

# 11.3 Find Probabilities Using Combinations



**Before**

You used permutations to count possibilities.

**Now**

You will use combinations to count possibilities.

**Why?**

So you can find the probability of an event, as in Example 3.

## Key Vocabulary

### • combination

A **combination** is a selection of objects in which order is *not* important. For instance, in a drawing for 3 identical prizes, you would use combinations, because the order of the winners would not matter. If the prizes were different, you would use permutations, because the order would matter.

COMMON CORE

CC.9-12.S.CP.9(+) Use permutations and combinations to compute probabilities of compound events and solve problems.\*

## EXAMPLE 1 Count combinations

Count the combinations of two letters from the list A, B, C, D.

### Solution

List all of the permutations of two letters in the list A, B, C, D. Because order is not important in a combination, cross out any duplicate pairs.

AB	AC	AD	<del>BA</del>	BC	<del>BD</del>	← BD and DB are the same pair.
<del>CA</del>	<del>CB</del>	CD	<del>DA</del>	<del>DB</del>	<del>DC</del>	

▶ There are 6 possible combinations of 2 letters from the list A, B, C, D.

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## GUIDED PRACTICE for Example 1

- Count the combinations of 3 letters from the list A, B, C, D, E.

**COMBINATIONS** In Example 1, you found the number of combinations of objects by making an organized list. You can also find the number of combinations using the following formula.

## KEY CONCEPT

## For Your Notebook

### Combinations

#### Formula

The number of combinations of  $n$  objects taken  $r$  at a time, where  $r \leq n$ , is given by:

$${}_n C_r = \frac{n!}{(n-r)! \cdot r!}$$

#### Example

The number of combinations of 4 objects taken 2 at a time is:

$${}_4 C_2 = \frac{4!}{(4-2)! \cdot 2!} = \frac{4 \cdot 3 \cdot \cancel{2} \cdot 1}{2 \cdot 1 \cdot (2 \cdot 1)} = 6$$

## EXAMPLE 2 Use the combinations formula

**LUNCH MENU** You order a sandwich at a restaurant. You can choose 2 side dishes from a list of 8. How many combinations of side dishes are possible?

### Solution

The order in which you choose the side dishes is not important. So, to find the number of combinations of 8 side dishes taken 2 at a time, find  ${}_8C_2$ .

$$\begin{aligned} {}_8C_2 &= \frac{8!}{(8-2)! \cdot 2!} && \text{Combinations formula} \\ &= \frac{8!}{6! \cdot 2!} && \text{Subtract.} \\ &= \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot (2 \cdot 1)} && \text{Expand factorials.} \\ & && \text{Divide out common factor, 6!.} \\ &= 28 && \text{Simplify.} \end{aligned}$$

► There are 28 different combinations of side dishes you can order.

## EXAMPLE 3 Find a probability using combinations

**PHOTOGRAPHY** A yearbook editor has selected 14 photos, including one of you and one of your friend, to use in a collage for the yearbook. The photos are placed at random. There is room for 2 photos at the top of the page. What is the probability that your photo and your friend's photo are the two placed at the top of the page?



### Solution

**STEP 1** Write the number of possible outcomes as the number of combinations of 14 photos taken 2 at a time, or  ${}_{14}C_2$ , because the order in which the photos are chosen is not important.

$${}_{14}C_2 = \frac{14!}{(14-2)! \cdot 2!} = \frac{14!}{12! \cdot 2!} = \frac{14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot (2 \cdot 1)} = 91$$

**STEP 2** Find the number of favorable outcomes. Only one of the possible combinations includes your photo and your friend's photo.

**STEP 3** Calculate the probability.

$$P(\text{your photo and your friend's photos are chosen}) = \frac{1}{91}$$



### GUIDED PRACTICE for Examples 2 and 3

- WHAT IF?** In Example 2, suppose you can choose 3 side dishes out of the list of 8 side dishes. How many combinations are possible?
- WHAT IF?** In Example 3, suppose there are 20 photos in the collage. Find the probability that your photo and your friend's photo are the two placed at the top of the page.

# 11.3 EXERCISES

## HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**  
Exs. 7 and 25

★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 14–20, and 25

### SKILL PRACTICE

**EXAMPLE 1**  
for Exs. 3, 4

**EXAMPLE 2**  
for Exs. 5–15

- VOCABULARY** Copy and complete: A(n)   ? is a selection of objects in which order is not important.
- ★ **WRITING** Explain how a combination differs from a permutation.
- COMBINATIONS** How many combinations of 3 letters from the list A, B, C, D, E, F are possible?
- ERROR ANALYSIS** Describe and correct the error in listing all of the possible combinations of 2 letters from the list A, B, C.
 

AB	BA	CA	✗
AC	BC	CB	
- ERROR ANALYSIS** Describe and correct the error in evaluating  ${}_{9}C_4$ .
 

${}_{9}C_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 3024$	✗
${}_{9}C_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 3024$	

#### COMBINATIONS Evaluate the expression.

- |                  |                  |                  |                  |
|------------------|------------------|------------------|------------------|
| 6. ${}_{5}C_1$   | 7. ${}_{8}C_5$   | 8. ${}_{9}C_9$   | 9. ${}_{8}C_6$   |
| 10. ${}_{12}C_3$ | 11. ${}_{11}C_4$ | 12. ${}_{15}C_8$ | 13. ${}_{20}C_5$ |
14. ★ **MULTIPLE CHOICE** What is the value of  ${}_{10}C_6$ ?
- (A) 7                       (B) 60                       (C) 210                       (D) 151,200
15. ★ **MULTIPLE CHOICE** You have the first season of your favorite television show on a set of DVDs. The set contains 13 episodes. You have time to watch 3 episodes. How many combinations of 3 episodes can you watch?
- (A) 286                       (B) 572                       (C) 1716                       (D) 589,680

★ **SHORT RESPONSE** In Exercises 16–19, tell whether the question can be answered using *combinations* or *permutations*. Explain your choice, then answer the question.

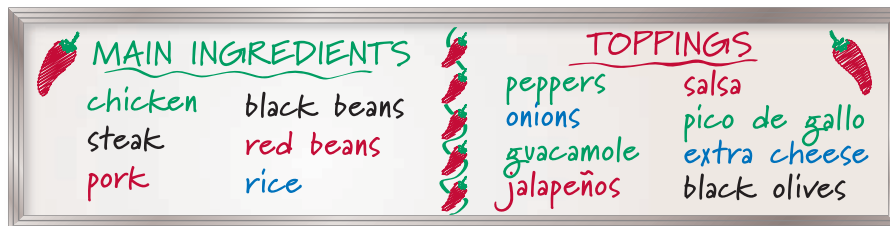
- Four students from your class of 120 students will be selected to organize a fundraiser. How many groups of 4 students are possible?
- Ten students are auditioning for 3 different roles in a play. In how many ways can the 3 roles be filled?
- To complete an exam, you must answer 8 questions from a list of 10 questions. In how many ways can you complete the exam?
- In how many ways can 5 people sit in a car that holds 5 passengers?
- ★ **WRITING** Which is greater,  ${}_6P_r$  or  ${}_6C_r$ ? Justify your answer.
- REASONING** Write an equation that relates  ${}_nP_r$  and  ${}_nC_r$ . Explain your reasoning.
- CHALLENGE** Prove that  ${}_nC_r = {}_nC_{n-r}$ . Explain why this makes sense.

## PROBLEM SOLVING

### EXAMPLE 2

for Ex. 23

23. **RESTAURANT** You are ordering a burrito with 2 main ingredients and 3 toppings. The menu below shows the possible choices. How many different burritos are possible?



### EXAMPLE 3

for Exs. 24–26

24. **WORK SCHEDULE** You work 3 evenings each week at a bookstore. Your supervisor assigns you 3 evenings at random from the 7 possibilities. What is the probability that your schedule this week includes working on Friday?

25. **★ SHORT RESPONSE** On a television game show, 9 members of the studio audience are randomly selected to be eligible contestants.
- Six of the 9 eligible contestants are randomly chosen to play a game on the stage. How many combinations of 6 players from the group of eligible contestants are possible?
  - You and your two friends are part of the group of 9 eligible contestants. What is the probability that all three of you are chosen to play the game on stage? *Explain* how you found your answer.
26. **REPRESENTATIVES** Your teacher chooses 2 students at random to represent your homeroom. The homeroom has a total of 30 students, including your best friend. What is the probability that you and your best friend are chosen? What is the probability that you are chosen first and your best friend is chosen second? Which event is more likely to occur?
27. **CHALLENGE** There are 30 students in your class. Your science teacher will choose 5 students at random to complete a group project. Find the probability that you and your 2 best friends in the science class are chosen to work in the group. *Explain* how you found your answer.





Use appropriate tools strategically.

## Find Permutations and Combinations

**QUESTION** How can you find combinations and permutations using a graphing calculator?

**EXAMPLE 1** Find the number of combinations

**STARTERS** There are 15 players on your softball team, but only 9 of them can be the starting players in one game. How many combinations of starting players are possible?

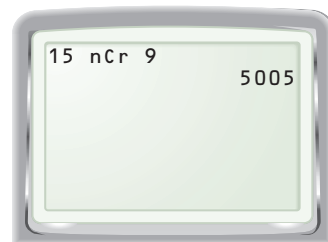
### Solution

You are finding  ${}_n C_r$  where  $n = 15$  and  $r = 9$ . Enter 15 for  $n$ .

Press **MATH**. Go to the PRB menu and select  ${}_n C_r$ .

Then enter 9 for  $r$ .

▶ There are 5005 possible combinations of starting players.



**EXAMPLE 2** Find the number of permutations

**BATTING ORDER** Before each softball game, your coach announces the batting order of the 9 starting players. This is the order in which the starting players will bat. How many batting orders can be formed using 9 players on your team of 15 players?

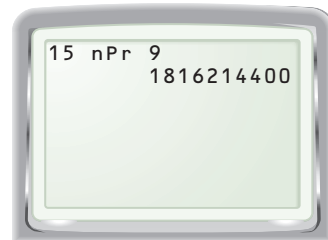
### Solution

You are finding  ${}_n P_r$  where  $n = 15$  and  $r = 9$ . Enter 15 for  $n$ .

Press **MATH**. Go to the PRB menu and select  ${}_n P_r$ .

Then enter 9 for  $r$ .

▶ There are 1,816,214,400 possible batting orders.



### PRACTICE

Evaluate the expression.

1.  ${}_7 C_4$

2.  ${}_6 C_6$

3.  ${}_{10} C_3$

4.  ${}_{16} C_8$

5.  ${}_9 P_5$

6.  ${}_7 P_6$

7.  ${}_{11} P_8$

8.  ${}_{12} P_5$

9. **GROUP PROJECT** Your teacher selects 3 students from a class of 28 students to work on a project in a group. Within the group, one member must be the writer, one must be the researcher, and one must be the presenter.

- How many different groups of 3 can your teacher select?
- After the group is formed, in how many ways can the roles in the group be assigned?

# MIXED REVIEW of Problem Solving



Make sense of problems and persevere in solving them.



The Puerto Rican crested toad is at risk of extinction.

1. **MULTI-STEP PROBLEM** There are 5743 known amphibian species in the world. Of these, 1856 species are judged to be at risk of extinction, and another 113 species may already be extinct.

- Find the probability that an amphibian species chosen at random is at risk of extinction.
- Find the probability that an amphibian species chosen at random may already be extinct.

4. **SHORT RESPONSE** A meteorologist reports that there is a 15% chance of snow tomorrow. What are the odds in favor of snow tomorrow? *Explain* how you found your answer.

5. **OPEN-ENDED** Describe a real-world situation in which the number of possible arrangements is given by  ${}_{10}P_2$ .

6. **EXTENDED RESPONSE** A survey asked a total of 400 students, 100 male students and 100 female students who were 13 and 15 years old, about their eating habits. The table shows the numbers of students who said that they eat fruit every day.

	13 years old	15 years old
Male	60	53
Female	61	58

2. **MULTI-STEP PROBLEM** You are ordering an omelet with two ingredients. You can choose from the following list: cheese, mushrooms, onions, tomatoes, peppers, sausage, ham, and steak.

- Make an organized list of all the possible omelets that you can order.
- Use a permutation or combination formula to find the number of possible omelets.

3. **MULTI-STEP PROBLEM** In NCAA women's basketball tournaments from 1982 to 2003, teams seeded, or ranked, number one have won 283 games and lost 71 games in the tournament. Suppose a team is chosen at random from all those that have been seeded number one.

- What is the probability that the team won a game in the tournament?
- What are the odds in favor of the team's having won a game in the tournament?

- Find the probability that a female student, chosen at random from the students surveyed, eats fruit every day.
- Find the probability that a 15-year-old student, chosen at random from the students surveyed, eats fruit every day.
- You select a student at random from the students surveyed. Find the odds against the student's eating fruit every day. *Explain* your reasoning.

7. **GRIDDED ANSWER** A music club gives you 6 free CDs for joining. You would like to own 11 of the free CDs that are offered. How many combinations of 6 CDs from the 11 CDs can you choose?



## Find Probabilities Using Venn Diagrams



Construct viable arguments and critique the reasoning of others.

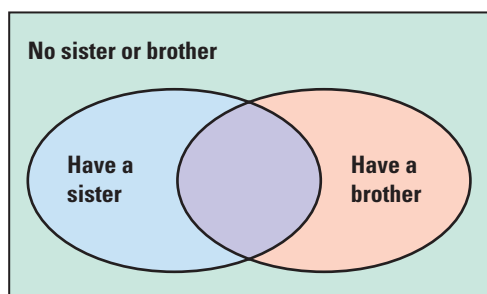
**QUESTION** How can you use a Venn diagram to find probabilities involving two events?

You have learned how to compute the probability of one event. In some situations, however, you might be interested in the probability that two events will occur simultaneously. You also might be interested in the probability that at least one of two events will occur. This activity demonstrates how a Venn diagram is useful for computing such probabilities.

**EXPLORE** Use a Venn diagram to collect data

**STEP 1** Complete a Venn diagram

Copy the Venn diagram shown below. Ask the members of your class if they have a sister, have a brother, have both, or have neither. Write their names in the appropriate part of the Venn diagram.



**STEP 2** Complete a table

Copy and complete the frequency table. When determining the frequency for a category, be sure to include all the students who are in the category. Note that a student can belong to more than one category.

Category	Number of students
Have a sister	?
Have a brother	?
Have both a sister and brother	?
Do not have a sister or brother	?

**DRAW CONCLUSIONS** Use your data to complete these exercises

- A student from your class is selected at random. Find the probability of each event. *Explain* how you found your answers.
  - The student has a sister.
  - The student has a brother.
  - The student has a sister and a brother.
  - The student does not have a sister or a brother.
- Find the probability that a randomly selected student from your class has either a sister or a brother. *Explain* how you found your answer.
- How could you calculate the answer to Exercise 2 using your answers from Exercise 1?