

2.3 Solve Two-Step Equations



Before

You solved one-step equations.

Now

You will solve two-step equations.

Why?

So you can find a scuba diver's depth, as in Example 4.

Key Vocabulary

- like terms
- input
- output

The equation $\frac{x}{2} + 5 = 11$ involves two operations performed on x : division by 2 and addition by 5. You typically solve such an equation by applying the inverse operations in the reverse order of the order of operations. This is shown in the table below.



CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Operations performed on x	Operations to isolate x
1. Divide by 2. 2. Add 5.	1. Subtract 5. 2. Multiply by 2.

EXAMPLE 1 Solve a two-step equation

Solve $\frac{x}{2} + 5 = 11$.

$$\frac{x}{2} + 5 = 11 \quad \text{Write original equation.}$$

$$\frac{x}{2} + 5 - 5 = 11 - 5 \quad \text{Subtract 5 from each side.}$$

$$\frac{x}{2} = 6 \quad \text{Simplify.}$$

$$2 \cdot \frac{x}{2} = 2 \cdot 6 \quad \text{Multiply each side by 2.}$$

$$x = 12 \quad \text{Simplify.}$$

► The solution is 12. Check by substituting 12 for x in the original equation.

$$\text{CHECK } \frac{x}{2} + 5 = 11 \quad \text{Write original equation.}$$

$$\frac{12}{2} + 5 \stackrel{?}{=} 11 \quad \text{Substitute 12 for } x.$$

$$11 = 11 \checkmark \quad \text{Simplify. Solution checks.}$$



GUIDED PRACTICE for Example 1

Solve the equation. Check your solution.

1. $5x + 9 = 24$

2. $4y - 4 = 16$

3. $-1 = \frac{z}{3} - 7$

EXAMPLE 2 Solve a two-step equation by combining like terms**REVIEW
LIKE TERMS**

You may want to review combining like terms before solving two-step equations.

Solve $7x - 4x = 21$.

$$7x - 4x = 21 \quad \text{Write original equation.}$$

$$3x = 21 \quad \text{Combine like terms.}$$

$$\frac{3x}{3} = \frac{21}{3} \quad \text{Divide each side by 3.}$$

$$x = 7 \quad \text{Simplify.}$$

EXAMPLE 3 Find an input of a function

The output of a function is 3 less than 5 times the input. Find the input when the output is 17.

Solution

STEP 1 Write an equation for the function. Let x be the input and y be the output.

$$y = 5x - 3 \quad \text{y is 3 less than 5 times x.}$$

STEP 2 Solve the equation for x when $y = 17$.

$$y = 5x - 3 \quad \text{Write original function.}$$

$$17 = 5x - 3 \quad \text{Substitute 17 for y.}$$

$$17 + 3 = 5x - 3 + 3 \quad \text{Add 3 to each side.}$$

$$20 = 5x \quad \text{Simplify.}$$

$$\frac{20}{5} = \frac{5x}{5} \quad \text{Divide each side by 5.}$$

$$4 = x \quad \text{Simplify.}$$

► An input of 4 produces an output of 17.

$$\text{CHECK} \quad y = 5x - 3 \quad \text{Write original function.}$$

$$17 \stackrel{?}{=} 5(4) - 3 \quad \text{Substitute 17 for y and 4 for x.}$$

$$17 \stackrel{?}{=} 20 - 3 \quad \text{Multiply 5 and 4.}$$

$$17 = 17 \checkmark \quad \text{Simplify. Solution checks.}$$

**GUIDED PRACTICE** for Examples 2 and 3

Solve the equation. Check your solution.

4. $4w + 2w = 24$

5. $8t - 3t = 35$

6. $-16 = 5d - 9d$

7. The output of a function is 5 more than -2 times the input. Find the input when the output is 11.

8. The output of a function is 4 less than 4 times the input. Find the input when the output is 3.

EXAMPLE 4 Solve a multi-step problem

SCUBA DIVING As a scuba diver descends into deeper water, the pressure of the water on the diver's body steadily increases.

The pressure at the surface of the water is 2117 pounds per square foot (lb/ft^2). The pressure increases at a rate of 64 pounds per square foot for each foot the diver descends. Find the depth at which a diver experiences a pressure of 8517 pounds per square foot.



ANOTHER WAY

For an alternative method for solving Example 4, see the **Problem Solving Workshop**.

Solution

STEP 1 Write a verbal model. Then write an equation.

Pressure at given depth (lb/ft^2)	=	Pressure at surface (lb/ft^2)	+	Rate of change of pressure (lb/ft^2 per foot of depth)	·	Diver's depth (ft)
↓		↓		↓		↓
P	=	2117	+	64	·	d

STEP 2 Find the depth at which the pressure is 8517 pounds per square foot.

$$P = 2117 + 64d$$

Write equation.

$$8517 = 2117 + 64d$$

Substitute 8517 for P .

$$8517 - 2117 = 2117 - 2117 + 64d$$

Subtract 2117 from each side.

$$6400 = 64d$$

Simplify.

$$\frac{6400}{64} = \frac{64d}{64}$$

Divide each side by 64.

$$100 = d$$

Simplify.

▶ A diver experiences a pressure of 8517 pounds per square foot at a depth of 100 feet.

CHECK $P = 2117 + 64d$

Write original equation.

$$8517 \stackrel{?}{=} 2117 + 64(100)$$

Substitute 8517 for P and 100 for d .

$$8517 \stackrel{?}{=} 2117 + 6400$$

Multiply 64 and 100.

$$8517 = 8517 \checkmark$$

Simplify. Solution checks.



GUIDED PRACTICE for Example 4

9. **WHAT IF?** In Example 4, suppose the diver experiences a pressure of 5317 pounds per square foot. Find the diver's depth.
10. **JOBS** Kim has a job where she makes \$8 per hour plus tips. Yesterday, Kim made \$53 dollars, \$13 of which was from tips. How many hours did she work?

2.3 EXERCISES

HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS**
Exs. 13, 19, and 39
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 21, 40, 41, and 44
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 43

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To solve the equation $2x + 3x = 20$, you would begin by combining $2x$ and $3x$ because they are ?.
2. ★ **WRITING** Describe the steps you would use to solve the equation $4x + 7 = 15$.

EXAMPLE 1

for Exs. 3–14

SOLVING TWO-STEP EQUATIONS Solve the equation. Check your solution.

- | | | |
|----------------------------|--|-----------------------------|
| 3. $3x + 7 = 19$ | 4. $5h + 4 = 19$ | 5. $7d - 1 = 13$ |
| 6. $2g - 13 = 3$ | 7. $10 = 7 - m$ | 8. $11 = 12 - q$ |
| 9. $\frac{a}{3} + 4 = 6$ | 10. $17 = \frac{w}{5} + 13$ | 11. $\frac{b}{2} - 9 = 11$ |
| 12. $-6 = \frac{z}{4} - 3$ | 13. $7 = \frac{5}{6}c - 8$ | 14. $10 = \frac{2}{7}n + 4$ |

EXAMPLE 2

for Exs. 15–23

COMBINING LIKE TERMS Solve the equation. Check your solution.

- | | | |
|--------------------|---|---------------------|
| 15. $8y + 3y = 44$ | 16. $2p + 7p = 54$ | 17. $11x - 9x = 18$ |
| 18. $36 = 9x - 3x$ | 19. $-32 = -5k + 13k$ | 20. $6 = -7f + 4f$ |

21. ★ **MULTIPLE CHOICE** What is the first step you can take to solve the equation $6 + \frac{x}{3} = -2$?

A Subtract 2 from each side.	B Add 6 to each side.
C Divide each side by 3.	D Subtract 6 from each side.

ERROR ANALYSIS Describe and correct the error in solving the equation.

22.

$$\begin{aligned} 7 - 3x &= 12 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$



23.

$$\begin{aligned} -2x + x &= 10 \\ \frac{-2x + x}{-2} &= \frac{10}{-2} \\ x &= -5 \end{aligned}$$



EXAMPLE 3

for Exs. 24–26

FINDING AN INPUT OF A FUNCTION Write an equation for the function described. Then find the input.

24. The output of a function is 7 more than 3 times the input. Find the input when the output is -8 .
25. The output of a function is 4 more than 2 times the input. Find the input when the output is -10 .
26. The output of a function is 9 less than 10 times the input. Find the input when the output is 11.

SOLVING EQUATIONS Solve the equation. Check your solution.

27. $5.6 = 1.1p + 1.2$ 28. $7.2y + 4.7 = 62.3$ 29. $1.2j - 4.3 = 1.7$
 30. $16 - 2.4d = -8$ 31. $14.4m - 5.1 = 2.1$ 32. $-5.3 = 2.2v - 8.6$
 33. $\frac{c}{5.3} + 8.3 = 11.3$ 34. $3.2 + \frac{x}{2.5} = 4.6$ 35. $-1.2 = \frac{z}{4.6} - 2.7$
36. **CHALLENGE** Solve the equations $3x + 2 = 5$, $3x + 2 = 8$, and $3x + 2 = 11$. Predict the solution of the equation $3x + 2 = 14$. *Explain.*

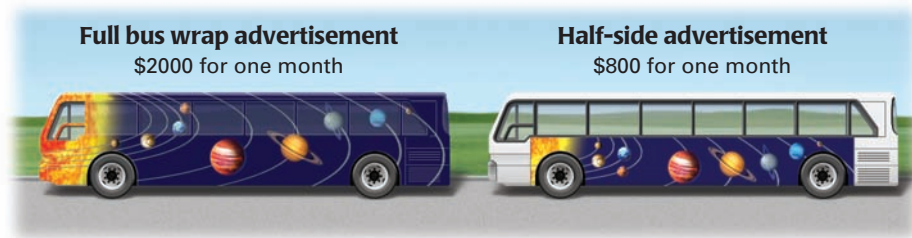
PROBLEM SOLVING

EXAMPLE 4
 for Exs. 37–40

37. **DANCE CLASSES** A dance academy charges \$24 per class and a one-time registration fee of \$15. A student paid a total of \$687 to the academy. Find the number of classes the student took.
38. **CAR REPAIR** Tyler paid \$124 to get his car repaired. The total cost for the repairs was the sum of the amount paid for parts and the amount paid for labor. Tyler was charged \$76 for parts and \$32 per hour for labor. Find the amount of time it took to repair his car.



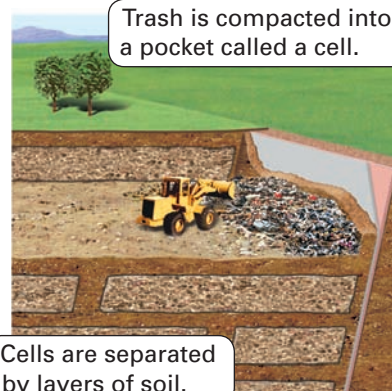
39. **ADVERTISING** A science museum wants to promote an upcoming exhibit by advertising on city buses for one month. The costs of the two types of advertisements being considered are shown. The museum has budgeted \$6000 for the advertisements. The museum decides to have 1 full bus wrap advertisement. How many half-side advertisements can the museum have?



40. **★ MULTIPLE CHOICE** A skateboarding park charges \$7 per session to skate and \$4 per session to rent safety equipment. Jared rents safety equipment every time he skates. During one year, he spends \$99 for skating charges and equipment rentals. Which equation can be used to find x , the number of sessions Jared attended?
- (A) $99 = 7x$ (B) $99 = 7x + 4x$ (C) $99 = 7x + 4$ (D) $99 = 4x + 7$
41. **★ SHORT RESPONSE** A guitar store offers a finance plan where you give a \$50 down payment on a guitar and pay the remaining balance in 6 equal monthly payments. You have \$50 and you can afford to pay up to \$90 per month for a guitar. Can you afford a guitar that costs \$542? *Explain.*

42. **MULTI-STEP PROBLEM** The capacity of a landfill is 4,756,505 tons. The landfill currently holds 2,896,112 tons. A cell is added to the landfill every day, and each cell averages 1600 tons.

- Write an equation that gives the amount y (in tons) in the landfill as a function of the number x of days from now.
- After how many days will the landfill reach capacity? Round your answer to the nearest day.
- Use estimation to check your answer to part (b).



43. **MULTIPLE REPRESENTATIONS** Two computer technicians are upgrading the software on the 54 computers in a school. On average, Marissa upgrades 5 computers in 1 hour and Ryan upgrades 7 computers in 1 hour.

- Writing an Equation** Write an equation that gives the total number y of computers upgraded as a function of the number x of hours worked.
- Making a Table** Make a table that shows the number of computers upgraded by each technician and the total number of computers upgraded after 1, 2, 3, 4, and 5 hours.
- Drawing a Graph** Graph the ordered pairs that represent the total number y of computers upgraded after x hours. Use the graph to estimate the number of hours it took to upgrade all of the computers.

44. **★ SHORT RESPONSE** At a restaurant, customers can dine inside the restaurant or pick up food at the take-out window. On an average day, 400 customers are served inside the restaurant, and 120 customers pick up food at the take-out window. After how many days will the restaurant have served 2600 customers? *Explain.*

45. **CHALLENGE** During a 1 mile race, one runner is running at a rate of 14.6 feet per second, and another runner is running at a rate of 11.3 feet per second. One lap around the track is 660 feet. After how many seconds will the faster runner be exactly one lap ahead of the other runner?

QUIZ

Solve the equation. Check your solution.

- Tell whether each of the following numbers is a real number, a rational number, an irrational number, an integer, or a whole number: -3 , $-\sqrt{5}$, -3.7 , $\sqrt{3}$. Then order the numbers from least to greatest.
- Rewrite the following conditional statement in if-then form: "No irrational numbers are negative numbers." Tell whether the statement is *true* or *false*. If it is false, give a counterexample.
- $-7b = -56$
- $\frac{z}{4} = 6$
- $-\frac{4}{3}t = -12$
- $9w - 4 = 14$
- $23 = 1 - d$
- $66 = 4m + 7m$

Another Way to Solve Example 4



Make sense of problems and persevere in solving them.

MULTIPLE REPRESENTATIONS In Example 4, you saw how to solve a problem about scuba diving by using an equation. You can also solve the problem using a table.

PROBLEM

SCUBA DIVING As a scuba diver descends into deeper water, the pressure of the water on the diver's body steadily increases. The pressure at the surface of the water is 2117 pounds per square foot (lb/ft^2). The pressure increases at a rate of 64 pounds per square foot for each foot the diver descends. Find the depth at which a diver experiences a pressure of 8517 pounds per square foot.

METHOD

Making a Table An alternative approach is to make a table.

STEP 1 **Make** a table that shows the pressure as the depth increases. Because you are looking for a fairly high pressure, use larger increments in depth, such as 20 feet.

Every 1 ft of depth increases the pressure by $64 \text{ lb}/\text{ft}^2$.

Every 20 ft of depth increases the pressure by $64(20) = 1280 \text{ lb}/\text{ft}^2$.

Depth (ft)	Pressure (lb/ft^2)
0	2117
1	2181
2	2245
~~~~~	
20	3397
40	4677
60	5957
80	7237
100	8517

**STEP 2** **Look** for the depth at which the pressure reaches 8517 pounds per square foot. This happens at a depth of 100 feet.

**PRACTICE**

- BASKETBALL** A sports club offers an organized basketball league. A team pays \$600 to join the league. In addition to paying their share of the \$600, team members who are not members of the sports club must pay a \$25 fee to play. A team pays a total of \$775. How many team members who are not club members are on the team? Solve this problem using two different methods.
- WHAT IF?** In Exercise 1, suppose you are on a team, but not a club member. The \$600 cost is divided equally among the team members. How many players must there be on your team for you to pay \$100 to play? Make a table to find the answer.
- FURNITURE** You have \$370 to spend on a dining table and chairs. A table costs \$220, and each chair costs \$35. How many chairs can you buy in addition to the table? Solve this problem using two different methods.