

Precision and Accuracy

GOAL Analyze and compare measurements for precision and accuracy.

You ask two friends for the time. Noah says that it is about 2:30. Mia says it is 2:28 and 19 seconds. Mia gives a more *precise* measurement of the time.

Precision is the level of detail an instrument can measure. Mia's watch is more precise than Noah's. A ruler marked in millimeters is more precise than a ruler marked only in centimeters, since a millimeter is a smaller unit than a centimeter.



EXAMPLE 1 Comparing Precision

Choose the more precise measurement in each pair.

a. 5 yd; 16 ft

b. 7 cm; 7.3 cm

c. 2830 g; 2.8 kg

SOLUTION

a. A foot is a smaller unit than a yard.

16 ft is more precise than 5 yd.

b. The units are the same, but tenths are smaller than ones. (Equivalently, $7.3 \text{ cm} = 73 \text{ mm}$ and a millimeter is a smaller unit than a centimeter.)

7.3 cm is more precise than 7 cm.

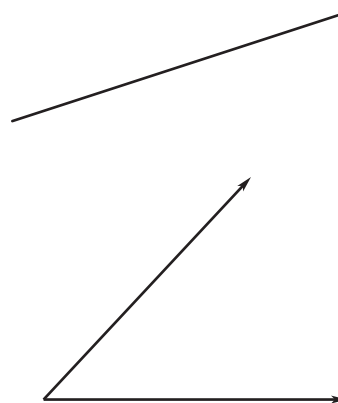
c. A gram is a smaller unit than a kilogram.

2830 g is more precise than 2.8 kg.

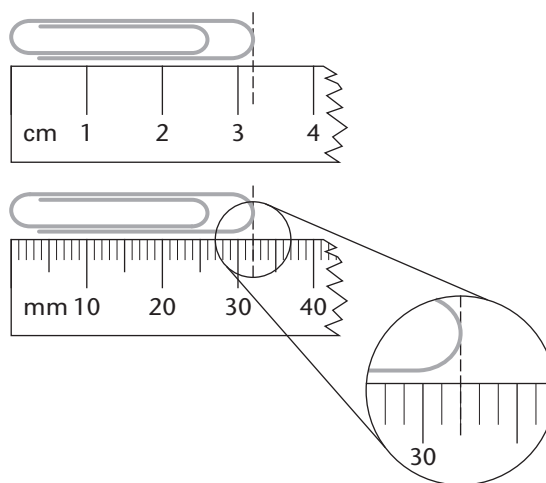
Activity

Making Precise Measurements

- Choose a measurement tool to measure the line segment and the angle. Give the measurements and describe the tools you used.
- What is the level of precision of each tool? That is, what is the smallest unit of measure on each tool?
- When you measured the line segment and angle, you used the nearest tick mark on the tool. What must be true about the actual segment length and the actual angle measure?



When you measure an object, the level of precision of the measurement tool tells you the range of possible values for the actual measurement. For example, when you measure the paper clip shown to the nearest centimeter, you would report its length as 3 cm. This means the actual length is between 2.5 cm and 3.5 cm (or $3 \text{ cm} \pm 0.5 \text{ cm}$). When you measure the paper clip to the nearest millimeter, you report its length as 32 mm. This means the actual length is between 31.5 mm and 32.5 mm (or $32 \text{ mm} \pm 0.5 \text{ mm}$).



EXAMPLE 2 Using Precision

Ann measures the length of a pencil to the nearest $\frac{1}{4}$ inch and reports its length as $6\frac{3}{4}$ inches. What is the least possible length of the pencil? What is the greatest possible length of the pencil?

SOLUTION

When you measure to the nearest $\frac{1}{4}$ inch, the actual length is within one-half of $\frac{1}{4}$ inch, or $\pm\frac{1}{8}$ inch, of the measured length.

$$6\frac{3}{4} - \frac{1}{8} = 6\frac{5}{8}$$

$$6\frac{3}{4} + \frac{1}{8} = 6\frac{7}{8}$$

The least possible length is $6\frac{5}{8}$ in. The greatest possible length is $6\frac{7}{8}$ in.

CHECK Examples 1 and 2

Choose the more precise measurement in each pair.

- 10,001 ft; 2 mi
- $5\frac{3}{8}$ in.; $5\frac{1}{2}$ in.
- 6 cm; 63 mm
4. A chemist measures the volume of acetone in a graduated cylinder to the nearest milliliter and finds that the volume is 772 mL. What is the least possible volume of the acetone? What is the greatest possible volume?

Accuracy describes how close a measurement is to the actual or accepted value. In the situation presented at the beginning of this lesson, suppose the actual time is exactly 2:34. This means that Noah's measurement is more accurate than Mia's, even though Mia's measurement is more precise.

Various factors can affect the accuracy of a measurement. For example, a measurement tool that is not calibrated properly is not likely to produce accurate measurements. Extreme heat or cold can affect the accuracy of some measurement tools. Human errors, such as misreading the markings on a ruler, also play a role in determining the accuracy of measurements.

EXAMPLE 3 Comparing Precision and Accuracy

The actual mass of a crystal is 1.8 kg. Three geologists use a scale to measure the crystal's mass. Their measurements are 1.7 kg, 1.92 kg, and 2 kg. Which measurement is most accurate? Which is most precise?

SOLUTION

Among the three measurements, 1.7 kg is closest to the actual mass of 1.8 kg.

Among the three measurements, 1.92 kg uses the smallest units (hundredths of a kilogram).

1.7 kg is the most accurate measurement; 1.92 kg is the most precise measurement.

The accuracy of a measurement can be described using *tolerance*. **Tolerance** is the amount by which a measurement may vary from a specified value. If Mia's watch is accurate and has a tolerance of ± 3 seconds, then the actual time lies between 2:28:16 and 2:28:22.

EXAMPLE 4 Comparing Tolerances

The table gives the diameters of hockey pucks produced by three different machines.

- Which machine produces hockey pucks with the least tolerance?
- A hockey puck has a diameter of 76.28 mm. Which machine produced it?

Hockey Puck Manufacturing	
Machine	Diameters (mm)
A	76 ± 0.25
B	75.9 ± 0.1
C	76.1 ± 0.2

SOLUTION

- Machine B has the least tolerance because $0.1 < 0.2 < 0.25$.
Machine B has the least tolerance.
- Write the range of diameters produced by each machine.
Machine A 75.75 mm to 76.25 mm
Machine B 75.8 mm to 76.0 mm
Machine C 75.9 mm to 76.3 mm

76.28 mm lies within the range of diameters for Machine C.

Machine C produced the hockey puck.

CHECK Examples 3 and 4

- Suppose the actual mass of the crystal in Example 3 is 1.98 kg. In this case, which of the given measurements is most accurate? Which is most precise?
- Machine D produces hockey pucks with diameters that range from 75.6 mm to 75.9 mm. What is the tolerance for this machine?

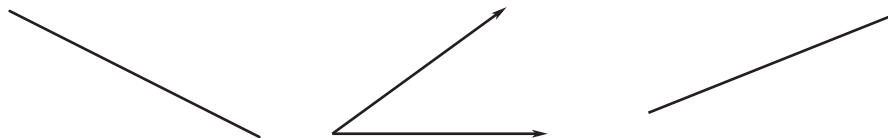
EXERCISES

Choose the more precise measurement in each pair.

1. 119 in.; 10 ft
2. 1 h; 62 min
3. 91.3 mL; 90.25 mL
4. 8 m; 823 cm
5. $12\frac{3}{8}$ mi; $12\frac{1}{4}$ mi
6. 7.3 g; 7013 mg
7. A website lists the heights of skyscrapers to the nearest foot. It gives the height of Taipei 101 as 1671 feet. What is the least possible height of the skyscraper? What is the greatest possible height of the skyscraper?
8. To the nearest $\frac{1}{8}$ oz, the weight of a package of rice is $8\frac{5}{8}$ oz. What is the least possible weight of the package? What is the greatest possible weight of the package?

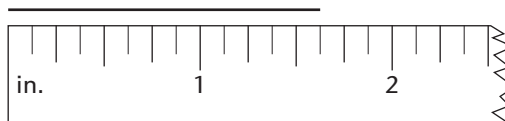
Measure each line segment or angle to the specified level of precision.

9. Nearest centimeter
10. Nearest 5°
11. Nearest $\frac{1}{8}$ inch



Use the figure to give the length of the line segment to the specified level of precision.

12. Nearest inch
13. Nearest $\frac{1}{2}$ inch
14. Nearest $\frac{1}{4}$ inch
15. Nearest $\frac{1}{8}$ inch

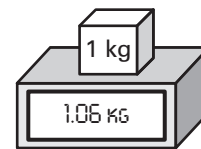


Tell what level of precision you would use in the given situation.
(Assume you are using U.S. customary units of measure.)

16. You measure a fishing rod to find out if it will fit the trunk of your car.
17. You measure a board so you can use it as a shelf in your closet.
18. You measure the height of a statue in front of your school so you can describe it in the school yearbook.
19. According to the U.S. Mint, a one-dollar coin has a mass of 8.1 grams. Justine finds the mass of a one-dollar coin and reports a mass of 8.05 g. Steven finds that the mass is 8.2 g. Whose measurement is more precise? Whose measurement is more accurate?
20. Three students are asked to measure a piece of string that has a length of exactly 15.1 cm. Their measurements are shown in the table.
 - a. Which student made the most precise measurement?
 - b. Which student made the most accurate measurement?
 - c. Which student made the least accurate measurement?

Student	Measurement
Alex	15.35 cm
Chandra	14.9 cm
Luis	154 mm

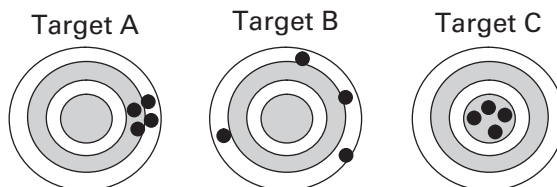
21. The figure shows the reading on a digital scale when you place a one-kilogram mass on the scale. What can you conclude about the scale?
22. Describe an example of a measurement that is precise but not accurate. Then describe an example of a measurement that is accurate but not precise.
23. Baseballs used in major league games must weigh between 5 and $5\frac{1}{4}$ ounces. Express the range of acceptable weights using a tolerance.



A popcorn manufacturer has four factories that produce bags of popcorn with the tolerances shown in the table. Use the table for Exercises 24–28.

Popcorn Packaging	
Factory	Bag weights (oz)
A	4 ± 0.2
B	3.9 ± 0.02
C	4.1 ± 0.03
D	3.9 ± 0.15

24. Which factory produces bags of popcorn with the least tolerance?
25. What is the greatest possible weight of a bag of popcorn produced at Factory C? the least possible weight?
26. A bag of popcorn weighs 3.79 ounces. Which factory produced it?
27. A bag of popcorn weighs more than 4 ounces. Which of the factories could have produced it?
28. Is it possible to say which factory produces bags of popcorn with the greatest accuracy? If so, name the factory and explain how you know. If not, describe any additional information you would need in order to answer the question.
29. Roberto claims that the mass of a fossil is $1 \text{ kg} \pm 0.1 \text{ kg}$. Lana claims that the mass of the fossil is $0.95 \text{ kg} \pm 0.05 \text{ kg}$. The actual mass is 0.98 kg. Roberto and Lana both state that their measurements are accurate. Do you agree? Why or why not?
30. The length of a photograph to the nearest inch is 5 inches. The width of the photograph to the nearest $\frac{1}{2}$ inch is $3\frac{1}{2}$ inches. What is the greatest possible area of the photograph? Explain.
31. Scientists and engineers sometimes define *precision* as follows. *Precision* is the degree to which repeated measurements show the same results. Thus, the closer repeated measurements are to each other, the more precise the measurement tool is. You can use a target to help you understand this definition of precision. The target's bull's-eye represents the accepted or true value of a measurement.



- a. Which target shows measurements that are not accurate and not very precise?
- b. Which target shows measurements that are not accurate, but very precise?
- c. Which target shows measurements that are both accurate and very precise?