

Factors and Greatest Common Factors

When two or more numbers are multiplied to form a product, each number is a **factor** of the product. **Prime numbers** are whole numbers greater than 1 that have exactly two factors, the number itself and 1. Whole numbers greater than 1 that have more than two factors are called **composite numbers**. When a whole number is expressed as a product of factors that are all prime numbers, the expression is called the **prime factorization** of the number. A monomial is in factored form when it is expressed as the product of prime numbers and no variable has an exponent greater than 1. You can use prime factorization to find the greatest **common factor (GCF)** of two or more integers, which is the greatest number that is a factor of all the integers.

Examples

a. Find the prime factorization of $72x^2$.

$72x^2$
 $= 2 \cdot 36 \cdot x \cdot x$ The least prime factor of 72 is 2.
 $= 2 \cdot 2 \cdot 18 \cdot x \cdot x$ The least prime factor of 36 is 2.
 $= 2 \cdot 2 \cdot 2 \cdot 9 \cdot x \cdot x$ The least prime factor of 18 is 2.
 $= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x$ The least prime factor of 9 is 3.
 The prime factorization of $72x^2$ is
 $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x$ or $2^3 \cdot 3^2 \cdot x^2$.

b. Find the GCF of $24a^2$ and $36a$.

Find the prime factorization of each number, then circle the common factors.

$24a^2 = \textcircled{2} \cdot \textcircled{2} \cdot 2 \cdot \textcircled{3} \cdot \textcircled{a} \cdot \textcircled{a}$
 $36a = \textcircled{2} \cdot \textcircled{2} \cdot \textcircled{3} \cdot 3 \cdot \textcircled{a}$

The GCF is the product of the common factors.
 $2 \cdot 2 \cdot 3 \cdot a = 12a$
 The GCF is $12a$.

Practice

State whether each number is *prime* or *composite*. If the number is composite, find its prime factorization.

1. 15 2. 27 3. 23 4. 31 5. 404 6. 1240

Find the GCF of the given monomials.

7. 18, 50 8. $12, 28$ 9. $56, 126$
 10. $5a^3, 20a$ 11. $12x^2y^4, 18y^2z$ 12. $4c, 16c^2, 28b^2c^5$

13. The Classroom Ms. Yip has 32, 36, and 24 students in each of her morning classes. What is the greatest number of desks can she place in each row of desks so that no row will be partially filled when the students from each of her classes are seated?

14. Standardized Test Practice Which list shows all the factors of 98?

- A** 1, 2, 4, 7, 14, 49 **B** 1, 2, 7, 8, 14, 49
C 1, 2, 7, 18, 49, 98 **D** 1, 2, 7, 14, 49, 98

Answers: 1. composite; 3 · 5 2. composite; 3 · 3 · 3 3. prime 4. prime 5. composite; 2 · 2 · 101
 6. composite; 2 · 2 · 2 · 5 · 31 7. 2 8. 4 9. 14 10. 5a 11. $6y^2$ 12. 4c 13. 4 desks per row 14. D