11.5 Find Probabilities of Independent and Dependent Events

Before You found probabilities of compound events.
Now You will examine independent and dependent events.
Why? So you can formulate coaching strategies, as in Ex. 37.

Key Vocabulary
- **independent events**
- **dependent events**
- **conditional probability**

Two events are **independent events** if the occurrence of one event does not affect the occurrence of the other. Two events are **dependent events** if the occurrence of one event *does* affect the occurrence of the other.

COMMON CORE

CC.9-12.S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*

EXAMPLE 1 Identify independent and dependent events

A jar contains red and blue marbles. You randomly choose a marble from the jar, and you do not replace it. Then you randomly choose another marble. Tell whether the events are *independent* or *dependent*.

Event A: The first marble you choose is red.
Event B: The second marble you choose is blue.

Solution

After you choose a red marble, fewer marbles remain in the jar. This affects the probability that the second marble is blue. So, the events are dependent.

CONDITIONAL PROBABILITIES

The conditional probability of B given A can be greater than, less than, or equal to the probability of B.

The probability that event B occurs given that event A has occurred is called the **conditional probability** of B given A and is written $P(B|A)$. Note that A and B are independent events if and only if $P(B) = P(B|A)$ since the probability of event B does not depend on the occurrence of event A.

KEY CONCEPT *For Your Notebook*

Probabilities of Independent and Dependent Events

Independent Events

For two independent events A and B, the probability that both events occur is the product of the probabilities of the events.

$P(A \text{ and } B) = P(A) \cdot P(B)$ *Events A and B are independent.*

Dependent Events

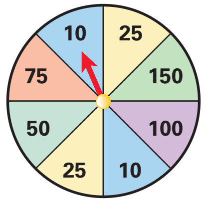
For two dependent events A and B, the probability that both occur is the product of the probability of the first event and the conditional probability of the second event given the first event.

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ *Events A and B are dependent.*

The formulas for finding the probabilities of independent and dependent events can be extended to three or more events.

EXAMPLE 2 Find probability of independent events

As part of a board game, you need to spin the spinner at the right, which is divided into equal parts. Find the probability that you get 25 on your first spin and 50 on your second spin.



Solution

Let event A be "get 25 on first spin" and let event B be "get 50 on second spin." Find the probability of each event. Then multiply the probabilities.

$P(A) = \frac{2}{8}$ "25" appears twice.

$P(B) = \frac{1}{8}$ "50" appears once.

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{2}{8} \cdot \frac{1}{8} = \frac{2}{64} \approx 0.031$$

▶ The probability that you get 25 on your first spin and 50 on your second spin is about 3.1%.

EXAMPLE 3 Find probability of dependent events

A bowl contains 36 green grapes and 14 purple grapes. You randomly choose a grape, eat it, and randomly choose another grape. Find the probability that both events *A* and *B* will occur.

Event *A*: The first grape is green.

Event *B*: The second grape is green.

Solution

Find $P(A)$ and $P(B|A)$. Then multiply the probabilities.

$P(A) = \frac{36}{50}$ Of the 50 grapes, 36 are green.

$P(B|A) = \frac{35}{49}$ Of the 49 remaining grapes, 35 are green.

$$P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{36}{50} \cdot \frac{35}{49} = \frac{1260}{2450} \approx 0.514$$

▶ The probability that both of the grapes are green is about 51.4%.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

Tell whether the situation describes *independent* or *dependent* events. Then answer the question.

1. **CLOTHING** A drawer contains 12 white socks and 8 black socks. You randomly choose one sock, and you do not replace it. Then you randomly choose another sock. What is the probability that both socks chosen are white?

2. **COIN FLIPS** Suppose you flip a coin twice. What is the probability that you get tails on the first flip and tails on the second flip?

CONDITIONAL PROBABILITY You can rewrite the formula for dependent events from the first page of this lesson to give a rule for finding conditional probabilities. Dividing both sides of the formula by $P(A)$ gives the following.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

EXAMPLE 4 Find a conditional probability

WEATHER The table shows the numbers of tropical cyclones that formed during the hurricane seasons from 1988 to 2004. Use the table to estimate **(a)** the probability that a future tropical cyclone in the Northern Hemisphere is a hurricane, and **(b)** the probability that a hurricane is in the Northern Hemisphere.

Type of Tropical Cyclone	Northern Hemisphere	Southern Hemisphere
Tropical depression	199	18
Tropical storm	398	200
Hurricane	545	215

Solution

a. $P(\text{hurricane}|\text{Northern Hemisphere})$

$$= \frac{\text{Number of hurricanes in Northern Hemisphere}}{\text{Total number of cyclones in Northern Hemisphere}} = \frac{545}{1142} \approx 0.477$$

b. $P(\text{Northern Hemisphere}|\text{hurricane})$

$$= \frac{\text{Number of hurricanes in Northern Hemisphere}}{\text{Total number of hurricanes}} = \frac{545}{760} \approx 0.717$$

EXAMPLE 5 Compare independent and dependent events

SELECTING CARDS You randomly select two cards from a standard deck of 52 cards. What is the probability that the first card is not a heart and the second is a heart if **(a)** you replace the first card before selecting the second, and **(b)** you do not replace the first card?

Solution

Let A be "the first card is not a heart" and B be "the second card is a heart."

AVOID ERRORS
It is important first to determine whether A and B are independent or dependent in order to calculate $P(A \text{ and } B)$ correctly.

a. If you replace the first card before selecting the second card, then A and B are independent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{39}{52} \cdot \frac{13}{52} = \frac{3}{16} \approx 0.188$$

b. If you do not replace the first card before selecting the second card, then A and B are dependent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{39}{52} \cdot \frac{13}{51} = \frac{13}{68} \approx 0.191$$

✓ GUIDED PRACTICE for Examples 4 and 5

3. **WHAT IF?** Use the information in Example 4 to find **(a)** the probability that a future tropical cyclone is a tropical storm and **(b)** the probability that a future tropical cyclone in the Southern Hemisphere is a tropical storm.

Find the probability of drawing the given cards from a standard deck of 52 cards (a) with replacement and (b) without replacement.

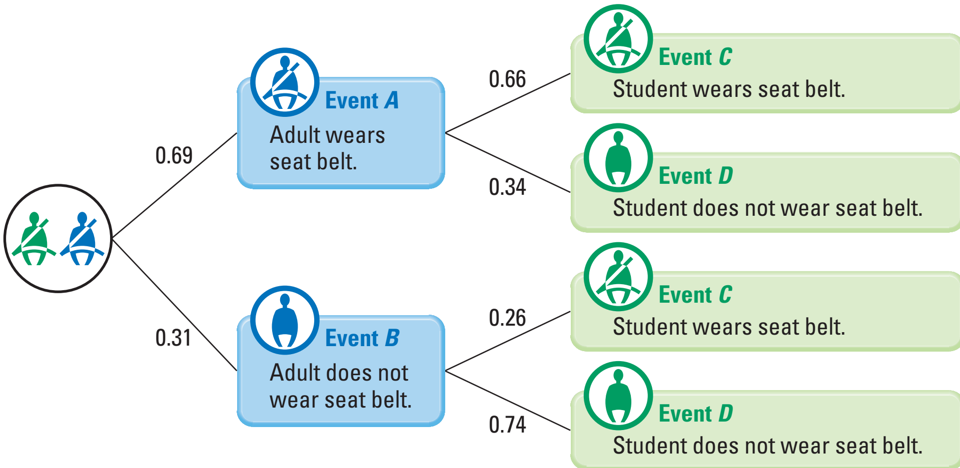
4. A spade, then a club
5. A jack, then another jack

EXAMPLE 6 Solve a multi-step problem

SAFETY Using observations made of drivers arriving at a certain high school, a study reports that 69% of adults wear seat belts while driving. A high school student also in the car wears a seat belt 66% of the time when the adult wears a seat belt, and 26% of the time when the adult does not wear a seat belt. What is the probability that a high school student in the study wears a seat belt?

Solution

A probability tree diagram, where the probabilities are given along the branches, can help you solve the problem. Notice that the probabilities for all branches from the same point must sum to 1.



So, the probability that a high school student wears a seat belt is:

$$P(C) = P(A \text{ and } C) + P(B \text{ and } C)$$
$$= P(A) \cdot P(C|A) + P(B) \cdot P(C|B)$$
$$= (0.69)(0.66) + (0.31)(0.26) = 0.536$$

✓ GUIDED PRACTICE for Example 6

6. **BASKETBALL** A high school basketball team leads at halftime in 60% of the games in a season. The team wins 80% of the time when they have the halftime lead, but only 10% of the time when they do not. What is the probability that the team wins a particular game during the season?

11.5 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 18 and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 10, 28, 29, and 37

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The probability that B will occur given that A has occurred is called the _?_ of B given A.

2. ★ **WRITING** *Explain* the difference between dependent events and independent events, and give an example of each.

EXAMPLE 1 for Exs. 3–6

INDEPENDENT AND DEPENDENT EVENTS **Tell whether the events are *independent* or *dependent*.**

3. A box of energy bars contains an assortment of flavors. You randomly choose an energy bar and eat it. Then you randomly choose another bar.

   Event A: You choose a honey-peanut bar first.

   Event B: You choose a chocolate chip bar second.

4. You roll a number cube and flip a coin.

   Event A: You get a 4 when rolling the number cube.

   Event B: You get tails when flipping the coin.

5. Your CD collection contains hip-hop and rock CDs. You randomly choose a CD, then choose another without replacing the first CD.

   Event A: You choose a hip-hop CD first.

   Event B: You choose a rock CD second.

6. There are 22 volumes of an encyclopedia on a shelf. You randomly choose a volume and put it back. Then you randomly choose another volume.

   Event A: You choose volume 7 first.

   Event B: You choose volume 5 second.

EXAMPLE 2 for Exs. 7–11

INDEPENDENT EVENTS **Events A and B are independent. Find the missing probability.**

7. $P(A) = 0.7$
   $P(B) = 0.3$
   $P(A \text{ and } B) =$ _?_

8. $P(A) = 0.22$
   $P(B) =$ _?_
   $P(A \text{ and } B) = 0.11$

9. $P(A) =$ _?_
   $P(B) = 0.4$
   $P(A \text{ and } B) = 0.13$

10. ★ **MULTIPLE CHOICE** Events A and B are independent. What is $P(A \text{ and } B)$ if $P(A) = 0.3$ and $P(B) = 0.2$?

    (A) 0.06 (B) 0.1 (C) 0.5 (D) 0.6

11. **REASONING** Let $P(A) = 0.3$, $P(B) = 0.2$, and $P(A \text{ and } B) = 0.06$. Find $P(A|B)$ and $P(B|A)$. Tell if A and B are dependent or independent. *Explain.*

EXAMPLE 3 for Exs. 12–14

DEPENDENT EVENTS **Events A and B are dependent. Find the missing probability.**

12. $P(A) = 0.5$
    $P(B|A) = 0.4$
    $P(A \text{ and } B) =$ _?_

13. $P(A) = 0.9$
    $P(B|A) =$ _?_
    $P(A \text{ and } B) = 0.72$

14. $P(A) =$ _?_
    $P(B|A) = 0.6$
    $P(A \text{ and } B) = 0.15$

EXAMPLE 4
for Exs. 15–18

CONDITIONAL PROBABILITY **Let n be a randomly selected integer from 1 to 20. Find the indicated probability.**

15. n is 2 given that it is even

16. n is 5 given that it is less than 8

17. n is prime given that it has 2 digits

18. n is odd given that it is prime

EXAMPLE 5
for Exs. 19–26

DRAWING CARDS **Find the probability of drawing the given cards from a standard deck of 52 cards (a) with replacement and (b) without replacement.**

19. A club, then a spade

20. A queen, then an ace

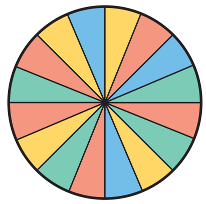
21. A face card, then a 6

22. A 10, then a 2

23. A king, then a queen, then a jack

24. A spade, then a club, then another spade

25. **REASONING** You are playing a game that involves spinning the wheel shown. Find **(a)** the probability of spinning blue and **(b)** the probability of first spinning green and then spinning blue. Are the events of spinning green and then blue dependent or independent? *Explain*.



26. ★ **MULTIPLE CHOICE** What is the approximate probability of drawing 3 consecutive hearts from a standard deck of 52 cards without replacement?

(A) 0.0122 (B) 0.0129 (C) 0.0156 (D) 0.0166

27. **ERROR ANALYSIS** Events A and B are independent. *Describe* and correct the error in finding $P(A \text{ and } B)$.

$P(A) = 0.4, P(B) = 0.5$

$P(A \text{ and } B) = 0.4 + 0.5 = 0.9$ ✗

28. ★ **OPEN-ENDED MATH** Flip a set of 3 coins and record the number of coins that come up heads. Repeat until you have a total of 10 trials.

a. What is the experimental probability that a trial results in 2 heads?

b. *Compare* your answer from part (a) with the theoretical probability that a trial results in 2 heads.

29. ★ **SHORT RESPONSE** A basket contains bottles of apple juice and orange juice in two sizes. The number of each type of bottle is shown in the table.

	6 oz	8 oz
Apple	4	8
Orange	6	9

a. What is the probability that a randomly chosen bottle of juice is a 6-ounce bottle given that the bottle contains apple juice?

b. What is the probability that a randomly chosen 6-ounce bottle contains orange juice?

c. What is the probability that a randomly chosen bottle of orange juice is a 6-ounce bottle?

d. Explain the difference between parts (b) and (c).

30. **REASONING** Let A and B be independent events. What is the relationship between $P(B)$ and $P(B|A)$? *Explain*.

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

31. CHALLENGE Bayes's Theorem states that $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$. Prove it by using the formula for the probability of dependent events and another version of it in which A and B are swapped.

PROBLEM SOLVING

EXAMPLE 4 for Exs. 32–34

32. ENVIRONMENT The table shows the numbers of species in the United States listed as endangered or threatened a few years ago. Find **(a)** the probability that a listed animal is a bird, **(b)** the probability that an endangered animal is a bird, and **(c)** the probability that a bird is endangered.

	Endangered	Threatened
Mammals	69	9
Birds	77	14
Reptiles	14	22
Amphibians	11	10
Other	219	74

In Exercises 33 and 34, use the following information.

TRANSPORTATION All the students at Allen High School were surveyed to find out how they get to school each day. The table shows the number of students at each grade level who walk, bike, or take the bus to school.

	Walk	Bike	Bus	Total
Freshman	52	16	72	140
Sophomore	41	18	61	120
Junior	43	35	72	150
Senior	28	40	52	120
Total	164	109	257	530

33. a. What is the probability that a randomly chosen student is a senior?

b. What is the probability that a randomly chosen student is a senior given that the student bikes to school?

c. Are the events "student is a senior" and "student bikes to school" independent events? Why or why not?

34. a. What is the probability that a randomly chosen student takes the bus given that the student is a junior?

b. What is the probability that a randomly chosen student who takes the bus is a junior?

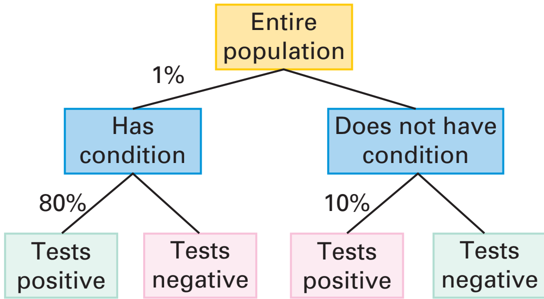
c. ★ **WRITING** Discuss the difference between the probabilities in parts (a) and (b).

EXAMPLE 6 for Ex. 35

35. TENNIS A tennis player wins a match 55% of the time when she serves first and 47% of the time when her opponent serves first. The player who serves first is determined by a coin toss before the match. What is the probability that the player wins a given match?

EXAMPLE 6
for Exs. 36–37

36. MEDICAL TESTING Suppose 1% of the population is known to have a medical condition. There is a test for the condition, and 80% of people with the condition test positive for it. Also, 10% of people without the condition test positive for it. Follow these steps to find the probability that a person actually has the condition given that he or she tests positive.



a. Let A be the event that a person has the condition and let B be the event that the person tests positive. What are $P(B|A)$ and $P(A)$?

b. Find $P(B)$. (*Hint:* Consider the portion of the population with the condition that tests positive and the portion of the population without the condition that tests positive.)

c. Use parts (a) and (b) and Bayes's Theorem to find the probability that a person has the condition given that he or she tests positive.

Bayes's Theorem: $P(A|B) = \dfrac{P(B|A) \cdot P(A)}{P(B)}$

37. ★ EXTENDED RESPONSE A football team is losing by 14 points near the end of a game. The team scores two touchdowns (worth 6 points each) before the end of the game. After each touchdown, the coach must decide whether to go for 1 point with a kick (which is successful 99% of the time) or 2 points with a run or pass (which is successful 45% of the time).

a. Calculate If the team goes for 1 point after each touchdown, what is the probability that the coach's team wins? loses? ties?

b. Calculate If the team goes for 2 points after each touchdown, what is the probability that the coach's team wins? loses? ties?

c. Reasoning Can you develop a strategy so that the coach's team has a probability of winning the game that is greater than the probability of losing? If so, explain your strategy and calculate the probabilities of winning and losing using your strategy.

QUIZ

Find the indicated probability.

1. $P(A) = 0.6$
$P(B) = 0.35$
$P(A \text{ or } B) = ?$
$P(A \text{ and } B) = 0.2$

2. $P(A) = ?$
$P(B) = 0.44$
$P(A \text{ or } B) = 0.56$
$P(A \text{ and } B) = 0.12$

3. $P(A) = 0.75$
$P(B) = ?$
$P(A \text{ or } B) = 0.83$
$P(A \text{ and } B) = 0.25$

Find the probability of randomly drawing the given marbles from a bag of 6 red, 9 green, and 5 blue marbles (a) with replacement and (b) without replacement.

4. red, then green

5. blue, then red

6. green, then green

See EXTRA PRACTICE in Student Resources

Extension Make and Analyze Decisions

COMMON CORE

CC.9-12.S.MD.6(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*

Probabilities can help in making fair decisions, as when using a process with equally likely outcomes to select a contest winner. Probabilities also underlie all kinds of real-world decisions in business, science, agriculture, and so on.

EXAMPLE 1 Use probability to make a decision

Twenty students, including Noe, volunteer to present the "Best Teacher" award at a school banquet. Describe a process that gives Noe a fair chance to be chosen, and find the probability, if (a) "fair" means equally likely, and (b) "fair" means proportional to how many banquet prep hours the volunteer worked. Each volunteer worked at least one hour, Noe worked four hours, and, in all, the 20 students worked 45 hours.

Solution

a. Write the names on slips of paper, place them in a box, and draw a slip at random. The probability is 1 out of 20, or 5%.

b. Write the names on slips of paper, but for each hour more than one that a student worked, write their name on an extra slip. Then draw as in part (a). The probability is 4 out of 45, or about 8.9%.

EXAMPLE 2 Use probability to make a decision

Your company must produce 50,000 non-defective cell phones using a component from one of the suppliers below.

	Price per 1000	*P*(defective)	*P*(working)
Supplier *X*	$740.00	4.0%	96.0%
Supplier *Y*	$800.00	1.9%	98.1%

Each defective component bought results in $2.20 in extra cost to your company. From which supplier should you buy?

Solution

Use the probability that a component is defective to estimate the total cost.

Total cost = Cost to get 50,000 working components + Extra cost from defective components

X: Solving $0.96x = 50{,}000$ gives $x = 52{,}083$. You must buy 53,000 components.
Total cost = 53,000($.74) + (0.04)(53,000)($2.20) = $39,220 + $4664 = $43,884

Y: Solving $0.981y = 50{,}000$ gives $y = 50{,}968$. You must buy 51,000 components.
Total cost = 51,000($.80) + (0.019)(51,000)($2.20) = $40,800 + $2132 = $42,932

▸ For the lowest total cost, you should buy from supplier Y.

PRACTICE

EXAMPLE 1 for Ex. 1

1. A teacher tells students, "For each puzzler you complete, I will assign you a prize entry." In all, 10 students complete 53 puzzlers. Leon completed 7. To award the prize, the teacher sets a calculator to generate a random integer from 1 to 53. Leon is assigned 18 to 24 as "winners." Is this fair to Leon according to the original instructions? *Explain.*

EXAMPLE 2 for Exs. 2–4

2. A company creates a new brand of a snack, N, and tests it against the current market leader, L. The table shows the results.

	Prefer L	Prefer N
Current L consumer	72	46
Not current L consumer	52	114

Use probability to explain how the company's decisions about whether to try to improve the snack before marketing it and to which consumers it should aim its marketing might differ if the total size of the snack's market is expected to (a) change very little, and (b) expand very rapidly.

3. The Redbirds trail the Bluebirds by 1 goal with 1 minute left in the hockey game. The coach must decide whether to remove the goalie and add a frontline player. The only way the Redbirds can tie the game is for them to score and for the Bluebirds not to score. The probabilities are shown below.

	Goalie	No Goalie
Redbirds score	0.1	0.3
Bluebirds score	0.1	0.6

a. Find the probability that the Redbirds score and the Bluebirds do not score if the coach leaves the goalie in.

b. Find the probability that the Redbirds score and the Bluebirds do not score if the coach takes the goalie out.

c. Based on parts (a) and (b), what should the coach do?

4. A farmer is offered a contract that guarantees him \$11.00 per bushel for his entire soybean crop when it is harvested in three months. Below are predictions for the market price m per of soybeans in three months.

$P(m \le \$9.00) = 10\%$ $P(m \ge \$10.50) = 50\%$
$P(m \ge \$12.50) = 20\%$

The farmer predicts a total crop of 20,000 bushels, with a 30% chance of less than 15,000 bushels, and a 20% chance of at least 25,000 bushels.

a. Find the probability and income range for (i) the best case: the farmer declines the contract, the price is highest, and the harvest is largest; and (ii) the worst case: he declines the contract, the price is lowest, and the harvest is smallest. (Assume harvest size and price are independent.)

b. How much will the farmer make if he accepts the contract and his total crop prediction is accurate? How might this and the answers to part (a) affect the decision of whether or not to accept the contract?