

CHAPTER
2**Unit Rates and Dimensional Analysis****KEY CONCEPT****Unit Rate**

A rate expressed with a denominator of 1 is called a unit rate. Any rate can be expressed as a unit rate.

A rate such as 50 miles per hour is already expressed as a unit rate: $\frac{50 \text{ miles}}{1 \text{ hour}}$. The following example shows how to rewrite other rates as unit rates.

EXAMPLE 1 Express a rate as a unit rate

Julian bought a 16-ounce bottle of soda for \$1.20. In this situation, the unit rate is the price per ounce of soda. To find the price per ounce, divide the price of the whole bottle by the number of ounces in the bottle.

$$\frac{\$1.20}{16 \text{ oz.}} = \frac{\$1.20 \div 16}{16 \text{ oz.} \div 16} = \frac{\$0.075}{1 \text{ oz.}} \text{ or } \$0.075 \text{ per ounce}$$

The unit rate is 0.075 dollar per ounce, or equivalently, 7.5 cents per ounce. ■

Unit rates are useful when making comparisons among different rates.

EXAMPLE 2 Compare rates

Mr. Nabors drove 460 miles in 8 hours, while Ms. Sanchez drove 560 miles in 10 hours. On average, who was driving faster?

Solution:

In order to compare the two speeds, we calculate the corresponding unit rates.

$$\text{Mr. Nabors: } \frac{460 \text{ miles}}{8 \text{ hours}} = \frac{460 \text{ miles} \div 8}{8 \text{ hours} \div 8} = \frac{57.5 \text{ miles}}{1 \text{ hour}} \text{ or } 57.5 \text{ miles per hour}$$

$$\text{Ms. Sanchez: } \frac{560 \text{ miles}}{10 \text{ hours}} = \frac{560 \text{ miles} \div 10}{10 \text{ hours} \div 10} = \frac{56 \text{ miles}}{1 \text{ hour}} \text{ or } 56 \text{ miles per hour}$$

On average, Mr. Nabors was driving faster than Ms. Sanchez. ■

KEY CONCEPT**Dimensional Analysis**

The units in which a rate is expressed can be changed using dimensional analysis.

EXAMPLE 3 Change the units in a rate

Miss Jenkins paid \$42.75 per square yard to have new carpet installed in her house. Express the rate in dollars per square foot.

Solution:

To express the rate in square feet instead of square yards, we recognize that 1 square yard = 9 square feet and multiply by the appropriate ratio.

$$\frac{\$42.75}{1 \text{ yd}^2} \cdot \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = \frac{\$42.75}{1 \text{ yd}^2} \cdot \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = \frac{\$42.75}{9 \text{ ft}^2} = \frac{\$42.75 \div 9}{9 \text{ ft}^2 \div 9} = \frac{\$4.75}{1 \text{ ft}^2} \text{ or } \$4.75 \text{ per square foot}$$

The equivalent rate is \$4.75 per square foot. ■

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Unit Rates and Dimensional Analysis *continued*

It is possible to change more than one unit at a time using dimensional analysis.

EXAMPLE 4 **Change the units in a rate**

Toby was traveling on his bicycle at a rate of 30 feet per second. Find his speed in miles per hour.

Solution:

In this example, we need to change feet to miles and seconds to hours. We will use 5280 feet = 1 mile, 60 seconds = 1 minute, and 60 minutes = 1 hour.

$$\begin{aligned} \frac{30 \text{ ft}}{1 \text{ sec}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} &= \frac{30 \cancel{\text{ft}}}{1 \cancel{\text{sec}}} \cdot \frac{1 \text{ mile}}{5280 \cancel{\text{ft}}} \cdot \frac{60 \cancel{\text{sec}}}{1 \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \\ &= \frac{30 \cdot 60 \cdot 60 \text{ miles}}{5280 \text{ hr}} = \frac{108,000 \text{ miles}}{5280 \text{ hr}} \\ &\approx 20.5 \text{ miles per hour} \end{aligned}$$

The equivalent rate is approximately 20.5 miles per hour. ■

We can use dimensional analysis to compare rates expressed in different units.

EXAMPLE 5 **Compare rates**

A meter on one pipe indicated that water was flowing through the pipe at a rate of 50 gallons per minute. A meter on a second pipe indicated that water was flowing through that pipe at a rate of 2750 gallons per hour. Which pipe had water flowing through it at the faster rate?

Solution:

In order to compare the two rates, we must first express them in the same units. There is no need to change both rates. We will express the rate of water flowing through the first pipe in gallons per hour.

$$\text{First pipe: } \frac{50 \text{ gal}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hour}} = \frac{50 \text{ gal}}{1 \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hour}} = \frac{3000 \text{ gal}}{1 \text{ hour}} \text{ or } 3000 \text{ gallons per hour}$$

Water was flowing through the first pipe at the faster rate. ■

Practice

Express each rate as a unit rate.

1. \$1.26 for 12 ounces of orange juice
2. \$1.59 for 2 liters of soda
3. \$10.80 for 3 pounds of cheese
4. \$35.70 for 6 pounds of roast beef
5. \$52,500 raised for charity in 14 days
6. 18,600 customers served in 15 weeks
7. 215 miles traveled in 4 hours
8. 518 kilometers traveled in 8 hours
9. 90 pages read in $2\frac{1}{2}$ hours
10. 182 words typed in $3\frac{1}{2}$ minutes
11. \$0.75 for 0.5 liter of water
12. \$3.50 for 0.25 hour of internet service
13. Mia paid \$6.32 for 8 candy bars at one store. Allison paid \$5.25 for 7 candy bars at another store. How much did each girl pay per candy bar? Who paid the higher price per candy bar?

Unit Rates and Dimensional Analysis *continued*

14. Nat paid \$4.75 for 50 minutes of long distance calling. Chi paid \$6.75 for 75 minutes of long distance calling. How much did each boy pay per minute? Who paid the higher price per minute?
15. Mr. Lighthorse drove 580 miles in 10 hours. Mr. Lopez drove 660 miles in 12 hours. How fast was each man driving? On average, who was driving faster?
16. A plane traveling west flew 1000 miles in 2.5 hours. A plane traveling east flew 1350 miles in 3 hours. How fast was each plane traveling? On average, which plane had the greater speed?
17. Renate bought a 32-ounce jar of mayonnaise for \$2.59. Dean bought a 48-ounce jar of mayonnaise for \$3.59. How much did each person pay per ounce of mayonnaise? Round your answers to the nearest cent. Who paid less per ounce?
18. Keisha bought a 16-ounce bottle of ketchup for \$1.19. Jules bought a 24-ounce bottle of ketchup for \$1.99. How much did each person pay per ounce of mayonnaise? Round your answers to the nearest cent. Who paid less per ounce?
19. Mrs. Billings drove 96 miles in $1\frac{1}{2}$ hours. Mr. Lucas drove 175 miles in $2\frac{1}{2}$ hours. Find each driver's average speed. On average, who was driving faster?
20. Dena read 45 pages of a novel in $1\frac{1}{2}$ hours. Jamison read 88 pages of the same novel in $2\frac{3}{4}$ hours. On average, how many pages did each student read per hour? Who was reading at the faster rate?
21. A fabric store sells denim for \$5.99 per yard. Find the price per foot. Round to the nearest cent.
22. A grocery store sells ground beef for \$2.99 per pound. Find the price per ounce. Round to the nearest cent. (1 pound = 16 ounces)
23. A flooring store is advertising laminate flooring at \$2.99 per square foot. Find the price per square yard. (1 square yard = 9 square feet)
24. A home improvement store sells carpet for as low as \$1.79 per square foot. Find the corresponding price per square yard. (1 square yard = 9 square feet)
25. A tile store sells ceramic tile for \$2.99 per square foot. Find the price per square inch. Round to the nearest cent. (1 square foot = 144 square inches)
26. A flooring store is selling laminate flooring for \$33.99 per square yard. Find the price per square foot. Round to the nearest cent. (1 square yard = 9 square feet)
27. Vernon bought square tiles measuring 6 inches on each side to tile his bathroom. Each tile cost \$1.19. Find the cost of the tile per square foot. (1 square foot = 144 square inches)
28. Halley bought square tiles measuring 10 inches on each side to tile her kitchen. Each tile cost \$2.39. Find the cost per square foot. Round to the nearest cent. (1 square foot = 144 square inches)

Unit Rates and Dimensional Analysis *continued*

Change each rate to miles per hour. Round to the nearest whole number, if necessary.

- 29.** 25 feet per second **30.** 35 feet per second
- 31.** 200 inches per second **32.** 275 inches per second
- 33.** 1500 feet per minute **34.** 7500 feet per minute
- 35.** 800 yards per minute **36.** 1000 yards per minute
- 37.** A meter on one pipe indicated that water was flowing through the pipe at a rate of 75 gallons per minute. A meter on a second pipe indicated that water was flowing through that pipe at a rate of 4350 gallons per hour. Express both rates in gallons per hour. Which pipe had water flowing through it at the faster rate?
- 38.** Water was flowing through a pipe filling one swimming pool at a rate of 95 gallons per minute. Water was flowing through a pipe filling a second swimming pool at a rate of 6100 gallons per hour. Express both rates in gallons per hour. Which swimming pool is filling at the faster rate?
- 39.** One store sells a particular chain for \$0.19 per inch. Another store charges \$2.29 per foot for the same chain. Express both rates in dollars per foot. Which store has the lower price?
- 40.** A flooring store sells a particular carpet for \$2.49 per square foot. A home improvement store sells the same carpet for \$17.99 per square yard. Express both rates in dollars per square yard. Which store has the lower price?