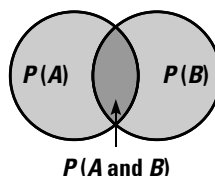


**CHAPTER
11****Conditional Probabilities**

The probability that an event B will occur, given that another event A has already occurred is called a **conditional probability**, and is written $P(B|A)$. The Venn diagram below illustrates the situation.



We can see that it is necessary to divide $P(A \text{ and } B)$ by $P(A)$ in order to find $P(B|A)$. The formula for conditional probability can also be derived from the formula for dependent events:

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad (\text{probability of dependent events})$$

We want to solve this equation for $P(B|A)$, as this is the conditional probability. If we divide both sides of the equation by $P(A)$, we obtain this result:

KEY CONCEPT**Conditional Probability**

When two events A and B are dependent, then the probability of event B , given that event A has already occurred is given by:

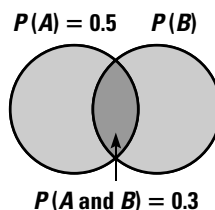
$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

EXAMPLE 1 Find the conditional probability

At Franklin High School, 50% of the student body is female. Also, 30% of the students ride a bicycle. What is the probability that a student rides a bicycle, given that she is female?

Solution:

The Venn diagram below represents this situation.



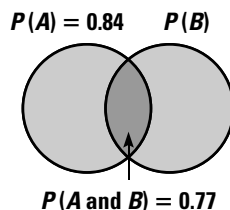
Let A represent the number of females at Franklin High School, and let B represent the number of students who ride a bicycle to school. It follows that $P(A) = 0.5$ and $P(A \text{ and } B) = 0.3$. Substituting into the formula, we have the following result:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.3}{0.5} = 0.6$$

So, the probability that a student rides a bicycle, given that she is female, is 0.6 ■

CHAPTER
11
Conditional Probabilities *continued*
EXAMPLE 2 Find the conditional probability

The local Weather Service office reports that there is a probability of 0.84 that it will rain on Saturday, and a probability of 0.77 that it will rain on Saturday and Sunday. What is the probability that it will rain on Sunday, given that it rained on Saturday?



Let A represent rain on Saturday and B represent rain on Sunday. It follows that $P(A) = 0.84$ and $P(A \text{ and } B) = 0.77$. Substituting into the formula, we have the following result:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.77}{0.84} \approx 0.92$$

The probability that it rains on Sunday, given that it rained on Saturday, is about 0.92. ■

The next example looks at another way to calculate a conditional probability.

EXAMPLE 3 An alternate solution

The table below shows the number of 6th, 7th, and 8th grade boys and girls at Lincoln Middle School.

	6 th grade	7 th grade	8 th grade
Boys	46	42	46
Girls	50	38	45

What is the probability that a student is a girl, given that the student is in 8th grade?

Solution:

Let A represent the set of 8th graders, and let B represent the set of girls at Lincoln Middle School. It follows that $P(A) = \frac{91}{267}$ and $P(A \text{ and } B) = \frac{45}{267}$. Substituting into the formula, we have the following result:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{45}{267} \div \frac{91}{267} = \frac{45}{91} \approx 0.49$$

Because of the way the data is presented in this problem, we can calculate this result by dividing the number of 8th grade girls (45) by the total number of 8th graders (133).

So, the probability that a student is a girl, given that she is in 8th grade is $\frac{45}{133}$ or about 0.34. ■

Conditional Probabilities *continued***Practice**

1. The probability that it is Monday and that it is a School holiday is 0.01. The probability that it is Monday is 0.2 (1 out of 5 school days). What is the probability that it is a school holiday, given that it is Monday?
2. At Washington High School, 12% of the student body play sports and are in the school play and 48% play sports. What percent of students who play sports are in the school play?
3. In a certain city, 60% of the households have a television set and a DVD player and 90% of the households have a television set. What percent of households in the city have a DVD player, given that they have a television set?
4. Ms. Weller gave two tests so far this semester. The probability that a student passed the first and the second test is 0.85 and the probability that a student passed the first test is 0.9. What is the probability that a student passed the second test, given that they passed the first test?

The table below shows the number of ninth grade boys and girls who are on the soccer and tennis teams. Use the table to answer Exercises 5–8.

	Boys	Girls
Soccer	18	12
Tennis	12	16

5. What is the probability that a student is on the soccer team, given that the student is a boy?
6. What is the probability that a student is on the tennis team, given that the student is a girl?