## Extension Make and Analyze Decisions

COMMON CORE
CC.9-12.S.MD.6(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).*

Probabilities can help in making fair decisions, as when using a process with equally likely outcomes to select a contest winner. Probabilities also underlie all kinds of real-world decisions in business, science, agriculture, and so on.

## EXAMPLE 1 Use probability to make a decision

Twenty students, including Noe, volunteer to present the "Best Teacher" award at a school banquet. Describe a process that gives Noe a fair chance to be chosen, and find the probability, if (a) "fair" means equally likely, and (b) "fair" means proportional to how many banquet prep hours the volunteer worked. Each volunteer worked at least one hour, Noe worked four hours, and, in all, the 20 students worked 45 hours.

## Solution

a. Write the names on slips of paper, place them in a box, and draw a slip at random. The probability is 1 out of 20 , or $5 \%$.
b. Write the names on slips of paper, but for each hour more than one that a student worked, write their name on an extra slip. Then draw as in part (a). The probability is 4 out of 45 , or about $8.9 \%$.

## EXAMPLE 2 Use probability to make a decision

Your company must produce $\mathbf{5 0 , 0 0 0}$ non-defective cell phones using a component from one of the suppliers below.

|  | Price per $\mathbf{1 0 0 0}$ | P(defective) | P(working) |
| :--- | :---: | :---: | :---: |
| Supplier $\boldsymbol{X}$ | $\$ 740.00$ | $4.0 \%$ | $96.0 \%$ |
| Supplier $\mathbf{Y}$ | $\$ 800.00$ | $1.9 \%$ | $98.1 \%$ |

Each defective component bought results in $\$ 2.20$ in extra cost to your company. From which supplier should you buy?

## Solution

Use the probability that a component is defective to estimate the total cost.

$$
\text { Total cost }=\underset{\text { working components }}{\text { Cost to get 50,000 }}+\underset{\text { Extra cost from }}{\text { defective components }}
$$

X: Solving $0.96 x=50,000$ gives $x=52,083$. You must buy 53,000 components.
Total cost $=53,000(\$ .74)+(0.04)(53,000)(\$ 2.20)=\$ 39,220+\$ 4664=\$ 43,884$
Y: Solving $0.981 y=50,000$ gives $y=50,968$. You must buy 51,000 components.
Total cost $=51,000(\$ .80)+(0.019)(51,000)(\$ 2.20)=\$ 40,800+\$ 2132=\$ 42,932$

- For the lowest total cost, you should buy from supplier Y.


## PRACTICE

EXAMPLE 1 for Ex. 1

EXAMPLE 2 for Exs. 2-4

1. A teacher tells students, "For each puzzler you complete, I will assign you a prize entry." In all, 10 students complete 53 puzzlers. Leon completed 7. To award the prize, the teacher sets a calculator to generate a random integer from 1 to 53 . Leon is assigned 18 to 24 as "winners." Is this fair to Leon according to the original instructions? Explain.
2. A company creates a new brand of a snack, $N$, and tests it against the current market leader, L. The table shows the results.

|  | Prefer L | Prefer $\mathbb{N}$ |
| :--- | :---: | :---: |
| Current L consumer | 72 | 46 |
| Not current L consumer | 52 | 114 |

Use probability to explain how the company's decisions about whether to try to improve the snack before marketing it and to which consumers it should aim its marketing might differ if the total size of the snack's market is expected to (a) change very little, and (b) expand very rapidly.
3. The Redbirds trail the Bluebirds by 1 goal with 1 minute left in the hockey game. The coach must decide whether to remove the goalie and add a frontline player. The only way the Redbirds can tie the game is for them to score and for the Bluebirds not to score. The probabilities are shown below.

|  | Goalie | No Goalie |
| :--- | :---: | :---: |
| Redbirds score | 0.1 | 0.3 |
| Bluebirds score | 0.1 | 0.6 |

a. Find the probability that the Redbirds score and the Bluebirds do not score if the coach leaves the goalie in.
b. Find the probability that the Redbirds score and the Bluebirds do not score if the coach takes the goalie out.
c. Based on parts (a) and (b), what should the coach do?
4. A farmer is offered a contract that guarantees him \$11.00 per bushel for his entire soybean crop when it is harvested in three months. Below are predictions for the market price $m$ per of soybeans in three months.

$$
\begin{aligned}
& P(m \leq \$ 9.00)=10 \% \quad P(m \geq \$ 10.50)=50 \% \\
& P(m \geq \$ 12.50)=20 \%
\end{aligned}
$$

The farmer predicts a total crop of 20,000 bushels, with a $30 \%$ chance of less than 15,000 bushels, and a $20 \%$ chance of at least 25,000 bushels.

a. Find the probability and income range for (i) the best case: the farmer declines the contract, the price is highest, and the harvest is largest; and (ii) the worst case: he declines the contract, the price is lowest, and the harvest is smallest. (Assume harvest size and price are independent.)
b. How much will the farmer make if he accepts the contract and his total crop prediction is accurate? How might this and the answers to part (a) affect the decision of whether or not to accept the contract?

