

Geometric Probability

GOAL Find the probability of an event using a geometric model.

In Lesson 7, you learned that the probability of an event is the number of favorable outcomes divided by the total number of outcomes. Calculating the probability of an event using geometric measures such as length, area, and volume is called **geometric probability**.

Length You can find the probability of an event using lengths. For example, if a randomly chosen point K lies on \overline{AD} , then the probability that it lies on \overline{AB} is

$$P(\text{point } K \text{ lies on } \overline{AB}) = \frac{\text{Length of } \overline{AB}}{\text{Length of } \overline{AD}}$$



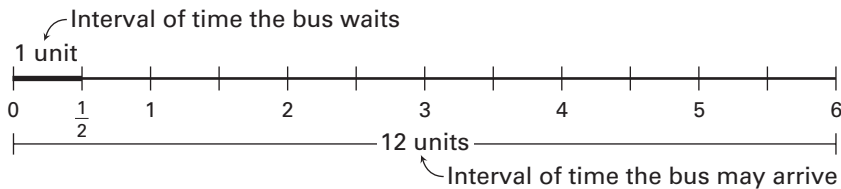
EXAMPLE 1 Using a Timeline to Find Probability

A bus arrives at your bus stop every 6 minutes. The driver waits for $\frac{1}{2}$ minute to let passengers on and off the bus. Then the bus continues on its route. You arrive at your bus stop at a random time. What is the probability that there is a bus waiting at your stop?

SOLUTION

Draw a timeline to show the bus's schedule divided into $\frac{1}{2}$ -minute intervals.

Because a bus arrives every 6 minutes, the timeline should show 6 minutes of time.



Find the ratio of the length of the segment for the interval of time the bus is waiting to the length of the segment for the interval of time the bus may arrive.

$$P(\text{bus waiting}) = \frac{\text{Length of segment for } 0 \text{ to } \frac{1}{2} \text{ min}}{\text{Length of segment for } 0 \text{ to } 6 \text{ min}} = \frac{1}{12}$$

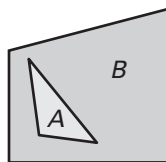
The probability that a bus is waiting at your stop is $\frac{1}{12}$, or about 8.3%.

CHECK Example 1

1. A news program on Channel 2 runs for 30 minutes with 3 commercial breaks of 2 minutes each and opening and closing credits of $\frac{1}{2}$ minute each. The remaining time is devoted to news. You turn to Channel 2 at a random time during the 30 minute news program. What is the probability that the program is showing news when you turn to Channel 2?

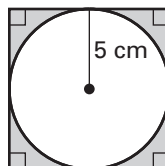
Area You can also find the probability of an event using area. For example, if a randomly chosen point P lies in region B , then the probability that it lies in region A is

$$P(\text{point } P \text{ lies in region } A) = \frac{\text{Area of } A}{\text{Area of } B}$$



EXAMPLE 2 Using Area to Find Probability

What is the probability that a randomly chosen point in the figure lies in the shaded region?



SOLUTION

Find the ratio of the area of the shaded region to the total area of the figure.

$$\begin{aligned} \text{Total area of figure} &= \text{Area of square} \\ &= s^2 \\ &= 10^2 \\ &= 100 \text{ cm}^2 \end{aligned}$$

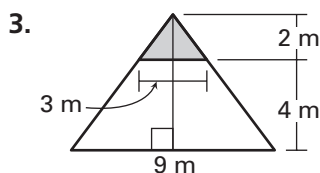
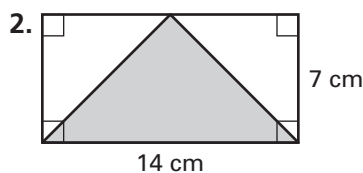
$$\begin{aligned} \text{Area of shaded region} &= \text{Area of square} - \text{Area of circle} \\ &= 100 - \pi r^2 \\ &= 100 - \pi(5)^2 \\ &= 100 - 25\pi \\ &\approx 21.46 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} P(\text{point lies in shaded region}) &= \frac{\text{Area of shaded region}}{\text{Total area of figure}} \\ &\approx \frac{21.46 \text{ cm}^2}{100 \text{ cm}^2} \\ &\approx 0.215 \end{aligned}$$

The probability is about 0.215, or 21.5%.

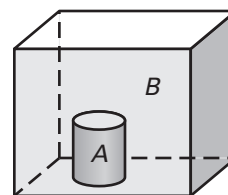
CHECK Example 2

Find the probability that a randomly chosen point in the figure lies in the shaded region.



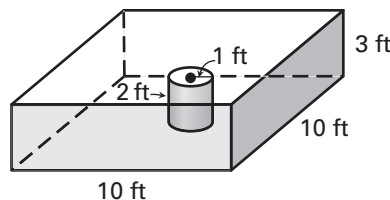
Volume You can also find the probability of an event using volume. For example, if a randomly chosen point P lies in solid B , then the probability that it lies in solid A is

$$P(\text{point } P \text{ lies in solid } A) = \frac{\text{Volume of } A}{\text{Volume of } B}$$



EXAMPLE 3 Using Volume to Find Probability

What is the probability that a randomly chosen point in the rectangular prism lies in the cylinder?



SOLUTION

Find the ratio of the volume of the cylinder to the total volume of the rectangular prism.

Total volume of rectangular prism **Volume of cylinder**

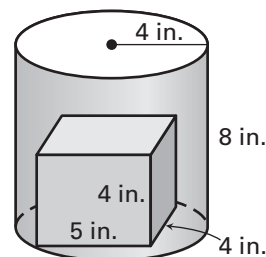
$$\begin{aligned} V &= \ell wh & V &= \pi r^2 h \\ &= (10)(10)(3) & &= \pi(2)^2(1) \\ &= 300 \text{ ft}^3 & &\approx 6.28 \text{ ft}^3 \end{aligned}$$

$$\begin{aligned} P(\text{point lies in cylinder}) &= \frac{\text{Volume of cylinder}}{\text{Total volume of rectangular prism}} \\ &\approx \frac{6.28 \text{ ft}^3}{300 \text{ ft}^3} \approx 0.021 \end{aligned}$$

The probability is about 0.021, or 2.1%.

CHECK Example 3

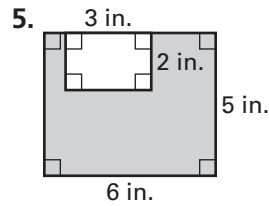
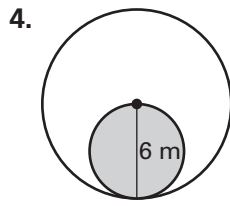
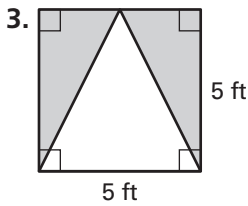
4. What is the probability that a randomly chosen point in the cylinder lies in the rectangular prism?



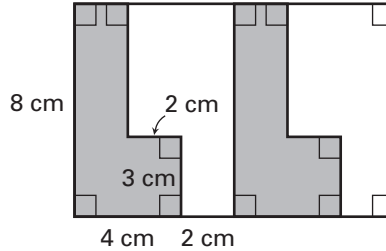
EXERCISES

- Mia does homework from 4:30 P.M. to 6:30 P.M. each day. She spends 45 minutes on her math homework. If Mia receives a phone call during her homework time, what is the probability that she will be working on math?
- You are auditioning for a role in a play. Actors are randomly assigned 20 minute time slots between the hours of 1:00 P.M. and 4:00 P.M. to audition. What is the probability that you will be assigned the time slot from 2:20 P.M. to 2:40 P.M.?

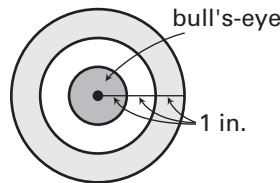
Find the probability that a randomly chosen point in the figure lies in the shaded region.



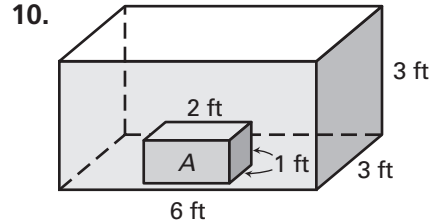
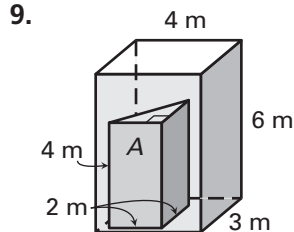
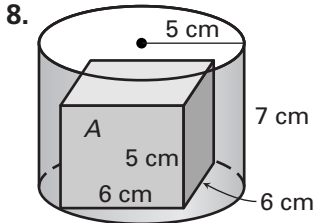
6. You drop a sewing needle on the floor and it randomly lands on one of the tiles shown. What is the probability that the needle lands on a shaded tile?



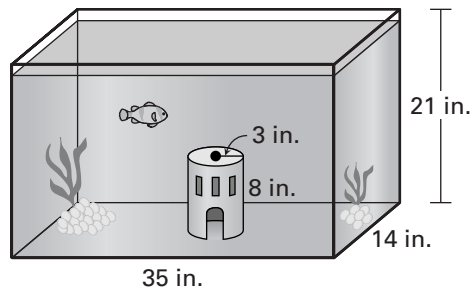
7. A dartboard has a bull's-eye and two rings with the same center as the bull's-eye. A dart is randomly thrown and hits the dartboard. What is the probability that it will land inside the bull's-eye?



Find the probability that a randomly chosen point in the figure lies in solid A.



11. A fish is swimming in a rectangular tank. The tank contains a hollow cylindrical castle, as shown. If the fish is equally likely to be anywhere inside the tank, what is the probability that the fish is inside the castle?



12. The term *Monte Carlo method* is a technique in which you generate random numbers or random points on a plane and then apply proportional reasoning. Draw an irregular shape in a coordinate plane and show how you can enclose the shape within a square that has a known area. Then explain how you can use a Monte Carlo method and geometric probability to estimate the area of the irregular shape. (*Hint*: Imagine that the square is a target and that a given number of darts hit the target randomly.)