2.2 Solve One-Step Equations

Before You solved equations using mental math.
Now You will solve one-step equations using algebra.
Why? So you can determine a weight limit, as in Ex. 56.

Key Vocabulary
- **inverse operations**
- **equivalent equations**
- **reciprocal**

Inverse operations are two operations that undo each other, such as addition and subtraction. When you perform the same inverse operation on each side of an equation, you produce an *equivalent equation*. **Equivalent equations** are equations that have the same solution(s).

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

KEY CONCEPT *For Your Notebook*

Addition Property of Equality

Words Adding the same number to each side of an equation produces an equivalent equation.

Algebra If $x - a = b$, then $x - a + a = b + a$, or $x = b + a$.

Subtraction Property of Equality

Words Subtracting the same number from each side of an equation produces an equivalent equation.

Algebra If $x + a = b$, then $x + a - a = b - a$, or $x = b - a$.

EXAMPLE 1 Solve an equation using subtraction

Solve $x + 7 = 4$.

$x + 7 = 4$ — Write original equation.

$x + 7 - 7 = 4 - 7$ — Use subtraction property of equality: Subtract 7 from each side.

$x = -3$ — Simplify.

▶ The solution is -3.

AVOID ERRORS To obtain an equivalent equation, be sure to subtract the same number from each side.

CHECK Substitute -3 for x in the original equation.

$x + 7 = 4$ — Write original equation.

$-3 + 7 \stackrel{?}{=} 4$ — Substitute -3 for x.

$4 = 4$ ✓ — Simplify. Solution checks.

EXAMPLE 2 Solve an equation using addition

USE HORIZONTAL FORMAT
In Example 2, both horizontal and vertical formats are used. In the rest of the book, equations will be solved using the horizontal format.

Solve $x - 12 = 3$.

Horizontal format		Vertical format
$x - 12 = 3$	Write original equation.	$x - 12 = 3$
$x - 12 + 12 = 3 + 12$	Add 12 to each side.	$+ 12 \quad + 12$
$x = 15$	Simplify.	$x = 15$

MULTIPLICATION AND DIVISION EQUATIONS Multiplication and division are inverse operations. So, the multiplication property of equality can be used to solve equations involving division, and the division property of equality can be used to solve equations involving multiplication.

KEY CONCEPT *For Your Notebook*

Multiplication Property of Equality

Words Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $\frac{x}{a} = b$ and $a \neq 0$, then $\boldsymbol{a} \cdot \frac{x}{a} = \boldsymbol{a} \cdot b$, or $x = ab$.

Division Property of Equality

Words Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $ax = b$ and $a \neq 0$, then $\frac{ax}{\boldsymbol{a}} = \frac{b}{\boldsymbol{a}}$, or $x = \frac{b}{a}$.

EXAMPLE 3 Solve an equation using division

Solve $-6x = 48$.

$-6x = 48$	Write original equation.
$\frac{-6x}{\boldsymbol{-6}} = \frac{48}{\boldsymbol{-6}}$	Divide each side by -6.
$x = -8$	Simplify.

GUIDED PRACTICE for Examples 1, 2, and 3

Solve the equation. Check your solution.

1. $y + 7 = 10$ **2.** $x - 5 = 3$ **3.** $q - 11 = -5$ **4.** $6 = t - 2$

5. $4x = 48$ **6.** $-65 = -5y$ **7.** $6w = -54$ **8.** $24 = -8n$

EXAMPLE 4 Solve an equation using multiplication

Solve $\frac{x}{4} = 5$.

Solution

$\frac{x}{4} = 5$ — Write original equation.

$4 \cdot \frac{x}{4} = 4 \cdot 5$ — Multiply each side by 4.

$x = 20$ — Simplify.

GUIDED PRACTICE for Example 4

Solve the equation. Check your solution.

9. $\frac{t}{-3} = 9$ **10.** $6 = \frac{c}{7}$ **11.** $13 = \frac{z}{-2}$ **12.** $\frac{a}{5} = -11$

USING RECIPROCALS Recall that the product of a number and its reciprocal is 1. You can isolate a variable with a fractional coefficient by multiplying each side of the equation by the reciprocal of the fraction.

EXAMPLE 5 Solve an equation by multiplying by a reciprocal

Solve $-\frac{2}{7}x = 4$.

REVIEW RECIPROCALS
For help with finding reciprocals, see p. SR7.

Solution

The coefficient of x is $-\frac{2}{7}$. The reciprocal of $-\frac{2}{7}$ is $-\frac{7}{2}$.

$-\frac{2}{7}x = 4$ — Write original equation.

$-\frac{7}{2}\left(-\frac{2}{7}x\right) = -\frac{7}{2}(4)$ — Multiply each side by the reciprocal, $-\frac{7}{2}$.

$x = -14$ — Simplify.

▶ The solution is -14. Check by substituting -14 for x in the original equation.

CHECK

$-\frac{2}{7}x = 4$ — Write original equation.

$-\frac{2}{7}(-14) \stackrel{?}{=} 4$ — Substitute -14 for x.

$4 = 4$ ✓ — Simplify. Solution checks.

GUIDED PRACTICE for Example 5

Solve the equation. Check your solution.

13. $\frac{5}{6}w = 10$ **14.** $\frac{2}{3}p = 14$ **15.** $9 = -\frac{3}{4}m$ **16.** $-8 = -\frac{4}{5}v$

EXAMPLE 6 Write and solve an equation

OLYMPICS In the 2004 Olympics, Shawn Crawford won the 200 meter dash. His winning time was 19.79 seconds. Find his average speed to the nearest tenth of a meter per second.

Solution

Let r represent Crawford's speed in meters per second. Write a verbal model. Then write and solve an equation.

Distance (meters)	=	Rate (meters/second)	·	Time (seconds)
↓		↓		↓
200	=	r	·	**19.79**

$$\frac{200}{19.79} = \frac{19.79r}{19.79}$$

$$10.1 \approx r$$

▶ Crawford's average speed was about 10.1 meters per second.

GUIDED PRACTICE for Example 6

17. **WHAT IF?** In Example 6, suppose Shawn Crawford ran 100 meters at the same average speed he ran the 200 meters. How long would it take him to run 100 meters? Round your answer to the nearest tenth of a second.

2.2 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 13 and 55
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 57, 58, and 61
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 59

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Two operations that undo each other are called __?__.

2. ★ **WRITING** Which property of equality would you use to solve the equation $14x = 35$? *Explain*.

EXAMPLES 1 and 2 for Exs. 3–14

SOLVING ADDITION AND SUBTRACTION EQUATIONS **Solve the equation. Check your solution.**

3. $x + 5 = 8$	**4.** $m + 9 = 2$	**5.** $11 = f + 6$	**6.** $13 = 7 + z$
7. $6 = 9 + h$	**8.** $-3 = 5 + a$	**9.** $y - 4 = 3$	**10.** $t - 5 = 7$
11. $14 = k - 3$	**12.** $6 = w - 7$	**13.** $-2 = n - 6$	**14.** $-11 = b - 9$

EXAMPLES 1 and 2 for Exs. 15, 16

15. ★ MULTIPLE CHOICE What is the solution of $-8 = d - 13$?

Ⓐ -21 Ⓑ -5 Ⓒ 5 Ⓓ 21

16. ★ MULTIPLE CHOICE What is the solution of $22 + v = -65$?

Ⓐ -87 Ⓑ -43 Ⓒ 43 Ⓓ 87

EXAMPLES 3 and 4 for Exs. 17–30

SOLVING MULTIPLICATION AND DIVISION EQUATIONS **Solve the equation. Check your solution.**

17. $5g = 20$ **18.** $-4q = 52$ **19.** $48 = 8c$

20. $-108 = 9j$ **21.** $15 = -h$ **22.** $187 = -17r$

23. $\frac{y}{3} = 5$ **24.** $\frac{m}{2} = 14$ **25.** $8 = \frac{x}{6}$

26. $7 = \frac{t}{-7}$ **27.** $-11 = \frac{z}{-2}$ **28.** $-3 = \frac{d}{14}$

In Exercises 29 and 30, refer to the method shown, which a student used to write a repeating decimal as a fraction.

29. *Explain* the student's method.

30. Write the repeating decimal as a fraction.

a. $0.\overline{7}$ **b.** $0.\overline{18}$

Let $x = 0.\overline{63}$. Then $100x = 63.\overline{63}$.

Subtract: $100x = 63.636363\ldots$

$-x = -0.636363\ldots$

$99x = 63$

$x = \frac{63}{99} = \frac{7}{11}$

SOLVING EQUATIONS **Solve the equation. Check your solution.**

31. $b - 0.4 = 3.1$ **32.** $-3.2 + z = -7.4$ **33.** $-5.7 = w - 4.6$

34. $-6.1 = p + 2.2$ **35.** $8.2 = -4g$ **36.** $-3.3a = 19.8$

37. $\frac{3}{4} = \frac{1}{8} + v$ **38.** $\frac{n}{4.6} = -2.5$ **39.** $-0.12 = \frac{y}{-0.5}$

EXAMPLE 5 for Exs. 40–48

40. $\frac{1}{2}m = 21$ **41.** $\frac{1}{3}c = 32$ **42.** $-7 = \frac{1}{5}x$

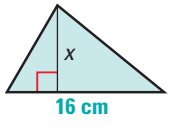
43. $\frac{3}{2}k = 18$ **44.** $-21 = -\frac{3}{5}t$ **45.** $-\frac{2}{7}v = 16$

46. $\frac{8}{5}x = \frac{4}{15}$ **47.** $\frac{1}{3}y = \frac{1}{5}$ **48.** $-\frac{4}{3} = \frac{2}{3}z$

GEOMETRY **The rectangle or triangle has area A. Write and solve an equation to find the value of x.**

49. $A = 54$ in.^{2}

50. $A = 72$ cm^2



CHALLENGE **Find the value of b using the given information.**

51. $4a = 6$ and $b = a - 2$

52. $a - 6.7 = 3.1$ and $b = 5a$

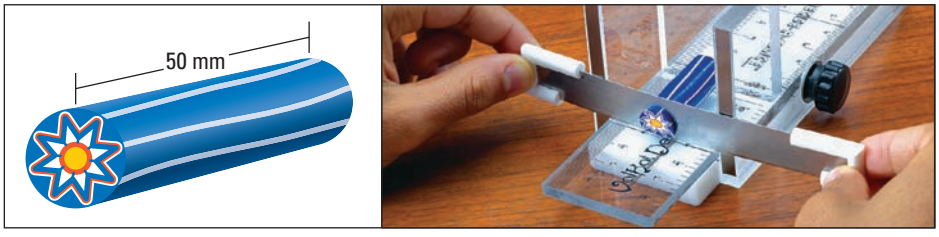
◯ = See WORKED-OUT SOLUTIONS in Student Resources ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

PROBLEM SOLVING

EXAMPLE 6 for Exs. 53–57

53. THE DEAD SEA For the period 1999–2004, the maximum depth of the Dead Sea decreased by 9.9 feet. The maximum depth in 2004 was 1036.7 feet. What was the maximum depth in 1999?

54. CRAFTS You purchase a cane of polymer clay to make pendants for necklaces. The cane is 50 millimeters long. How thick should you make each pendant so that you will have 20 pendants of uniform thickness?



55. TRAMPOLINES A rectangular trampoline has an area of 187 square feet. The length of the trampoline is 17 feet. What is its width?

56. WHEELCHAIRS The van used to transport patients to and from a rehabilitation facility is equipped with a wheelchair lift. The maximum lifting capacity for the lift is 300 pounds. The wheelchairs used by the facility weigh 55 pounds each. What is the maximum weight of a wheelchair occupant who can use the lift?

57. ★ SHORT RESPONSE In Everglades National Park in Florida, there are 200 species of birds that migrate. This accounts for $\frac{4}{7}$ of all the species of birds sighted in the park.

a. Write an equation to find the number of species of birds that have been sighted in Everglades National Park.

b. There are 600 species of plants in Everglades National Park. Are there more species of birds or of plants in the park? *Explain*.

58. ★ OPEN-ENDED *Describe* a real-world situation that can be modeled by the equation $15x = 135$. Solve the equation and explain what the solution means in this situation.

59. MULTIPLE REPRESENTATIONS A box jellyfish can travel at a rate of 6.5 feet per second.

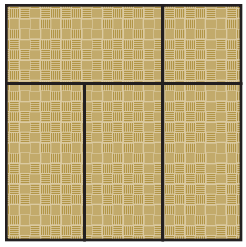
a. Making a Table Make a table that shows the distance d the jellyfish can travel after 1, 2, 3, 4, and 5 seconds.

b. Drawing a Graph Graph the ordered pairs from the table in a coordinate plane. How long does it take the jellyfish to travel 26 feet?

c. Writing an Equation Write and solve an equation to find the time it takes the jellyfish to travel 26 feet.

Animated Algebra at my.hrw.com

60. MULTI-STEP PROBLEM Tatami mats are a floor covering used in Japan. Tatami mats are equal in size, unless they are cut in half. The floor shown has an area of 81 square feet and is covered with 4.5 tatami mats.



a. What is the area of one tatami mat?

b. What is the length of one tatami mat if it has a width of 3 feet?

61. ★ EXTENDED RESPONSE In baseball, a player's batting average is calculated by dividing the number of hits by the number of at bats.

a. Calculate Use the information in the table to find the number of hits Bill Mueller had in the 2003 Major League Baseball regular season. Round your answer to the nearest whole number.

Player	Team	Batting average	At bats
Bill Mueller	Boston Red Sox	0.326	524

b. Calculate The number of hits Bill Mueller had was 44 less than the number of hits Vernon Wells of the Toronto Blue Jays had in the 2003 regular season. How many hits did Vernon Wells have?

c. Compare In the 2003 regular season, Mueller had a higher batting average than Wells. Did Wells have fewer at bats than Mueller? *Explain* your reasoning.

62. AMERICAN FLAGS An American flag has a length that is 1.9 times its width. What is the area of a flag that has a length of 9.5 feet?

63. CHALLENGE At a farm where you can pick your own strawberries, the cost of picked strawberries is calculated using only the weight of the strawberries. The total weight of a container full of strawberries is 2.1 pounds. The cost of the strawberries is $4.68. The weight of the container is 0.3 pound. What is the cost per pound for strawberries?

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com