Solving Quadratic Equations by Completing the Square

You can solve some quadratic equations by taking the square root of each side. To do so, the quadratic expression on one side of the equation must be a perfect square. However, few quadratic expressions are perfect squares. To make any quadratic expression a perfect square, use the method called **completing the square**.

Completing the Square	To complete the square for a quadratic expression of the form $x^2 + bx$, follow the steps below. 1. Find $\frac{1}{2}$ of b, the coefficient of x. 2. Square the result of step 1. 3. Add the result of step 2 to $x^2 + bx$, the original expression.

Examples

a. Find the value of c that makes $x^2 + 12x + c$ a perfect square.

1. Find $\frac{1}{2}$ of 12. $\qquad\qquad \frac{12}{2} = 6$
2. Square the result of step 1. $\qquad 6^2 = 36$
3. Add the result of step 2 to $x^2 + 12x$. $\quad x^2 + 12x + 36$

So, $c = 36$.
Notice that $x^2 + 12x + 36 = (x + 6)^2$.

b. Solve $x^2 + 16x - 10 = 0$ by completing the square.

Notice that $x^2 + 16x - 10$ is not a perfect square.

$x^2 + 16x - 10 = 0$

$x^2 + 16x = 10$ $\qquad$ Add 10 to each side.

$x^2 + 16x + 64 = 74$ $\qquad$ Since $\left(\frac{16}{2}\right)^2$ is 64, add 64 to each side.

$(x + 8)^2 = 74$ $\qquad$ Factor $x^2 + 16x + 64$.

$x + 8 = \pm\sqrt{74}$ $\qquad$ Take the square root of each side.

$x = -8 \pm \sqrt{74}$

Solution set: $\{-8 + \sqrt{74}, -8 - \sqrt{74}\}$

Practice

Find the value of c that makes each trinomial a perfect square.

1. $y^2 + 8y + c$ $\qquad\qquad$ **2.** $a^2 + 6a + c$ $\qquad\qquad$ **3.** $x^2 + 10x + c$

4. $x^2 + 9x + c$ $\qquad\qquad$ **5.** $s^2 + 11s + c$ $\qquad\qquad$ **6.** $z^2 + 7z + c$

Solve each equation by completing the square. Leave irrational roots in simplest radical form.

7. $x^2 + 8x + 12 = 0$ $\qquad$ **8.** $y^2 + 6y - 15 = 0$ $\qquad$ **9.** $z^2 + 12z - 25 = 0$

10. $a^2 + 14a - 18 = 0$ $\qquad$ **11.** $x^2 + 10x + 16 = 0$ $\qquad$ **12.** $x^2 + 18x + 17 = 0$

13. | Standardized Test Practice | Which expression shows the solutions of $x^2 + 16x + 32 = 0$?

A $8 + 4\sqrt{2}$ $\qquad$ **B** $-8 + 4\sqrt{2}$ $\qquad$ **C** $8 \pm 4\sqrt{2}$ $\qquad$ **D** $-8 \pm 4\sqrt{2}$

Answers: 1. 16 **2.** 9 **3.** 25 **4.** 20.25 **5.** 30.25 **6.** 12.25 **7.** −2, −6 **8.** −3 ± 2√6 **9.** −6 ± √61 **10.** −7 ± √67 **11.** −2, −8 **12.** −1, −17 **13.** D