5.3 Solving Equations with Variables on Both Sides

Another concept that we will need to solve equations is knowing how to deal with equations where there are variables on both sides of the equation. Since we only know how to solve equations where the variable is on one side, we need to get the equations into that form. Let's start with the following example:

$$4(x-2)+2x=3(x-1)-11$$

Note that there are variables on both sides of the equation, but we also need to do some simplification. It is almost always easiest to simplify first, so let's do so.

$$4x-8+2x=3x-3-11$$

$$6x-8=3x-14$$

Now there are still variables on both sides. We have the $6x$ on the left side and $3x$ on the other side. It would be nice if the $3x$ were not on the right. So how do we get rid of it? We can subtract $3x$ from both sides of the equation because we know that $3x-3x=0$ which will eliminate the variable on the right side of the equation. Observe:

$$\begin{array}{l} \quad 6x-8=3x-14 \\ -3x \qquad\quad -3x \\ \quad 3x-8=-14 \end{array}$$

Now we have it down to a two-step equation which we know how to solve. We'll leave it as an exercise to verify the solution is $x=-2$.

Let's look at one more example of getting the variable on one side of the equation where the solution should be $x=7$:

$$\begin{array}{l} -2x-6=-4x+8 \\ +4x \qquad\quad +4x \\ \quad 2x-6=8 \end{array}$$

Add $4x$ to both sides to eliminate the $-4x$ on the right side.

Lesson 5.3

Solve each equation by using the distributive property, combining like terms, and eliminating the variable on one side of the equation.

1. $2y + 3 + 4 = 5y + 10$

2. $2p + 4p - 3 = 2p + 1$

3. $8k + 5 + 2k = 23 + k$

4. $4r + \frac{9}{4}r + 14 = 5r - \frac{3}{4}r + 1 - 3$

5. $2x + 3 = 2x - (3 + 2x) + 6$

6. $4(x - 1) + 2x = 2(x + 2)$

7. $-5(f + 2) = 3f + 2$

8. $\frac{3}{2}c - 3c + 4 = \frac{5}{2}c + 7 - 3$

9. $10(a + 1) = 2(a + 2) - 2$

10. $5x - 3x + 7 = 3x - 1$

11. $5d - 25 + 2d = 2d$

12. $4(2t + 1) + t = 3(t + 2)$

13. $\frac{1}{2}q + 2(q + 5) = -4(q + 1) + 1$

14. $4(1 - 2u) = 2(u + 2)$

15. $5z - z + 3 = z + 3 + 1$

16. $6x - 3x + 26 = 5(x + 8)$

17. $6(x + 1) = 4\left(1 + \frac{1}{4}x\right) + 6 + 3x$

18. $9m - m + 3 = -2(m + 1)$

19. $-(y - 4) + 3y = 4(y + 1)$

20. $-2(j + 5) + 6 = 4(j + 2)$

Write an equation for each situation and then solve by using the distributive property, combining like terms, and eliminating the variable on one side of the equation.

21. Tao is making a 7 feet high door. If the height is 1 foot more than twice its width, what is its width?

22. Terikka bought three bags of popcorn at the concession and a drink for $1.50. If she paid $3.75 total, how much was each bag of popcorn?

23. Naphtali's cell phone company charges $0.25 per text plus a $10 flat fee. Asher's cell phone company charges $0.10 per text plus a $25 flat fee. At how many texts are Naphtali and Asher paying exactly the same amount?

24. Stanley bought five packs of Yu-Gi-Oh cards, $7 worth of bubble gum, and then eight more packs of Yu-Gi-Oh cards. Simon bought four packs of Yu-Gi-Oh cards, $10 worth of Cheetos, $12 worth of Mt. Dew, and then six more packs of Yu-Gi-Oh cards. If they paid the same amount, how much was each pack of Yu-Gi-Oh cards?

25. Toby sells his framed paintings for $20 each. Ishmael sells his paintings for $14 each and charges a flat fee of $18 for framing. How many paintings need to be purchased for Toby and Ishmael to charge the same amount?

26. The original price of Doritos is the same at both Wal-Mart and County Market. Jon found out that Wal-Mart had Doritos on sale at $0.50 off per bag and bought four bags. Later that day, he found out that County Market had Doritos on sale at $1 off per bag and bought six bags. If he paid the same amount at both stores, what was the original price of Doritos?