Name ____________________ Date ____________

CHAPTER 2 Writing Repeating Decimals as Fractions

You can rewrite a repeating decimal as a fraction by writing and solving a linear equation.

KEY CONCEPT

Representations of Rational Numbers

A rational number can be written both as a fraction, where the numerator and denominator are both integers, and as a decimal, where the decimal expansion either terminates or has a repeating block of digits.

EXAMPLE 1

Write a repeating decimal as a fraction

Write the repeating decimal 0.55555… in its fraction form.

Solution:

Let $x = 0.55555\ldots$ Then we want to write x in fraction form. We start by multiplying each side of the equation by 10. We get $10x = 5.55555\ldots$ Since the fives continue indefinitely, we can write as many as we want to the right of the decimal.

Now we line up the two equations in the paragraph above and subtract:

$$\begin{aligned} 10x &= 5.55555\ldots \\ (-)\underline{\ \ x} &= \underline{0.55555\ldots} \\ 9x &= 5 \end{aligned}$$

We finish by dividing each side of the equation $9x = 5$ by 9:

$$\begin{aligned} \frac{9x}{9} &= \frac{5}{9} \\ x &= \frac{5}{9} \end{aligned}$$

The repeating decimal 0.55555… is equivalent to the fraction $\frac{5}{9}$. ■

In Example 1, we multiplied each side of the equation $x = 0.55555\ldots$ by 10 because the repeating block has just one digit, 5. In general, if the repeating block of the decimal form of a rational number has n digits, multiply each side of the equation by 10^n.

EXAMPLE 2

Write a repeating decimal as a mixed number

Write the repeating decimal $2.\overline{36}$ as a mixed number.

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CHAPTER 2

Writing Repeating Decimals as Fractions *continued*

Solution:

Let $x = 2.\overline{36} = 2.363636\ldots$ We multiply each side of the equation by 10^2 or 100 because the repeating block has two digits. We get $100x = 236.363636\ldots$ Then we subtract equations and solve for x.

$$\begin{aligned} 100x &= 236.363636\ldots \\ (-)\underline{\quad x} &= \underline{\quad 2.363636\ldots} \\ 99x &= 234 \\ \frac{99x}{99} &= \frac{234}{99} \\ x &= \frac{234}{99} \\ x &= \frac{26}{11} \quad \text{or} \quad 2\frac{4}{11} \end{aligned}$$

The repeating decimal $2.\overline{36}\ldots$ is equivalent to the mixed number $2\frac{4}{11}$. ■

Practice

Write each repeating decimal as a fraction.

1. 0.22222…	**2.** 0.44444…	**3.** $0.\overline{7}$	**4.** $0.\overline{8}$
5. 0.171717…	**6.** 0.595959…	**7.** $0.\overline{26}$	**8.** $0.\overline{71}$
9. 0.636363…	**10.** 0.818181…	**11.** $0.\overline{205}$	**12.** $0.\overline{477}$

Write each repeating decimal as a mixed number.

13. 5.11111…	**14.** 3.55555…	**15.** $6.\overline{6}$	**16.** $7.\overline{92}$
17. 4.232323…	**18.** 9.191919…	**19.** $6.\overline{516}$	**20.** $9.\overline{755}$

21. Use the techniques of Example 1 to rewrite 0.99999… What value do you get? Do you believe that your answer and 0.99999… are equivalent? Explain.

22. Rewrite 8.515151… as a mixed number two ways. First follow the method of Example 2. Then form a mixed number by taking 8 as the whole part and rewriting 0.515151… as a fraction to get the fractional part. Do you get the same answer? Describe a method of rewriting a repeating decimal as a mixed number that is different than that in Example 2.

23. Suppose that a repeating decimal has a block of three repeating digits. Would the methods of this lesson still work if you multiplied by 10^6 instead of 10^3? Explain. You may want to try it for a few numbers such as $0.\overline{123}$, $0.\overline{456}$, and $0.\overline{789}$ before you answer.

24. Can the methods of this lesson be used to rewrite the decimal expansion of π as a fraction where both the numerator and the denominator are integers? Explain.