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CHAPTER 2

Counting Sets of Rational Numbers

The number of objects in a set is called the cardinality of the set.

EXAMPLE 1

Find the cardinality of a set

The cardinality of the set $\{a, b, c, d, e\}$ is 5, because the set contains five letters: a, b, c, d, and e.

The cardinality of the set of positive integers less than 10 is 9, because there are nine positive integers less than 10: 1, 2, 3, 4, 5, 6, 7, 8, and 9. ■

To find the cardinality of a set, you may need to list the objects in the set as in Example 1. This is especially true when the set is expressed in set-builder notation as in Example 2.

EXAMPLE 2

Find the cardinality of a set expressed in set-builder notation

Find the cardinality of the set $\{x \mid x$ is an integer and $-6 \le x < 1\}$.

Solution:

Notice that the set includes -6 but not 1. So, the set contains the numbers -6, -5, -4, -3, -2, -1, and 0. Since there are seven numbers in the set, its cardinality is 7. ■

In the examples so far, we have been able to list the objects in the set. We can do this because the sets are small. Larger sets require different counting techniques.

EXAMPLE 3

Find the cardinality of a large set

Find the cardinality of the set $\{x \mid x$ is an integer and $-50 \le x \le 100\}$.

Solution:

Let's break the set up into smaller sets, count the smaller sets, and add the totals of the smaller sets. There 50 integers between -50 and -1, inclusive, 100 integers between 1 and 100, inclusive, and the number 0. The cardinality of the set $\{x \mid x$ is an integer and $-50 \le x \le 100\}$ is $50 + 100 + 1$ or 151. ■

Some sets have no limits or bounds; these sets are said to be **infinite**. The set of integers itself has no bounds. When we write the integers as . . . , -3, -2, -1, 0, 1, 2, 3, . . . , the three dots at each end indicate that the numbers continue without end in both directions. So, the set of integers is infinite.

The cardinality of an infinite set can not be described by any real number.

EXAMPLE 4

Identify an infinite set

Find the cardinality of the set $\{x \mid x$ is an integer and $x > 4\}$.

Solution:

This set contains the numbers 5, 6, 7, 8, 9, . . . , which continue without end. The set $\{x \mid x$ is an integer and $x > 4\}$ is infinite. ■

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Counting Sets of Rational Numbers *continued*

Other sets are infinite but bounded. A set of numbers is **bounded** when there is some number that is less than or equal to every number in the set and another number that is greater than or equal to every number in the set.

EXAMPLE 5 Recognize a bounded infinite set

The set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\right\}$ is infinite because the fractions of the form $\frac{1}{n}$ continue without end. However, all of the numbers in the set are greater than 0 and less than or equal to $\frac{1}{2}$. So, this set is *bounded below* by 0 and *bounded above* by $\frac{1}{2}$. The set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\right\}$ is a bounded infinite set. ■

Example 5 illustrates that there are infinitely many rational numbers between 0 and $\frac{1}{2}$.

EXAMPLE 6 Determine whether an infinite set is bounded or unbounded

Determine whether each of the following infinite sets is bounded or unbounded.

a. $\{x \mid x \text{ is a positive multiple of } 5\}$

b. $\{\ldots, -10, -9.5, -9, -8.5, -8\}$

c. $\{0.9, 0.99, 0.999, 0.9999, 0.99999, \ldots\}$

Solution:

a. The set of positive multiples of 5 contains 5, 10, 15, 20, 25, ... There is no real number greater than every number in this set. Since this set is not bounded above, the set is unbounded.

b. There is no number less than all of the numbers in the set $\{\ldots, -10, -9.5, -9, -8.5, -8\}$, so the set is unbounded.

c. All of the numbers in the set $\{0.9, 0.99, 0.999, 0.9999, 0.99999, \ldots\}$ are greater than 0 and less than 1, so the set is bounded. ■

Practice

Find the cardinality of the set.

1. {▲, ■, ●, ▼}

2. {f, g, k, l, t, y}

3. $\{u, v, w, x, y, z\}$

4. {F, G, H, I, J}

5. {A, B, C, . . . , X, Y, Z}

6. $\{m, n, o, \ldots, r, s, t\}$

List the numbers in the set. Then find its cardinality.

7. the set of positive integers less than 5

8. the set of positive integers less than or equal to 8

9. $\{x \mid x \text{ is an integer and } 0 < x < 15\}$

10. $\{x \mid x \text{ is an integer and } 5 \le x \le 15\}$

11. $\{x \mid x \text{ is an integer and } -9 \le x < -1\}$

12. $\{x \mid x \text{ is an integer and } -4 \le x \le 4\}$

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Find the cardinality of the set without listing the numbers in the set.

13. $\{x \mid x$ is an integer and $-10 \le x \le 10\}$

14. $\{x \mid x$ is an integer and $-100 \le x \le 100\}$

15. $\{x \mid x$ is an integer and $0 \le x < 1000\}$

16. $\{x \mid x$ is an integer and $-500 < x \le 0\}$

Find the cardinality of the set or state that the set is infinite.

17. $\{x \mid x$ is an integer and $x < 12\}$

18. $\{x \mid x$ is a positive integer and $x < 12\}$

19. $\{x \mid x$ is a negative integer and $x \ge -8\}$

20. $\{x \mid x$ is an integer and $x \ge -8\}$

Determine whether the infinite set is bounded or unbounded.

21. $\left\{1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}, 5\frac{1}{2}, \ldots\right\}$

22. $\left\{3\frac{1}{2}, 3\frac{1}{3}, 3\frac{1}{4}, 3\frac{1}{5}, 3\frac{1}{6}, \ldots\right\}$

23. $\{0.1, 0.01, 0.001, 0.0001, 0.00001, \ldots\}$

24. $\{1, 10, 100, 1000, 10{,}000\}$

Problem Solving

25. Greg has listed the names of all the students in his Algebra class on a piece of paper. Is the set of names infinite? Explain.

26. Hannah recorded the ages, in years, of five hundred people attending a concert. Find one age that could reasonably be less than or equal to all the ages in the set. Then find another age that could reasonably be greater than or equal to all the ages in the set.

Pre-AP Copymasters