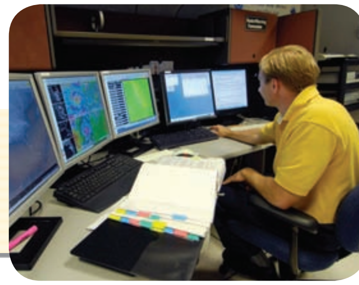
11.4 Find Probabilities of Disjoint and Overlapping Events



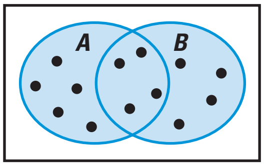
Before	You found probabilities of simple events.
Now	You will find probabilities of compound events.
Why?	So you can solve problems about meteorology, as in Ex. 44.

Key Vocabulary
- **compound event**
- **overlapping events**
- **disjoint or mutually exclusive events**

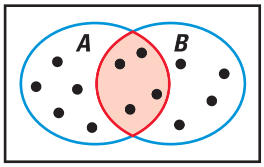
COMMON CORE

CC.9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*

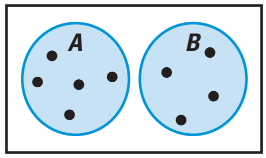
When you consider all the outcomes for either of two events A and B, you form the *union* of A and B. When you consider only the outcomes shared by both A and B, you form the *intersection* of A and B. The union or intersection of two events is called a **compound event**.



Union of A and B



Intersection of A and B



Intersection of A and B is empty.

To find $P(A \text{ or } B)$ you must consider what outcomes, if any, are in the intersection of A and B. Two events are **overlapping** if they have one or more outcomes in common, as shown in the first diagram. Two events are **disjoint**, or **mutually exclusive**, if they have no outcomes in common, as shown in the third diagram.

KEY CONCEPT *For Your Notebook*

Probability of Compound Events

If A and B are any two events, then the probability of A or B is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are disjoint events, then the probability of A or B is:

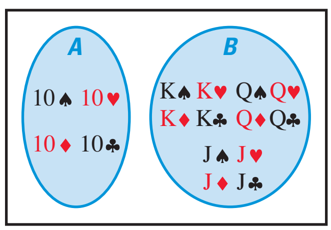
$$P(A \text{ or } B) = P(A) + P(B)$$

EXAMPLE 1 Find probability of disjoint events

A card is randomly selected from a standard deck of 52 cards. What is the probability that it is a 10 *or* a face card?

Solution

Let event A be selecting a 10 and event B be selecting a face card. A has 4 outcomes and B has 12 outcomes. Because A and B are disjoint, the probability is:



$$P(A \text{ or } B) = P(A) + P(B) = \frac{4}{52} + \frac{12}{52} = \frac{16}{52} = \frac{4}{13} \approx 0.308$$

★ EXAMPLE 2 Standardized Test Practice

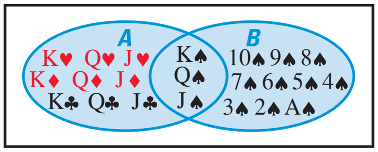
A card is randomly selected from a standard deck of 52 cards. What is the probability that it is a face card *or* a spade?

Ⓐ $\frac{3}{52}$ Ⓑ $\frac{11}{26}$ Ⓒ $\frac{25}{52}$ Ⓓ $\frac{7}{13}$

Solution

AVOID ERRORS
When two events A and B overlap, as in Example 2, $P(A \text{ or } B)$ does not equal $P(A) + P(B)$.

Let event A be selecting a face card and event B be selecting a spade. A has 12 outcomes and B has 13 outcomes. Of these, 3 outcomes are common to A and B. So, the probability of selecting a face card *or* a spade is:



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$$

▸ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 3 Use a formula to find *P*(*A* and *B*)

SENIOR CLASS Out of 200 students in a senior class, 113 students are either varsity athletes *or* on the honor roll. There are 74 seniors who are varsity athletes and 51 seniors who are on the honor roll. What is the probability that a randomly selected senior is both a varsity athlete *and* on the honor roll?

Solution

Let event A be selecting a senior who is a varsity athlete and event B be selecting a senior on the honor roll. From the given information you know that $P(A) = \frac{74}{200}$, $P(B) = \frac{51}{200}$, and $P(A \text{ or } B) = \frac{113}{200}$. Find $P(A \text{ and } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$ **Write general formula.**

$$\frac{113}{200} = \frac{74}{200} + \frac{51}{200} - P(A \text{ and } B)$$ **Substitute known probabilities.**

$$P(A \text{ and } B) = \frac{74}{200} + \frac{51}{200} - \frac{113}{200}$$ **Solve for *P*(*A* and *B*).**

$$P(A \text{ and } B) = \frac{12}{200} = \frac{3}{50} = 0.06$$ **Simplify.**

✓ GUIDED PRACTICE for Examples 1, 2, and 3

A card is randomly selected from a standard deck of 52 cards. Find the probability of the given event.

1. Selecting an ace *or* an eight
2. Selecting a 10 *or* a diamond
3. **WHAT IF?** In Example 3, suppose 32 seniors are in the band and 64 seniors are in the band *or* on the honor roll. What is the probability that a randomly selected senior is both in the band *and* on the honor roll?

COMPLEMENTS The event $\overline{A}$, called the *complement* of event A, consists of all outcomes that are not in A. The notation $\overline{A}$ is read as "A bar."

KEY CONCEPT — *For Your Notebook*

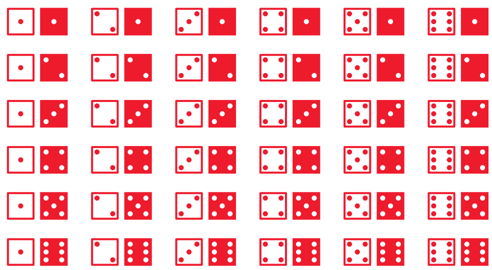
Probability of the Complement of an Event

The probability of the complement of A is $P(\overline{A}) = 1 - P(A)$.

EXAMPLE 4 Find probabilities of complements

ANOTHER WAY
For an alternative method for solving the problem in Example 4, see the **Problem Solving Workshop**.

DICE When two six-sided dice are rolled, there are 36 possible outcomes, as shown. Find the probability of the given event.



a. The sum is not 6.

b. The sum is less than or equal to 9.

Solution

a. $P(\text{sum is not }6) = 1 - P(\text{sum is }6) = 1 - \frac{5}{36} = \frac{31}{36} \approx 0.861$

b. $P(\text{sum} \le 9) = 1 - P(\text{sum} > 9) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6} \approx 0.833$

EXAMPLE 5 Use a complement in real life

FORTUNE COOKIES A restaurant gives a free fortune cookie to every guest. The restaurant claims there are 500 different messages hidden inside the fortune cookies. What is the probability that a group of 5 people receive at least 2 fortune cookies with the same message inside?

Solution

The number of ways to give messages to the 5 people is 500^5. The number of ways to give *different* messages to the 5 people is $500 \cdot 499 \cdot 498 \cdot 497 \cdot 496$. So, the probability that at least 2 of the 5 people have the same message is:

$$\begin{aligned} P(\text{at least 2 are the same}) &= 1 - P(\text{none are the same}) \\ &= 1 - \frac{500 \cdot 499 \cdot 498 \cdot 497 \cdot 496}{500^5} \\ &\approx 0.0199 \end{aligned}$$

✓ GUIDED PRACTICE for Examples 4 and 5

Find $P(\overline{A})$.

4. $P(A) = 0.45$ **5.** $P(A) = \frac{1}{4}$ **6.** $P(A) = 1$ **7.** $P(A) = 0.03$

8. WHAT IF? In Example 5, how does the answer change if there are only 100 different messages hidden inside the fortune cookies?

11.4 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 11, 21, and 45

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 34, 39, 40, 44, and 47

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The union or intersection of two events is called a(n) _?_.

2. ★ **WRITING** Are the events A and $\overline{A}$ disjoint? *Explain*. Then give an example of a real-life event and its complement.

EXAMPLE 1 for Exs. 3–8

DISJOINT EVENTS **Events A and B are disjoint. Find $P(A \text{ or } B)$.**

3. $P(A) = 0.3, P(B) = 0.1$
4. $P(A) = 0.55, P(B) = 0.2$
5. $P(A) = 0.41, P(B) = 0.24$
6. $P(A) = \frac{2}{5}, P(B) = \frac{3}{5}$
7. $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$
8. $P(A) = \frac{2}{3}, P(B) = \frac{1}{5}$

EXAMPLES 2 and 3 for Exs. 9–15

OVERLAPPING EVENTS **Find the indicated probability.**

9. $P(A) = 0.5, P(B) = 0.35$
   $P(A \text{ and } B) = 0.2$
   $P(A \text{ or } B) =$ _?_
10. $P(A) = 0.6, P(B) = 0.2$
    $P(A \text{ or } B) = 0.7$
    $P(A \text{ and } B) =$ _?_
11. $P(A) = 0.28, P(B) = 0.64$
    $P(A \text{ or } B) = 0.71$
    $P(A \text{ and } B) =$ _?_
12. $P(A) = 0.46, P(B) = 0.37$
    $P(A \text{ and } B) = 0.31$
    $P(A \text{ or } B) =$ _?_
13. $P(A) = \frac{2}{7}, P(B) = \frac{4}{7}$
    $P(A \text{ and } B) = \frac{1}{7}$
    $P(A \text{ or } B) =$ _?_
14. $P(A) = \frac{6}{11}, P(B) = \frac{3}{11}$
    $P(A \text{ or } B) = \frac{7}{11}$
    $P(A \text{ and } B) =$ _?_

15. ★ **MULTIPLE CHOICE** What is $P(A \text{ or } B)$ if $P(A) = 0.41$, $P(B) = 0.53$, and $P(A \text{ and } B) = 0.27$?

    Ⓐ 0.12　　Ⓑ 0.67　　Ⓒ 0.80　　Ⓓ 0.94

EXAMPLE 4 for Exs. 16–19

FINDING PROBABILITIES OF COMPLEMENTS **Find $P(\overline{A})$.**

16. $P(A) = 0.5$
17. $P(A) = 0$
18. $P(A) = \frac{1}{3}$
19. $P(A) = \frac{5}{8}$

CHOOSING CARDS **A card is randomly selected from a standard deck of 52 cards. Find the probability of drawing the given card.**

20. A king *and* a diamond
21. A king *or* a diamond
22. A spade *or* a club
23. A 4 *or* a 5
24. A 6 *and* a face card
25. *Not* a heart

ERROR ANALYSIS ***Describe*** **and correct the error in finding the probability of randomly drawing the given card from a standard deck of 52 cards.**

26.

P(heart or face card)

$= P(\text{heart}) + P(\text{face card})$

$= \frac{13}{52} + \frac{12}{52}$

$= \frac{25}{52}$

27.

P(club or 9)

$= P(\text{club}) + P(9) + P(\text{club and } 9)$

$= \frac{13}{52} + \frac{4}{52} + \frac{1}{52}$

$= \frac{9}{26}$

FINDING PROBABILITIES **Find the indicated probability. State whether A and B are disjoint events.**

28. $P(A) = 0.25$
$P(B) = 0.4$
$P(A \text{ or } B) = 0.50$
$P(A \text{ and } B) = \underline{\ ?\ }$

29. $P(A) = 0.6$
$P(B) = 0.32$
$P(A \text{ or } B) = \underline{\ ?\ }$
$P(A \text{ and } B) = 0.25$

30. $P(A) = \underline{\ ?\ }$
$P(B) = 0.38$
$P(A \text{ or } B) = 0.65$
$P(A \text{ and } B) = 0$

31. $P(A) = \frac{8}{15}$
$P(B) = \underline{\ ?\ }$
$P(A \text{ or } B) = \frac{3}{5}$
$P(A \text{ and } B) = \frac{2}{15}$

32. $P(A) = \frac{1}{2}$
$P(B) = \frac{1}{6}$
$P(A \text{ or } B) = \frac{2}{3}$
$P(A \text{ and } B) = \underline{\ ?\ }$

33. $P(A) = 16\%$
$P(B) = \underline{\ ?\ }$
$P(A \text{ or } B) = 32\%$
$P(A \text{ and } B) = 8\%$

34. ★ **OPEN-ENDED MATH** *Describe* a real-life situation that involves two disjoint events A and B. Then describe a real-life situation that involves two overlapping events C and D.

ROLLING DICE **Two six-sided dice are rolled. Find the probability of the given event. (Refer to Example 4 for the possible outcomes.)**

35. The sum is 3 or 4.

36. The sum is not 7.

37. The sum is greater than or equal to 5.

38. The sum is less than 8 or greater than 11.

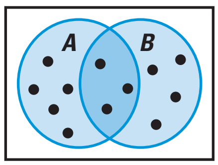
39. ★ **MULTIPLE CHOICE** Two six-sided dice are rolled. What is the probability that the sum is a prime number?

(A) $\frac{13}{36}$ (B) $\frac{7}{18}$ (C) $\frac{5}{12}$ (D) $\frac{5}{11}$

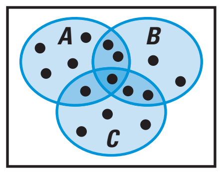
40. ★ **SHORT RESPONSE** Use the first diagram at the right to explain why this equation is true:

$$P(A) + P(B) = P(A \text{ or } B) + P(A \text{ and } B)$$

41. **CHALLENGE** Use the second diagram at the right to derive a formula for $P(A \text{ or } B \text{ or } C)$.



Ex. 40



Ex. 41

PROBLEM SOLVING

EXAMPLES 1, 2, and 3 for Exs. 42–44

42. **CLASS ELECTIONS** You and your best friend are among several candidates running for class president. You estimate that there is a 45% chance you will win and a 25% chance your best friend will win. What is the probability that either you or your best friend win the election?

43. **BIOLOGY** You are performing an experiment to determine how well plants grow under different light sources. Out of the 30 plants in the experiment, 12 receive visible light, 15 receive ultraviolet light, and 6 receive both visible and ultraviolet light. What is the probability that a plant in the experiment receives either visible light or ultraviolet light?

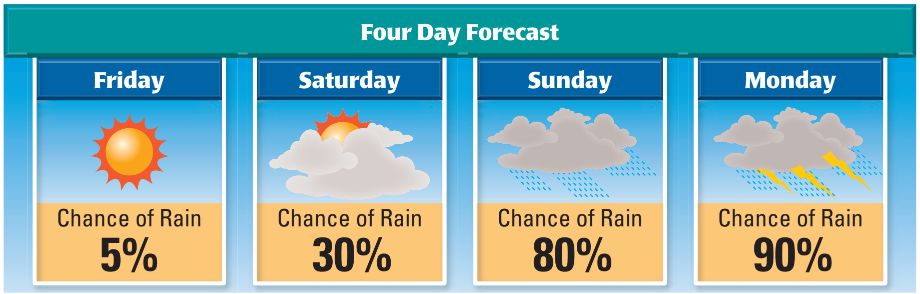
Animated Algebra at my.hrw.com

EXAMPLES 4 and 5 for Exs. 44–46

44. ★ **MULTIPLE CHOICE** Refer to the chart below. Which of the following probabilities is greatest?

Ⓐ P(rains on Sunday)　　Ⓑ P(does not rain on Saturday)

Ⓒ P(rains on Monday)　　Ⓓ P(does not rain on Friday)



45. **DRAMA CLUB** The organizer of a cast party for a drama club asks each of 6 cast members to bring one food item from a list of 10 items. What is the probability that at least 2 of the 6 cast members bring the same item?

46. **HOME ELECTRONICS** A development has 6 houses with the same model of garage door opener. Each opener has 4096 possible transmitter codes. What is the probability that at least 2 of the 6 houses have the same code?

47. ★ **EXTENDED RESPONSE** Use the given information about a farmer's tomato crop to complete parts (a)–(c).

a. 40% of the tomatoes are partially rotten, 30% of the tomatoes have been fed on by insects, and 12% are partially rotten *and* have been fed on by insects. What is the probability that a randomly selected tomato is partially rotten *or* has been fed on by insects?

b. 20% of the tomatoes have bite marks from a chipmunk and 7% have bite marks *and* are partially rotten. What is the probability that a randomly selected tomato has bite marks *or* is partially rotten?

c. Suppose the farmer finds out that 6% of the tomatoes have bite marks *and* have been fed on by insects. Do you have enough information to determine the probability that a randomly selected tomato has been fed on by insects *or* is partially rotten *or* has bite marks from a chipmunk? If not, what other information do you require?

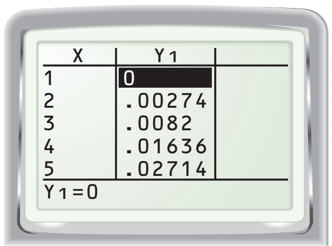
48. **MULTI-STEP PROBLEM** Follow the steps below to explore a famous probability problem called the *birthday problem*. (Assume that there are 365 possible birthdays.)

a. Calculate Suppose that 6 people are chosen at random. Find the probability that at least 2 of the people share the same birthday.

b. Calculate Suppose that 10 people are chosen at random. Find the probability that at least 2 of the people share the same birthday.

c. Model Generalize the results from parts (a) and (b) by writing a formula for the probability $P(x)$ that at least 2 people in a group of x people share the same birthday. (*Hint:* Use $_nP_r$ notation in your formula.)

d. Analyze Enter the formula from part (c) into a graphing calculator. Use the *table* feature to make a table of values. For what group size does the probability that at least 2 people share the same birthday first exceed 50%?



○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

49. PET STORE A pet store has 8 black Labrador retriever puppies (5 females and 3 males) and 12 yellow Labrador retriever puppies (4 females and 8 males). You randomly choose one of the Labrador retriever puppies. What is the probability that it is a female or a yellow Labrador retriever?

50. CHALLENGE You own 50 DVDs consisting of 25 comedies, 15 dramas, and 10 thrillers. You randomly pick 4 movies to watch during a long train ride. What is the probability that you pick at least one DVD of each type of movie?

PROBLEM SOLVING WORKSHOP
LESSON 11.4

Using ALTERNATIVE METHODS

Another Way to Solve Example 4

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

MULTIPLE REPRESENTATIONS In Example 4, you found theoretical probabilities involving the sum of two dice. You can also perform a *simulation* to estimate these probabilities.

PROBLEM

DICE When two six-sided dice are rolled, there are 36 possible outcomes. Find the probability of the given event.

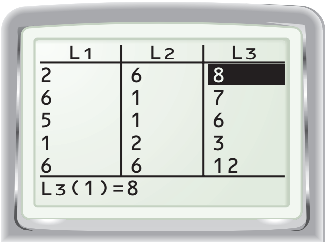
a. The sum is not 6.

b. The sum is less than or equal to 9.

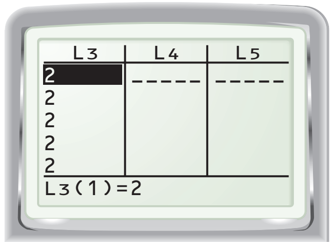
METHOD

Using a Simulation An alternative approach is to use the random number feature of a graphing calculator to simulate rolling two dice. You can then use the results of the simulation to find the experimental probabilities for the problem.

STEP 1 **Generate** two lists of 120 random integers from 1 to 6 by entering randInt(1,6,120) into lists L_1 and L_2. Define list L_3 to be the sum of lists L_1 and L_2.



STEP 2 **Sort** the sums in list L_3 in ascending order using the command SortA(L_3). Scroll through the list and count the frequency of each sum.



STEP 3 **Find** the probabilities.

a. Divide the number of times the sum was 6 by the total number of simulated rolls, then subtract the result from 1.

b. Divide the number of times the sum was greater than 9 by the total number of simulated rolls, then subtract the result from 1.

PRACTICE

1. **WRITING** *Compare* the probabilities found in the simulation above with the theoretical probabilities found in Example 4.

2. **SIMULATIONS** Use the results of the simulation above to find the experimental probability that the sum is greater than or equal to 4. *Compare* this to the theoretical probability of the event.

3. **SIMULATIONS** Use the results of the simulation above to find the experimental probability that the sum is not 8 or 9. *Compare* this to the theoretical probability of the event.

4. **REASONING** How could you change the simulation above so that the results would be closer to the theoretical probabilities of the events? *Explain.*