:



What is the probability distribution for this experiment?

Solution:

Spinning the spinner is an experiment with the following outcomes: red, blue, and green. After spinning, the spinner has an equal chance of landing on either the blue or the green sections. However, the spinner has twice the likelihood of landing in the red section because of its relative size (50% of the spinner outcome space).

Probability Distributions and Expected Value *continued*

Below is a probability distribution for this situation.

Section	Probability of Outcome
blue	$\frac{1}{4}$
green	$\frac{1}{4}$
red	$\frac{1}{2}$

Again, notice that the sum of the probabilities is equal to 1. ■

Law #2 is useful when finding missing probabilities in a probability distribution.

EXAMPLE 4 Find missing probabilities in a distribution

A deck of cards used for a game contains blue, yellow and orange cards. The probability distribution for randomly selecting each of the cards from the deck is shown below:

Card Color	Probability of Randomly Selecting the Card
blue	0.2
yellow	?
orange	0.3

What is the probability of randomly selecting a yellow card from the deck?

Solution:

Law #2 ensures that the sum of the probabilities is equal to 1. So, the probability of randomly selecting a yellow card is equal to 0.5. ■

The **expected value** of an experiment can be calculated when the outcomes are numerical. Numerical outcomes, such as dollar amounts or points, vary depending on the outcome, but the expected value gives us an "average" that weighs all of the outcomes.

Expected value is calculated by multiplying each outcome by its probability, then adding all of the products.

CHAPTER 11 Probability Distributions and Expected Value *continued*

EXAMPLE 5 Calculate expected value

A bag contains 10 quarters, 14 dimes, and 16 nickels. We want to calculate the expected value if a coin is randomly chosen from the bag.

The chart below shows the outcomes, the probabilities of each outcome, and the product of the outcome and its probability.

Outcome	Probability of Outcome	Product
Quarter ($0.25)	$\frac{10}{40} = \frac{1}{4}$	$\frac{1}{4} \cdot (\$0.25)$
Dime ($0.10)	$\frac{14}{40} = \frac{7}{20}$	$\frac{7}{20} \cdot (\$0.10)$
Nickel ($0.05)	$\frac{16}{40} = \frac{2}{5}$	$\frac{2}{5} \cdot (\$0.05)$

The expected value is calculated by adding up the three products:

$$\frac{1}{4} \cdot (\$0.25) + \frac{7}{20} \cdot (\$0.10) + \frac{2}{5} \cdot (\$0.05) \approx \$0.1175$$

So, the expected value is approximately equal to *12 cents*. ■

It is important to remember that the expected value is unlikely to be equal to the actual outcome. Suppose a quarter was blindly chosen in the last example. Here, the actual value (25 cents) differs from the expected value by 13 cents. However, if 5 coins were randomly selected from the bag, their average value is likely to be close to the expected value.

Practice

A bag contains 20 red, 40 blue, 15 white, and 5 green marbles. A marble is selected at random from the bag. Find the probability of the outcome.

1. A blue marble is selected.
2. A green marble is selected.
3. A white marble is not selected.
4. A red, blue, white, or green marble is selected.
5. An orange marble is selected.

Make a table that shows the distribution of probabilities for each possible outcome of the experiment.

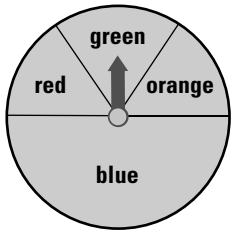
6. A coin is tossed.
7. Two coins are tossed.

Name _____ Date _____

CHAPTER 11

Probability Distributions and Expected Value *continued*

8. The spinner shown below is spun.



9. A student is randomly chosen from a class that has 6 ninth graders, 8 tenth graders, and 5 eleventh graders.

Problem Solving

10. An experiment has four possible outcomes: A, B, C, and D. The probability that outcome A will occur is 0.24 and the probability that outcome D will occur is 0.46. If the probability of outcomes B and C are equal, what is the probability that outcome C will occur?

11. Valerie can be assigned one of three jobs by a temporary-worker agency. The table below shows the probability that she will get each job, along with the hourly wage for each job.

Job	Probability of getting the job each day	Hourly Wage
Filing	0.6	$6.60
Receptionist	0.1	$8.40
Typing	0.3	$7.80

What is the expected value for Valerie's hourly wage?

12. A card game involves a deck containing 20 red, 10 blue, 18 green, and 6 yellow cards. The table below shows the number of points that a player will get for randomly selecting each card color.

Card Color	Number of Points
Red	40
Blue	25
Green	35
Yellow	60

What is the expected value for the number of points earned from a card that is chosen at random?