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LESSON 11.5 Study Guide
For use with the lesson "Find Probabilities of Independent and Dependent Events"
GOAL Examine independent and dependent events.

## Vocabulary

Two events are independent events if the occurrence of one event does not affect the occurrence of the other.

Two events are dependent events if the occurrence of one event does affect the occurrence of the other.

The probability that event $B$ occurs given that event $A$ has occurred is called the conditional probability of $B$ given $A$ and is written $P(B \mid A)$.

## EXAMPLE 1 Identify independent and dependent events

A drawer contains red markers and blue markers. You randomly choose a marker from the drawer and do not replace it. Then you randomly choose another marker. Tell whether the events are independent or dependent.
Event $A$ : The first marker you choose is blue.
Event $B$ : The second marker you choose is also blue.

## Solution

After choosing a blue marker, fewer markers remain. This affects the probability that the second marker you choose is blue. So, the events are dependent.

## Exercises for Example 1

Tell whether the events are independent or dependent.

1. A bucket contains tulip bulbs and daffodil bulbs. You randomly choose a bulb, plant it, then choose another bulb.
Event $A$ : The first bulb you choose is a tulip bulb.
Event $B$ : The second bulb you choose is a daffodil bulb.

## EXAMPLE2 Find probability of independent events

Kenesha and Ramón each have a standard deck of 52 cards. Each draws a card at random from his or her deck. Find the probability that Kenesha draws a heart and Ramón draws a 6.

## Solution

Let event $A$ be "draw a heart" and event $B$ be "draw a 6." Because there are two decks, the events are independent, so multiply the probabilities of the events.
$P(A$ and $B)=P(A) \cdot P(B)=\frac{13}{52} \cdot \frac{4}{52}=\frac{1}{52} \approx 0.019$, or about $1.9 \%$

## Exercises for Example 2

2. You have 3 red sweaters, 2 black sweaters, and 4 gray sweaters. You randomly pick one sweater, put it back, then randomly pick another. Find the probability that the first sweater chosen is red and the second is gray.
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## EXAMPLE3 Find probability of dependent events

A bucket contains 40 yellow golf balls and 30 white golf balls. You randomly choose a ball, hit it toward the flag, then randomly choose another ball. Find the probability that the first ball is white and the second ball is yellow.

## Solution

Let event $A$ be "the first ball is white" and event $B$ be "the second ball is yellow." The events are dependent. Find $P(A)$ and $P(B \mid A)$. Then multiply the probabilities.
$P(A)=\frac{30}{70} \quad$ Of the 70 golf balls, 30 are white.
$P(B \mid A)=\frac{40}{69} \quad$ Of the 69 remaining golf balls, 40 are yellow.
$P(A$ and $B)=\frac{30}{70} \cdot \frac{40}{69}=\frac{40}{161} \approx 0.248$, or about $24.8 \%$

## Exercises for Example 3

3. A bowl contains 10 peaches and 8 nectarines. You randomly choose a piece of fruit, eat it, then randomly choose another. Find the probability that the first is a peach and the second is a nectarine.

## EXAMPLE 4 Find a Conditional Probability

The table shows the number of juniors and seniors who have different window treatments in their room.

|  | Juniors | Seniors |
| :--- | :---: | :---: |
| Blinds | 41 | 26 |
| Shades | 96 | 84 |
| Curtains | 25 | 36 |

Use the table to estimate (a) the probability that a randomly chosen junior has blinds in his or her room, and (b) the probability that a randomly chosen student who has blinds in his or her room is a junior.

Solution
a. $\quad P($ blinds $\mid$ junior $)=\frac{\text { number of juniors with blinds }}{\text { total number of juniors }}=\frac{41}{162} \approx 0.253$
b. $\quad P($ junior $\mid$ blinds $)=\frac{\text { number of juniors with blinds }}{\text { total number of students with blinds }}=\frac{41}{67} \approx 0.612$

## Exercise for Example 4

4. Use the table for Example 4. Find (a) the probability that a randomly chosen student who has curtains is a senior and (b) the probability that a randomly chosen senior has curtains in his or her room.
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## EXAMPLE 5 Compare independent and dependent events

You randomly select two cards from a standard 52-card deck. What is the probability that the first card is a queen and the second card is not a queen if (a) you replace the first card before selecting the second card, and (b) you do not replace the first card?

Let $A$ be "the first card is a queen" and $B$ be "the second card is not a queen."
a. $A$ and $B$ are independent events. So, the probability is:

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P(A \text { and } B)=P(A) \cdot P(B)=\frac{4}{52} \cdot \frac{48}{52}=\frac{12}{169} \approx 0.071
$$

b. If you do not replace the first card before selecting the second card, then $A$ and $B$ are dependent events. So, the probability is:
$P(A$ and $B)=P(A) \cdot P(B \mid A)=\frac{4}{52} \cdot \frac{48}{51}=\frac{16}{221} \approx 0.072$

## Exercise for Example 5

5. You randomly select two cards from a standard deck of 52 cards. What is the probability the first card is a ten and the second is a jack if (a) you replace the first card before selecting the second, and (b) you do not replace the first card?

## EXAMPLE 6 Solve a multi-step problem

A survey shows that $\mathbf{3 2 \%}$ of the surveyed fathers are at least 6 feet tall and $\mathbf{4 1 \%}$ of their sons are at least 6 feet tall. When a father is less than $\mathbf{6}$ feet tall, $\mathbf{3 5 \%}$ of the sons are at least 6 feet tall. What is the probability that a son in this survey is at least $\mathbf{6}$ feet tall?

Make a tree diagram. The probability that a boy in this survey is at least 6 feet tall is:


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\begin{aligned}
P(C)=P(A \text { and } C)+P(B \text { and } C) & =P(A) \cdot P(C \mid A)+P(B) \cdot P(C \mid B) \\
& =(0.32)(0.41)+(0.68)(0.35)=0.3692
\end{aligned}
$$

## Exercise for Example 6

6. A survey shows that $80 \%$ of the students write with their right hand. Of these, $83 \%$ throw a ball with their right hand, while $13 \%$ of the students who write left-handed throw a ball with their right hand. What is the probability that a student in this survey throws a ball with the left hand?
