Name ______________________________ Date ______________

CHAPTER 11 Permutations

A permutation is an arrangement of objects in a certain order.

EXAMPLE 1 List the permutations: set of three

List all of the possible permutations of the letters X, Y, and Z.

Solution:

$XYZ \quad XZY \quad YXZ \quad YZX \quad ZXY \quad ZYX$

Notice that there are exactly 6 permutations. There are a total of 3 letters to choose from for the first letter, 2 letters to choose from for the second letter, and 1 letter to choose from for the third letter. The counting principle (page 931 in the textbook) allows us to multiply these numbers to obtain our answer of $3 \cdot 2 \cdot 1 = 6$ permutations. ■

We can now see that a formula for finding the number of permutations of n objects is obtained by multiplying n by $(n - 1) \cdot (n - 2) \cdot \ldots$ until we reach the number 1. The product of the numbers 1 to n is called ***n* factorial**, and is written $n!$.

EXAMPLE 2 Find the number of permutations: set of four

In how many ways can you arrange the letters X, Y, Z, and W?

Solution:

Here, there are $n = 4$ elements in the set. There are a total of $n! = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ permutations of the letters X, Y, Z, and W. ■

EXAMPLE 3 List the permutations: subset of a set of four

List the permutations of the letters X, Y, Z, and W using only 2 of these letters at a time.

Solution:

$XY \quad XZ \quad XW \quad YX \quad YZ \quad YW \quad ZX \quad ZY \quad ZW \quad WX \quad WY \quad WZ$

Notice that there exactly 12 such permutations. There are a total of 4 letters to choose from for the first letter and 3 letters to choose from for the second letter. So, the answer is obtained by multiplying $4 \cdot 3$ to obtain the answer of 12. ■

From the last example, notice that $12 = 4 \cdot 3 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{4!}{2!} = \frac{4!}{(4 - 2)!}$. This observation leads us to the following formula:

KEY CONCEPT

Permutations

Consider a set with n objects, and a subset of this set with r objects. The number of permutations of n objects taken r at a time is given by:

$$\frac{n!}{(n - r)!}$$

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CHAPTER 11

Permutations *continued*

EXAMPLE 4 **Use the permutation formula**

Find the number of permutations of the word FOREST taken 4 letters at a time.

Solution:

Here, $n = 6$ and $r = 4$. Substituting into the formula, we have:

$$\frac{n!}{(n-r)!} = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 6 \cdot 5 \cdot 4 \cdot 3 = 360$$

So, there are 360 permutations of the word FOREST, taken 4 letters at a time. ■

Practice

List all of the possible permutations for the situation.

1. The letters of the word RED.
2. Two of the letters of the word BOY.
3. Two of the letters of the word READ.

Find the number of permutations of a) all of the letters in the given word, b) 3 of the letters of the word, and c) 2 of the letters of the word.

4. SPACE
5. NUMBER
6. FIND
7. FIGURES

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