

LESSON
11.2**Study Guide***For use with the lesson "Find Probabilities Using Permutations"***GOAL** Use the formula for the number of permutations.**Vocabulary**

A **permutation** is an arrangement of objects in which order is important.

For any positive integer n , the product of the integers from 1 to n is called **n factorial** and is written as $n!$.

EXAMPLE 1 **Count permutations****Consider the number of permutations of the letters in the word APRIL.**

- In how many ways can you arrange all of the letters?
- In how many ways can you arrange 3 of the letters?

Solution

- Use the counting principle to find the number of permutations of the letters in the word APRIL.

$$\begin{array}{rcl}
 \begin{array}{|c|} \hline \text{Number} \\ \text{of} \\ \text{Permutations} \\ \hline \end{array} & = & \begin{array}{|c|} \hline \text{Choices} \\ \text{for 1st} \\ \text{letter} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \text{Choices} \\ \text{for 2nd} \\ \text{letter} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \text{Choices} \\ \text{for 3rd} \\ \text{letter} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \text{Choices} \\ \text{for 4th} \\ \text{letter} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \text{Choices} \\ \text{for 5th} \\ \text{letter} \\ \hline \end{array} \\
 & = & 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\
 & = & 120
 \end{array}$$

There are 120 ways you can arrange all of the letters in the word APRIL.

- When arranging 3 letters of the word APRIL, you have 5 choices for the first letter, 4 for the second letter, and 3 for the third letter.

$$\begin{array}{rcl}
 \begin{array}{|c|} \hline \text{Number} \\ \text{of} \\ \text{Permutations} \\ \hline \end{array} & = & \begin{array}{|c|} \hline \text{Choices} \\ \text{for 1st} \\ \text{letter} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \text{Choices} \\ \text{for 2nd} \\ \text{letter} \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \text{Choices} \\ \text{for 3rd} \\ \text{letter} \\ \hline \end{array} \\
 & = & 5 \cdot 4 \cdot 3 \\
 & = & 60
 \end{array}$$

There are 60 ways you can arrange 3 of the letters in the word APRIL.

Exercises for Example 1**Count the permutations.**

- In how many ways can you arrange the letters in the word FLOWER?
- In how many ways can you arrange 4 of the letters in the word PANTHER?
- In how many ways can you arrange 2 of the letters in the word COMPUTER?

LESSON
11.2**Study Guide** *continued*
*For use with the lesson "Find Probabilities Using Permutations"***EXAMPLE 2** Use permutation formula

Packing You have 11 pairs of shorts and plan to pack 5 of them for a vacation. In how many ways can you choose the shorts you pack for your vacation?

Solution

To find the number of permutations of 5 pairs of shorts chosen from 11, find ${}_{11}P_5$.

$$\begin{aligned}
 {}_{11}P_5 &= \frac{11!}{(11-5)!} && \text{Permutation formula} \\
 &= \frac{11!}{6!} && \text{Subtract.} \\
 &= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}} && \text{Expand factorials. Divide out common factorial, } 6!. \\
 &= 55,440 && \text{Multiply.}
 \end{aligned}$$

There are 55,440 ways to arrange 5 pairs of shorts out of 11.

EXAMPLE 3 Find a probability using permutations

Softball There are 10 players on a softball team. Each game the batting order is randomly fixed. Find the probability that you are chosen to bat first, and your best friend is chosen to bat second.

Solution

STEP 1 Write the number of possible outcomes as the number of permutations of the 10 players on the team. This is ${}_{10}P_{10} = 10!$.

STEP 2 Write the number of favorable outcomes as the number of permutations of the players given that you are the first batter, and your best friend is the second. This is ${}_8P_8 = 8!$.

STEP 3 Calculate the probability.

$$\begin{aligned}
 P\left(\begin{array}{l} \text{You are first batter,} \\ \text{Best friend is second batter} \end{array}\right) &= \frac{8!}{10!} && \text{Form a ratio of favorable to possible outcomes.} \\
 &= \frac{\cancel{8!}}{10 \cdot 9 \cdot \cancel{8!}} && \text{Expand factorial.} \\
 &= \frac{1}{90} && \text{Divide out common factor, } 8!. \\
 &&& \text{Simplify.}
 \end{aligned}$$

Exercises for Examples 2 and 3

- What if?** In Example 2, suppose you have 9 pairs of shorts. You pack 4 of them for your vacation. In how many ways can you choose the shorts you pack for vacation?
- What if?** In Example 3, suppose there are 15 players on the team and that everyone gets to bat. Find the probability that you are the first batter and that your best friend is the second batter.