Unit 18: Introduction to Probability

SUMMARY OF VIDEO

There are lots of times in everyday life when we want to predict something in the future. Rather than just guessing, probability is the mathematical way to make these kinds of predictions. Here are some examples that use the language of probability: a 50% percent chance of snow, a 20% percent chance of complications from surgery, a one in one hundred seventy five million chance of winning the lottery. We encounter statements such as these all the time in daily life – but what do they really mean?

These statements are attempts to quantify uncertainty. For example, how likely is it to rain later in the day – the answer to this question helps people decide if they should carry an umbrella. When meteorologist Kevin Skarupa issues his forecast for the residents of New Hampshire, he doesn't know for sure what is going to happen. Weather is an example of a random phenomenon. It is an event with an uncertain outcome, but it does have a regular pattern over time.

Today's meteorologists rely on multiple complicated mathematical models to make their predictions for the public. The models churn through tons of weather related data – from current weather balloon information and surface observations, to historic patterns. These models combine all these weather inputs to create maps predicting what will happen in the next few days based on what has happened in the past when similar scenarios have been observed. Over time, the weather exhibits patterns; but for any one particular instance in the future, the weather is not completely predictable with perfect accuracy. That is why forecasters talk in probabilities – for example, "70% chance of rain."

When a station announces a 70% chance of rain, it generally means that 70% of the viewers, if equally spread out, will see precipitation. In this case, the forecaster feels sure that 70% of the area will see rain that day – so the 70% refers to area coverage. The percentage number is also used to quantify the likelihood of any precipitation at all – the degree of confidence of getting rain. So, when reporting a percentage, meteorologists are usually expressing a combination of degree of confidence and area coverage.

The probability of any event is the proportion, or percentage, of times it would occur in a long series of repetitions. Random phenomena like weather events are not chaotic; they are unpredictable in the short run, but they have a regular pattern in the long run. Take the example of flipping a coin. The toss can come up heads or it can come up tails. Suppose we start flipping a coin and then record the proportion of heads. Say, on the first flip we get tails, the proportion of heads is 0. On the next toss we get heads, and the proportion of heads goes up to 0.5. On the next two flips we get tails, and now the proportion of heads in the four flips is down to 0.25. In the short term, the proportion of heads is quite variable. But, suppose we continue flipping the coin – 50 times, 100 times, 1000 times. Over the long term a pattern emerges – the proportions hover around 0.5 – as can be seen in Figure 18.1.

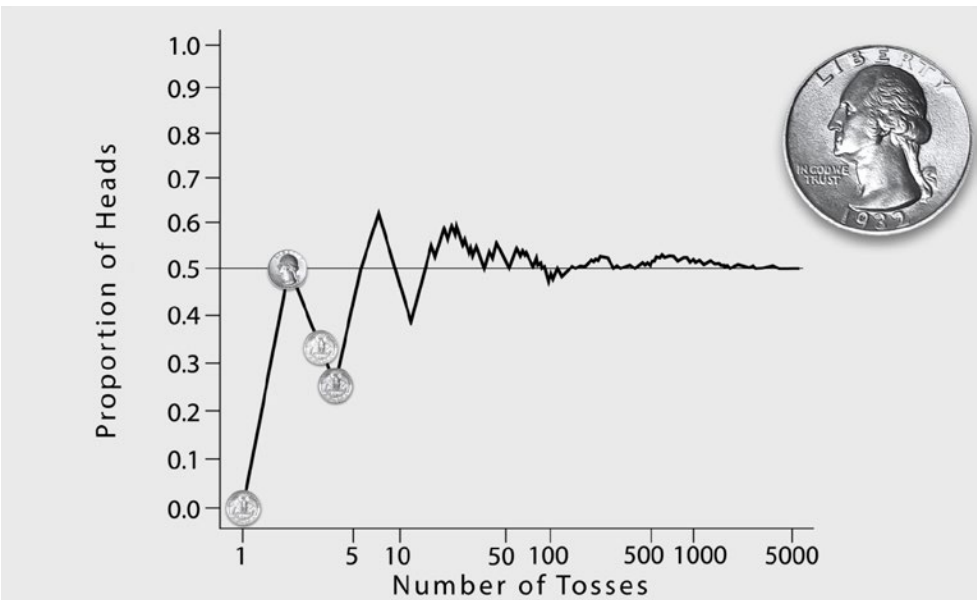

Figure 18.1. Proportion of heads in flipping a coin.

While we don't know what is going to happen on any one toss, over time we can predict that we will get a proportion of heads around 0.5.

Like the proportions on the graph in Figure 18.1, probabilities are between zero and one. Events with a probability closer to zero are less likely and those with a probability closer to one are more likely to happen. Our probability of 0.5 for heads in a coin toss means that either outcome is equally likely – 50/50.

Probabilities are assigned to more than just coin tosses. NASA's Near Earth Object Program is closely monitoring asteroids with the potential to do serious damage to our planet. While "near-Earth space" is home to over 9,000 known asteroids, only about half of them are large enough and have orbits that come close enough to Earth to classify them as PHAs – Potentially Hazardous Asteroids. Scientists track these PHAs, collecting data on them, so they can determine the likelihood that any might be on a collision course with Earth. As more and more data come in about an asteroid's orbit, scientists refine their predictions of its orbit, which allows them to start work on computing the probability that it will collide with Earth.

The poster child for near-Earth objects is an asteroid called Apophis. It is coming very close to Earth in 2029. When it was first discovered, scientists didn't know how close it would come to Earth. In fact, the uncertainty region during its passage by Earth was so large, that Earth was right in the middle of it. (See Figure 18.2.)

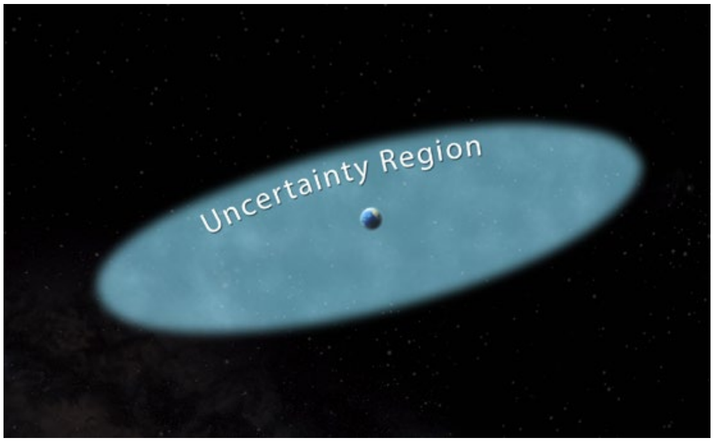

Figure 18.2. Initial uncertainty region.

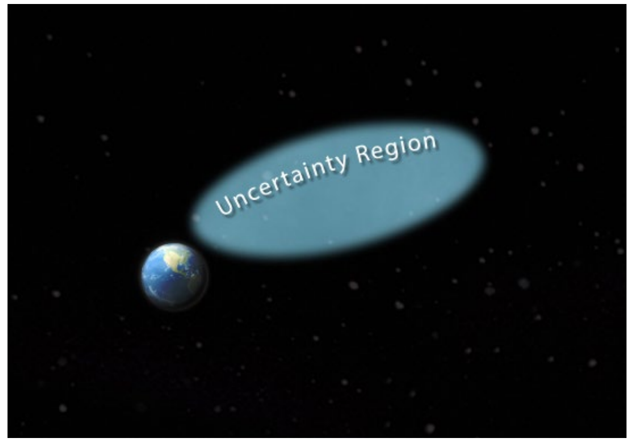

Figure 18.3. Refined uncertainty region

As we got more and more observations on the asteroid, scientists were able to refine their projections of its path and shrink the uncertainty region so that it no longer intersects with Earth. (See Figure 18.3.) So, we now know that Apophis will pass by Earth in 2029 and get very close, but the probability that it will hit Earth is essentially zero. However, Apophis' close encounter with Earth's gravity in 2029 will bend its trajectory, which makes the job of predicting where it will go from there much more difficult. Apophis' next passage by Earth will be in 2036 and scientists will once again be collecting data, predicting its orbit, and assessing the likelihood that Apophis is on a collision course with Earth.