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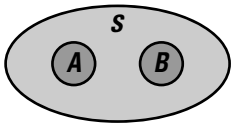
# CHAPTER 11 Distinguishing Between Mutually Exclusive and Independent Events

Distinguishing between mutually exclusive events and independent events is useful when solving problems that involve probability.

**KEY CONCEPT**

## Mutually Exclusive Events

Mutually exclusive events are subsets of the same sample space as shown in the Venn diagram.

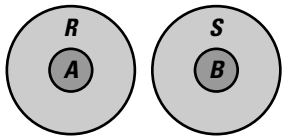



For example, let the sample space $S$ be all of the students that attend a school. Then let subset $A$ be all male students who have brown eyes, and subset $B$ be all female students with green eyes. Both $A$ and $B$ are subsets of $S$, but have no common elements. $A$ and $B$ are mutually exclusive events.

**KEY CONCEPT**

## Independent Events

Independent events have different sample spaces as shown in the Venn diagram.




For example let the sample space $R$ be the 6 outcomes when a die is tossed, and subset $A$ be the desired outcome. Similarly, let sample space $S$ be the 2 outcomes when a coin is tossed, and subset $B$ be the desired outcome. Both $A$ and $B$ are independent because they have different sample spaces. If event $A$ occurs, it does not affect event $B$ and vice versa.

**EXAMPLE 1**

## Compare mutually exclusive and independent events

The table below shows the number of $9^{th}$, $10^{th}$, $11^{th}$, and $12^{th}$ grade boys and girls at a certain high school.

| | $9^{th}$ | $10^{th}$ | $11^{th}$ | $12^{th}$ |
|---|---|---|---|---|
| **Girls** | 90 | 115 | 100 | 95 |
| **Boys** | 110 | 85 | 100 | 105 |

# CHAPTER 11 Distinguishing Between Mutually Exclusive and Independent Events *continued*

**a.** Out of all of the boys at the school, a boy is selected at random. Out of all of the girls at the school, a girl is selected at random. Find the probability that the boy is in $11^{th}$ grade and the girl is in $12^{th}$ grade.

**Solution:**

These events are independent because they have different sample spaces (boys and girls).

The probability is $\underbrace{\frac{100}{400}}_{P(\text{boy})} \cdot \underbrace{\frac{95}{400}}_{P(\text{girl})} = \frac{1}{4} \cdot \frac{19}{80} = \frac{19}{320} \approx 0.059$

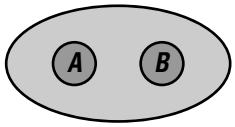
**b.** A student is selected from the student body at random. Find the probability that the student is in $9^{th}$ grade and 11th grade.

**Solution:**

Although each event is part of the same sample space (the student body), these events are mutually exclusive because there is no way a student could be in both grades.

The probability is 0. ■

It is important to note that mutually exclusive events are never independent. Look at the diagram below, which shows two mutually exclusive events $A$ and $B$:




Events $A$ and $B$ do not need to have any common elements for us to calculate $P(A \text{ or } B)$. However, it is impossible to calculate $P(A \text{ and } B)$, because events $A$ and $B$ cannot happen at the same time.

## EXAMPLE 2 Find probabilities

A drawer contains 6 red paper clips, 10 blue paper clips, and 2 yellow paper clips. Selecting a red or yellow paper clip are mutually exclusive events (e.g. a paper clip cannot be red and yellow). A paper clip is chosen at random.

**a.** Find the probability that the paper clip is red or yellow:

$$P(\text{Red or Yellow}) = \frac{6}{18} + \frac{2}{18} = \frac{8}{18} = \frac{4}{9}$$

**b.** Find the probability that the paper clip is red and yellow:

$$P(\text{Red and Yellow}) = 0 \ ■$$

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CHAPTER 11

# Distinguishing Between Mutually Exclusive and Independent Events *continued*

The information above is also useful in finding probabilities that involve inclusive compound events $A$ and $B$, where knowing $P(A)$ and $P(B)$ is necessary in order to calculate $P(A \text{ or } B)$.

**KEY CONCEPT**

## Probability of Inclusive Events

If two events $A$ and $B$ are inclusive, then the probability that event $A$ or event $B$ will occur can be calculated as follows:

$$P(A \text{ or } B) = P(A) + P(B) - P(A) \cdot P(B)$$

**EXAMPLE 3**

## Use the formula for inclusive events

Suppose the probability that it will rain on Saturday is 0.5, and the probability that it will rain on Sunday is 0.7. Find the probability that it will rain on Saturday or Sunday, but not both days.

**Solution:**

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.5 + 0.7 - (0.5) \cdot (0.7) = 0.85$

There is an 85% chance of rain on Saturday or Sunday. ■

## Practice

1. **Challenge** Give an example of a situation that involves mutually exclusive events. Draw a Venn diagram that illustrates the situation.

2. **Challenge** Give an example of a situation that involves independent events. Draw a Venn diagram that illustrates the situation.

**Identify the events as either *mutually exclusive* or *independent*.**

3. A number cube and a coin are tossed. The events are: getting a 4 on the die and a tails on the coin.

4. Mr. Wong's class has 28 students. A student leaves the room for a music lesson, then returns. A second student does the same thing. The events are: the first student is a boy and the second student is a girl.

5. Two number cubes are tossed. The events are: the sum of the numbers is 7 and each die has the same number.

6. A bag contains red and blue marbles. A marble is selected, and then it is replaced. A second marble is selected, and then it is replaced. The events are: both marbles are red.