

2.4 Solve Multi-Step Equations



Before

You solved one-step and two-step equations.

Now

You will solve multi-step equations.

Why?

So you can solve a problem about lifeguarding, as in Ex. 40.

Key Vocabulary

- like terms
- distributive property
- reciprocal

Solving a linear equation may take more than two steps. Start by simplifying one or both sides of the equation, if possible. Then use inverse operations to isolate the variable.

EXAMPLE 1 Solve an equation by combining like terms

Solve $8x - 3x - 10 = 20$.

$$8x - 3x - 10 = 20$$

Write original equation.

$$5x - 10 = 20$$

Combine like terms.

$$5x - 10 + 10 = 20 + 10$$

Add 10 to each side.

$$5x = 30$$

Simplify.

$$\frac{5x}{5} = \frac{30}{5}$$

Divide each side by 5.

$$x = 6$$

Simplify.

COMMON CORE

CC.9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

EXAMPLE 2 Solve an equation using the distributive property

Solve $7x + 2(x + 6) = 39$.

Solution

When solving an equation, you may feel comfortable doing some steps mentally. Method 2 shows a solution where some steps are done mentally.

METHOD 1 Show All Steps

$$7x + 2(x + 6) = 39$$

$$7x + 2x + 12 = 39$$

$$9x + 12 = 39$$

$$9x + 12 - 12 = 39 - 12$$

$$9x = 27$$

$$\frac{9x}{9} = \frac{27}{9}$$

$$x = 3$$

METHOD 2 Do Some Steps Mentally

$$7x + 2(x + 6) = 39$$

$$7x + 2x + 12 = 39$$

$$9x + 12 = 39$$

$$9x = 27$$

$$x = 3$$

REVIEW PROPERTIES

You may want to review the distributive property before studying these methods.

**EXAMPLE 3** Standardized Test Practice

Which equation represents Step 2 in the solution process?

Step 1 $5x - 4(x - 3) = 17$

Step 2

Step 3 $x + 12 = 17$

Step 4 $x = 5$

ELIMINATE CHOICES

You can eliminate choices B and C because -4 has not been distributed to *both* terms in the parentheses.

(A) $5x - 4x - 12 = 17$

(B) $5x - 4x - 3 = 17$

(C) $5x - 4x + 3 = 17$

(D) $5x - 4x + 12 = 17$

Solution

In Step 2, the distributive property is used to simplify the left side of the equation. Because $-4(x - 3) = -4x + 12$, Step 2 should be $5x - 4x + 12 = 17$.

▶ The correct answer is D. (A) (B) (C) (D)

**GUIDED PRACTICE** for Examples 1, 2, and 3

Solve the equation. Check your solution.

1. $9d - 2d + 4 = 32$

2. $2w + 3(w + 4) = 27$

3. $6x - 2(x - 5) = 46$

USING RECIPROCAL Although you can use the distributive property to solve an equation such as $\frac{3}{2}(3x + 5) = -24$, it is easier to multiply each side of the equation by the reciprocal of the fraction.

EXAMPLE 4 Multiply by a reciprocal to solve an equation

Solve $\frac{3}{2}(3x + 5) = -24$.

$$\frac{3}{2}(3x + 5) = -24$$

Write original equation.

$$\frac{2}{3} \cdot \frac{3}{2}(3x + 5) = \frac{2}{3}(-24)$$

Multiply each side by $\frac{2}{3}$, the reciprocal of $\frac{3}{2}$.

$$3x + 5 = -16$$

Simplify.

$$3x = -21$$

Subtract 5 from each side.

$$x = -7$$

Divide each side by 3.

**GUIDED PRACTICE** for Example 4

Solve the equation. Check your solution.

4. $\frac{3}{4}(z - 6) = 12$

5. $\frac{2}{5}(3r + 4) = 10$

6. $-\frac{4}{5}(4a - 1) = 28$

EXAMPLE 5 Write and solve an equation

SUMMER CAMP You are planning a scavenger hunt for 21 campers. You plan to have 5 teams. One camper from each team will be the recorder and the rest will be searchers. How many searchers will each team have?

Solution

Let s be the number of searchers on each team. Then $1 + s$ is the total number of campers on each team.

Number of campers	=	Number of teams	·	Number of campers on each team	
↓		↓		↓	
21	=	5	·	$(1 + s)$	
		$21 = 5(1 + s)$		Write equation.	
		$21 = 5 + 5s$		Distributive property	
		$16 = 5s$		Subtract 5 from each side.	
		$3.2 = s$		Divide each side by 5.	

CHECK REASONABLENESS

The number of searchers must be a whole number.

▶ Because 4 searchers per team would require a total of $5(1 + 4) = 25$ campers, 4 teams will have 3 searchers and 1 team will have 4 searchers.



GUIDED PRACTICE for Example 5

7. **WHAT IF?** In Example 5, suppose you decide to use only 4 teams. How many searchers should there be on each team?

2.4 EXERCISES

HOMEWORK KEY

- = See **WORKED-OUT SOLUTIONS**
Exs. 17 and 39
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 18, 35, 36, and 41
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 42

SKILL PRACTICE

1. **VOCABULARY** What is the reciprocal of the fraction in the equation

$$\frac{3}{5}(2x + 8) = 18?$$

2. ★ **WRITING** Describe the steps you would use to solve the equation

$$3(4y - 7) = 6.$$

COMBINING LIKE TERMS Solve the equation. Check your solution.

3. $p + 2p - 3 = 6$

4. $12v + 14 + 10v = 80$

5. $11w - 9 - 7w = 15$

6. $5a + 3 - 3a = -7$

7. $6c - 8 - 2c = -16$

8. $9 = 7z - 13z - 21$

9. $-2 = 3y - 18 - 5y$

10. $23 = -4m + 2 + m$

11. $35 = -5 + 2x - 7x$

EXAMPLE 1
for Exs. 3–11

EXAMPLES
2 and 3

for Exs. 12–18, 25

USING THE DISTRIBUTIVE PROPERTY Solve the equation. Check your solution.

12. $3 + 4(z + 5) = 31$ 13. $14 + 2(4g - 3) = 40$ 14. $5m + 2(m + 1) = 23$
15. $5h + 2(11 - h) = -5$ 16. $27 = 3c - 3(6 - 2c)$ 17. $-3 = 12y - 5(2y - 7)$
18. **★ MULTIPLE CHOICE** What is the solution of $7v - (6 - 2v) = 12$?
Ⓐ -3.6 Ⓑ -2 Ⓒ 2 Ⓓ 3.6

EXAMPLE 4

for Exs. 19–24,
26

MULTIPLYING BY A RECIPROCAL Solve the equation. Check your solution.

19. $\frac{1}{3}(d + 3) = 5$ 20. $\frac{3}{2}(x - 5) = -6$ 21. $\frac{4}{3}(7 - n) = 12$
22. $4 = \frac{2}{9}(4y - 2)$ 23. $-32 = \frac{8}{7}(3w - 1)$ 24. $-14 = \frac{2}{5}(9 - 2b)$

ERROR ANALYSIS Describe and correct the error in solving the equation.

25.

$$5x - 3(x - 6) = 2$$

$$5x - 3x - 18 = 2$$

$$2x - 18 = 2$$

$$2x = 20$$

$$x = 10$$

26.

$$\frac{1}{2}(2x - 10) = 4$$

$$2x - 10 = 2$$

$$2x = 12$$

$$x = 6$$

SOLVING EQUATIONS Solve the equation. Check your solution.

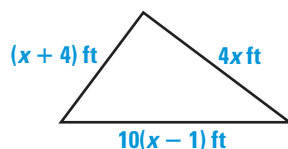
27. $8.9 + 1.2(3a - 1) = 14.9$ 28. $-11.2 + 4(2.1 + q) = -0.8$
29. $1.3t + 3(t + 8.2) = 37.5$ 30. $1.6 = 7.6 - 5(k + 1.1)$
31. $0.5 = 4.1x - 2(1.3x - 4)$ 32. $8.7 = 3.5m - 2.5(5.4 - 6m)$

REVIEW
CONVERTING
UNITS

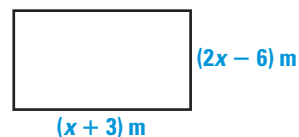
For help with converting units of measurement, see SR19.

GEOMETRY Find the value of x for the triangle or rectangle. Be sure to use the same units for the side lengths and the perimeters.

33. Perimeter = 288 inches



34. Perimeter = 2600 centimeters



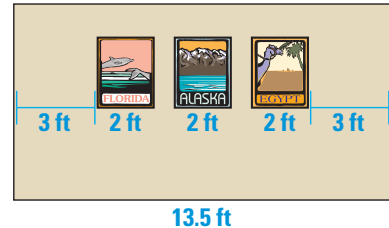
35. **★ WRITING** The length of a rectangle is 3.5 inches more than its width. The perimeter of the rectangle is 31 inches. Find the length and the width of the rectangle. *Explain* your reasoning.
36. **★ SHORT RESPONSE** Solve each equation by first dividing each side of the equation by the number outside the parentheses. When would you recommend using this method to solve an equation? *Explain*.
a. $9(x - 4) = 72$ b. $8(x + 5) = 60$
37. **CHALLENGE** An even integer can be represented by the expression $2n$. Find three consecutive even integers that have a sum of 54.

PROBLEM SOLVING

EXAMPLE 5
for Exs. 38–40

38. **BASKETBALL** A ticket agency sells tickets to a professional basketball game. The agency charges \$32.50 for each ticket, a convenience charge of \$3.30 for each ticket, and a processing fee of \$5.90 for the entire order. The total charge for an order is \$220.70. How many tickets were purchased?

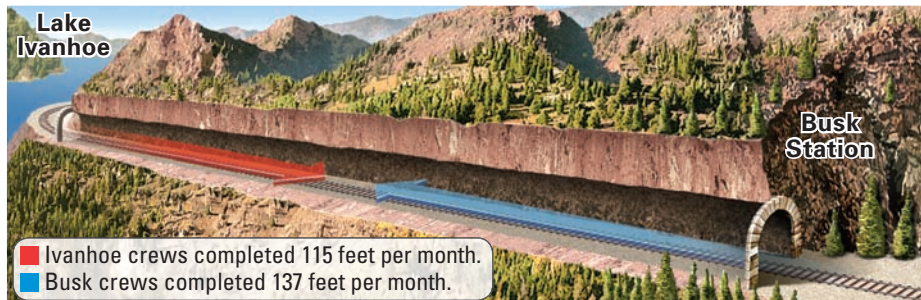
39. **HANGING POSTERS** You want to hang 3 equally-sized travel posters on the wall in your room so that the posters on the ends are each 3 feet from the end of the wall. You want the spacing between posters to be equal. How much space should you leave between the posters?



40. **LIFEGUARD TRAINING** To qualify for a lifeguard training course, you have to swim continuously for 500 yards using either the front crawl or the breaststroke. You swim the front crawl at a rate of 45 yards per minute and the breaststroke at a rate of 35 yards per minute. You take 12 minutes to swim 500 yards. How much time did you spend swimming the front crawl? Use the verbal model below.


$$\text{Distance} = \text{Rate for front crawl} \cdot \text{Time for front crawl} + \text{Rate for breaststroke} \left(\text{Total time} - \text{Time for front crawl} \right)$$

41. **★ EXTENDED RESPONSE** The Busk-Ivanhoe Tunnel on the Colorado Midland Railway was built in the 1890s with separate work crews starting on opposite ends at different times. The crew working from Ivanhoe started 0.75 month later than the crew working from Busk.



Cutaway of Busk-Ivanhoe Tunnel

- Starting at the time construction began on the Busk end, find the time it took to complete a total of 8473 feet of the tunnel. Round your answer to the nearest month.
- After 8473 feet were completed, the work crews merged under the same supervision. The combined crew took 3 months to complete the remaining 921 feet of the tunnel. Find the rate at which the remainder of the tunnel was completed.
- Was the tunnel being completed more rapidly before or after the work crews merged? *Explain* your reasoning.

42.  **MULTIPLE REPRESENTATIONS** A roofing contractor gives estimates for shingling a roof in cost per square, where a square is a 10 foot by 10 foot section of roof. The contractor estimates \$27.50 per square for materials, \$17 per square for labor, \$30 per square for overhead and profit, and a total of \$750 for miscellaneous expenses.
- a. **Writing an Equation** Write an equation that gives the estimate y (in dollars) as a function of the number x of squares of a roof. The contractor gives an estimate of \$2314.50. About how many squares does the roof have?
- b. **Making a Table** Make a table that shows the estimates for shingling a roof that has 5, 10, 15, 20, or 25 squares. Use your table to check your answer to part (a).
43. **CHALLENGE** Jan says that she has quarters and dimes that total \$2.80, and that the number of dimes is 8 more than the number of quarters. Demonstrate algebraically that Jan must be mistaken.