11.2 Find Probabilities Using Permutations

Before You used the counting principle.

Now You will use the formula for the number of permutations.

Why? So you can find the number of possible arrangements, as in Ex. 38.

Key Vocabulary
- **permutation**
- ***n* factorial**

A **permutation** is an arrangement of objects in which order is important. For instance, the 6 possible permutations of the letters A, B, and C are shown.

ABC ACB BAC BCA CAB CBA

COMMON CORE

CC.9-12.S.CP.9(+) Use permutations and combinations to compute probabilities of compound events and solve problems.*

EXAMPLE 1 Count permutations

Consider the number of permutations of the letters in the word JULY.

a. In how many ways can you arrange all of the letters?

b. In how many ways can you arrange 2 of the letters?

Solution

a. Use the counting principle to find the number of permutations of the letters in the word JULY.

$$\begin{array}{l}\text{Number of permutations} = \text{Choices for 1st letter} \cdot \text{Choices for 2nd letter} \cdot \text{Choices for 3rd letter} \cdot \text{Choices for 4th letter} \\ = 4 \cdot 3 \cdot 2 \cdot 1 \\ = 24\end{array}$$

▶ There are 24 ways you can arrange all of the letters in the word JULY.

b. When arranging 2 letters of the word JULY, you have 4 choices for the first letter and 3 choices for the second letter.

$$\begin{array}{l}\text{Number of permutations} = \text{Choices for 1st letter} \cdot \text{Choices for 2nd letter} \\ = 4 \cdot 3 \\ = 12\end{array}$$

▶ There are 12 ways you can arrange 2 of the letters in the word JULY.

REVIEW COUNTING PRINCIPLE
For help with using the counting principle, see p. SR22.

GUIDED PRACTICE for Example 1

1. In how many ways can you arrange the letters in the word MOUSE?

2. In how many ways can you arrange 3 of the letters in the word ORANGE?

FACTORIAL In Example 1, you evaluated the expression $4 \cdot 3 \cdot 2 \cdot 1$. This expression can be written as 4! and is read "4 *factorial*." For any positive integer n, the product of the integers from 1 to n is called ***n* factorial** and is written as $n!$. The value of 0! is defined to be 1.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \text{ and } 0! = 1$$

In Example 1, you also found the permutations of four objects taken two at a time. You can find the number of permutations using the formulas below.

KEY CONCEPT *For Your Notebook*

Permutations

Formulas	Examples
The number of permutations of n objects is given by: $${}_nP_n = n!$$	The number of permutations of 4 objects is: $${}_4P_4 = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$
The number of permutations of n objects taken r at a time, where $r \leq n$, is given by: $${}_nP_r = \frac{n!}{(n-r)!}$$	The number of permutations of 4 objects taken 2 at a time is: $${}_4P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}} = 12$$

EXAMPLE 2 Use a permutations formula

CD RECORDING Your band has written 12 songs and plans to record 9 of them for a CD. In how many ways can you arrange the songs on the CD?

Solution

To find the number of permutations of 9 songs chosen from 12, find ${}_{12}P_9$.

$${}_{12}P_9 = \frac{12!}{(12-9)!}$$ **Permutations formula**

$$= \frac{12!}{3!}$$ **Subtract.**

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}}$$ **Expand factorials. Divide out common factor, 3!.**

$$= 79{,}833{,}600$$ **Multiply.**

▶ There are 79,833,600 ways to arrange 9 songs out of 12.

DIVIDE COMMON FACTORS
When you divide out common factors, remember that 3! is a factor of 12!.

GUIDED PRACTICE for Example 2

3. WHAT IF? In Example 2, suppose your band has written 15 songs. You will record 9 of them for a CD. In how many ways can you arrange the songs on the CD?

EXAMPLE 3 Find a probability using permutations

PARADE For a town parade, you will ride on a float with your soccer team. There are 12 floats in the parade, and their order is chosen at random. Find the probability that your float is first and the float with the school chorus is second.

Solution

STEP 1 **Write** the number of possible outcomes as the number of permutations of the 12 floats in the parade. This is ${}_{12}P_{12} = 12!$.

STEP 2 **Write** the number of favorable outcomes as the number of permutations of the other floats, given that the soccer team is first and the chorus is second. This is ${}_{10}P_{10} = 10!$.

STEP 3 **Calculate** the probability.

$$P\left(\begin{array}{c}\text{soccer team is first}\\ \text{chorus is second}\end{array}\right) = \frac{10!}{12!}$$ Form a ratio of favorable to possible outcomes.

$$= \frac{\cancel{10!}}{12 \cdot 11 \cdot \cancel{10!}}$$ Expand factorials. Divide out common factor, 10!.

$$= \frac{1}{132}$$ Simplify.

GUIDED PRACTICE for Example 3

4. **WHAT IF?** In Example 3, suppose there are 14 floats in the parade. Find the probability that the soccer team is first and the chorus is second.

11.2 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 21 and 35
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 11, 30, 33, and 35
- = **MULTIPLE REPRESENTATIONS** Ex. 34

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: An arrangement of objects in which order is important is called a(n) __?__.

2. ★ **WRITING** *Explain* what the notation ${}_9P_2$ means. What is the value of this expression?

EXAMPLES 1 and 2 for Exs. 3–11

COUNTING PERMUTATIONS **Find the number of ways you can arrange (a) all of the letters in the given word and (b) 2 of the letters in the word.**

3. AT
4. TRY
5. GAME
6. CAT
7. WATER
8. ROCK
9. APRIL
10. FAMILY

11. ★ **OPEN-ENDED** *Describe* a real-world situation where the number of possibilities is given by ${}_5P_2$.

EXAMPLE 2 for Exs. 12–30

FACTORIALS AND PERMUTATIONS **Evaluate the expression.**

12. $1!$ **13.** $3!$ **14.** $0!$ **15.** $5!$

16. $8!$ **17.** $10!$ **18.** $12!$ **19.** $13!$

20. ${}_5P_2$ **21.** ${}_7P_3$ **22.** ${}_9P_1$ **23.** ${}_6P_5$

24. ${}_8P_8$ **25.** ${}_{12}P_0$ **26.** ${}_{30}P_2$ **27.** ${}_{25}P_5$

ERROR ANALYSIS ***Describe*** **and correct the error in evaluating the expression.**

28. ${}_{11}P_7 = \frac{11!}{(11-7)} = \frac{11!}{4} = 9{,}979{,}200$ ✗

29. ${}_5P_3 = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3!}{3!} = 20$ ✗

30. ★ **MULTIPLE CHOICE** The judges in an art contest award prizes for first, second, and third place out of 11 entries. Which expression gives the number of ways the judges can award first, second, and third place?

(A) $\frac{3!}{11!}$ (B) $\frac{8!}{11!}$ (C) $\frac{11!}{8!}$ (D) $\frac{11!}{3!}$

31. **CHALLENGE** Consider a set of 4 objects and a set of n objects.

a. Are there more permutations of all 4 of the objects or of 3 of the 4 objects? *Justify* your answer using an organized list.

b. In general, are there more permutations of n objects taken n at a time or of n objects taken $n - 1$ at a time? *Justify* your answer using the formula for the number of permutations.

PROBLEM SOLVING

EXAMPLE 2 for Exs. 32–33

32. **MOVIES** Six friends go to a movie theater. In how many different ways can they sit together in a row of 6 empty seats?

33. ★ **MULTIPLE CHOICE** You plan to visit 4 stores during a shopping trip. In how many orders can you visit these stores?

(A) 4 (B) 16 (C) 24 (D) 256

EXAMPLE 3 for Exs. 34–38

34. **MULTIPLE REPRESENTATIONS** You and your friend are two of 4 servers working a shift in a restaurant. The host assigns tables of new diners to the servers in a particular order. This order remains the same, so that all servers are likely to wait on the same number of tables by the end of the shift.

a. **Making a List** List all the possible orders in which the host can assign tables to the servers.

b. **Using a Formula** Use the formula for permutations to find the number of ways in which the host can assign tables to the servers.

c. **Describe in Words** What is the likelihood that you and your friend are assigned the first 2 tables? *Explain* your answer using probability.

35. ★ **SHORT RESPONSE** Every student in your history class is required to present a project in front of the class. Each day, 4 students make their presentations in an order chosen at random by the teacher. You make your presentation on the first day.

a. What is the probability that you are chosen to be the first or second presenter on the first day? *Explain* how you found your answer.

b. What is the probability that you are chosen to be the second or third presenter on the first day? *Compare* your answer with that in part (a).

36. **HISTORY EXAM** On an exam, you are asked to list 5 historical events in the order in which they occurred. You guess the order of the events at random. What is the probability that you choose the correct order?

37. **SPIRIT** You make 6 posters to hold up at a basketball game. Each poster has a letter of the word TIGERS. You and 5 friends sit next to each other in a row. The posters are distributed at random. What is the probability that TIGERS is spelled correctly when you hold up the posters?

38. **BAND COMPETITION** Seven marching bands will perform at a competition. The order of the performances is determined at random. What is the probability that your school band will perform first, followed by the band from the one other high school in your town?

39. **CHALLENGE** You are one of 10 students performing in a school talent show. The order of the performances is determined at random. The first five performers go on stage before the intermission, while the remaining five performers go on stage after the intermission.

a. What is the probability that you are the last performer before the intermission and your rival performs immediately before you?

b. What is the probability that you are *not* the first performer?

Quiz

1. **MARBLES** A bag contains 16 red marbles and 8 white marbles. You select a marble at random.

a. What is the probability that you select a red marble?

b. What are the odds in favor of selecting a red marble?

2. **PASSWORD** The password for an e-mail account is the word FISH followed by a 3-digit number. The 3-digit number contains the digits 1, 2, and 3. How many different passwords are possible?