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CHAPTER 11 Binomial Probabilities

When a coin is tossed, the outcome is either heads or tails. Each outcome (heads or tails) has the same chance of occurring, $\frac{1}{2}$. A probability distribution that involves a number of independent trials that has only two possible outcomes (success or failure), with each trial having the same chance of success, is a **binomial distributions**.

EXAMPLE 1 Two coin toss

Two coins are tossed at the same time. Graph the binomial distribution.

Solution:

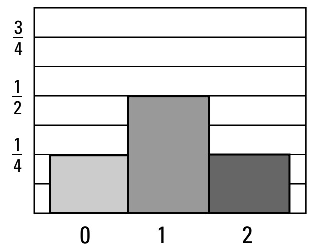
The outcomes are listed in the table below.

1st toss	Tails	Tails	Heads	Heads
2nd toss	Tails	Heads	Tails	Heads

Notice that there are a total of $2^2 = 4$ outcomes. If "tails" is the desired outcome, then the probabilities of these outcomes are shown in the table below.

Number of Tails	Probability
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

The graph of this binomial distribution is:



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EXAMPLE 2 Three coin toss

Three coins are tossed at the same time. Graph the binomial distribution.

Solution:

All of the outcomes are listed in the table below.

1st coin	Tails	Tails	Tails	Heads	Heads	Heads	Tails	Heads
2nd coin	Tails	Tails	Heads	Heads	Heads	Tails	Heads	Tails
3rd coin	Tails	Heads	Heads	Heads	Tails	Tails	Tails	Heads

Notice that there are a total of $2^3 = 8$ outcomes. Given that "tails" is the desired outcome, then the probabilities can be shown in a table.

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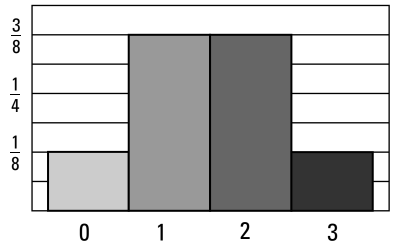
Binomial Probabilities *continued*

Number of Tails	Probability
0	$\frac{1}{8}$
1	$\frac{3}{8}$
2	$\frac{3}{8}$
3	$\frac{1}{8}$

The graph of this binomial distribution is:



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A probability calculated from a binomial distribution is called a **binomial probability**. Calculating a binomial probability from a binomial distribution like the one in the last example is fairly simple. We can answer simple probability questions by looking at the distribution in Example 2.

If three coins are tossed:

- the probability that one of the coins will show “tails” is $\frac{3}{8}$.
- the probability that 1 *or more* of the coins will show “tails” is $\frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$.
- the probability of getting 0 or 1 tails is $\frac{1}{8} + \frac{3}{8} = \frac{1}{2}$.

The first two examples listed the outcomes for 2 coins and 3 coins. As the number of trials increase, however, the need for a formula becomes necessary. This is due to the fact that the sample space grows exponentially with each new trial, depending on the number of outcomes in each trial.

CHAPTER 11

Binomial Probabilities *continued*

In general, we want to know the probability of n successes out of m trials in a binomial distribution, and will assume that the probability of success in a given trial is p.

Consider the example of the three coin toss from Example 2. Let's say that we want to know the probability of getting two heads. Here, $n = 2$ and $m = 3$. Also, the probability of getting a heads (success) is 0.5.

The probability of getting two heads in a three coin toss is obtained by multiplying the number of combinations of 2 out of 3 by the product of each independent event.

$$P(\text{2 out of 3 heads}) = (\text{number of combinations of 2 out of 3}) \cdot (P(\text{success})) \cdot (P(\text{failure}))$$

$$= \underbrace{\frac{3!}{2!(3-2)!}}_{\textbf{number of combinations}} \cdot \underbrace{(0.5)(0.5)}_{\textbf{success}}\underbrace{(0.5)}_{\textbf{failure}}$$

$$= 0.375 = \frac{3}{8}$$

So, the answer is $\frac{3}{8}$. We can generalize this calculation for any m, n and p with this formula.

KEY CONCEPT

Binomial Distribution

The probability of n successes out of m trials in a binomial distribution is given by:

$$P(n \text{ out of } m) = \frac{m!}{n!(m-n)!} \cdot p^n \cdot (1-p)^{m-n}$$

where p represents the probability of success in a given trial.

EXAMPLE 3

Use the Binomial Distribution formula

Consider a number cube rolled four times. There are a total of $6^4 = 1296$ outcomes! We want to calculate the probability that we will get a 3 on two of the rolls. Here, there are $n = 2$ successes out of $m = 4$ trials. The probability of success is $p = \frac{1}{6}$.

Substituting into our formula, we have:

$$P(\text{2 out of 4}) = \frac{4!}{2!(4-2)!} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(1 - \frac{1}{6}\right)^2$$

$$= \frac{4!}{2! \cdot 2!} \cdot \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= \frac{25}{216} \approx 0.1157 \blacksquare$$

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CHAPTER 11

Binomial Probabilities *continued*

EXAMPLE 4 Find expected value

Two coins are tossed at the same time. Find the expected value for the number of heads that result.

Solution:

$$0 \cdot \underbrace{\frac{1}{4}}_{\substack{\text{probability}\\\text{of getting}\\\text{0 head}}} + 1 \cdot \underbrace{\frac{1}{2}}_{\substack{\text{probability}\\\text{of getting}\\\text{1 head}}} + 2 \cdot \underbrace{\frac{1}{4}}_{\substack{\text{probability}\\\text{of getting}\\\text{2 heads}}} = \frac{1}{2} + \frac{1}{2} = 1$$

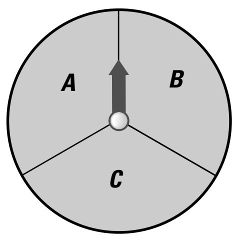
So, the expected value for the number of heads in a 2 coin toss is 1. ■

Practice

Round each answer to the nearest thousandths of a decimal place.

1. Two coins are tossed. Calculate the probability that both coins show heads.
2. Six number cubes are tossed. Calculate the probability of getting a 5 on 4 of the number cubes.
3. Four coins are tossed. Calculate the probability that three of the coins show tails.
4. Four number cubes are tossed. Calculate the probability that 2 or more of the cubes show a 1.

Use the dial shown below to answer Exercises 5–6.



5. The dial is spun three times in a row. Calculate the probability that the dial lands on section *B* two of the times.
6. The dial is spun four times in a row. Calculate the probability that the dial lands on section *A* or section *B*, three of the times.

Find the expected value in Exercises 7–8.

7. Calculate the expected value for the number of tails in a 4 coin toss.
8. Three number cubes are tossed. Calculate the expected value for getting a 4.