2.6 Write Ratios and Proportions

Key Vocabulary
• ratio
• proportion
• simplest form

Throughout this book you have been using rates, such as 50 miles per hour. A rate is a special type of *ratio*.

KEY CONCEPT *For Your Notebook*

Ratios

A **ratio** uses division to compare two quantities. You can write the ratio of two quantities a and b, where b is not equal to 0, in three ways.

$$a \text{ to } b \qquad\qquad a:b \qquad\qquad \frac{a}{b}$$

Each ratio is read "the ratio of a to b." Ratios should be written in simplest form.

EXAMPLE 1 Write a ratio

VOLLEYBALL A volleyball team plays 14 home matches and 10 away matches.

  a. Find the ratio of home matches to away matches.

  b. Find the ratio of home matches to all matches.

Solution

  a. $\dfrac{\text{home matches}}{\text{away matches}} = \dfrac{14}{10} = \dfrac{7}{5}$

  b. $\dfrac{\text{home matches}}{\text{all matches}} = \dfrac{14}{14 + 10} = \dfrac{14}{24} = \dfrac{7}{12}$

✓ **GUIDED PRACTICE** for Example 1

Derek and his brother decide to combine their CD collections. Derek has 44 CDs, and his brother has 52 CDs. Find the specified ratio.

  1. The number of Derek's CDs to the number of his brother's CDs

  2. The number of Derek's CDs to the number of CDs in the entire collection

PROPORTIONS A **proportion** is an equation that states that two ratios are equivalent. The general form of a proportion is given below.

$$\frac{a}{b} = \frac{c}{d} \text{ where } b \neq 0, d \neq 0$$

If one of the numbers in a proportion is unknown, you can solve the proportion to find the unknown number. To solve a proportion with a variable in the numerator, you can use the same methods you used to solve equations.

EXAMPLE 2 **Solve a proportion**

Solve the proportion $\frac{11}{6} = \frac{x}{30}$.

$\frac{11}{6} = \frac{x}{30}$	**Write original proportion.**
$30 \cdot \frac{11}{6} = 30 \cdot \frac{x}{30}$	**Multiply each side by 30.**
$\frac{330}{6} = x$	**Simplify.**
$55 = x$	**Divide.**

✓ **GUIDED PRACTICE** **for Example 2**

Solve the proportion. Check your solution.

3. $\frac{w}{35} = \frac{4}{7}$ **4.** $\frac{9}{2} = \frac{m}{12}$ **5.** $\frac{z}{54} = \frac{5}{9}$

SETTING UP A PROPORTION There are different ways to set up a proportion. Consider the following problem.

A recipe for tomato salsa calls for 30 tomatoes to make 12 pints of salsa. How many tomatoes are needed to make 4 pints of salsa?

The tables below show two ways of arranging the information from the problem. In each table, x represents the number of tomatoes needed to make 4 pints of salsa. The proportions follow from the tables.

	Tomatoes	Pints
Smaller recipe	x	4
Normal recipe	30	12

Proportion: $\frac{x}{30} = \frac{4}{12}$

	Smaller recipe	Normal recipe
Tomatoes	x	30
Pints	4	12

Proportion: $\frac{x}{4} = \frac{30}{12}$

EXAMPLE 3 **Solve a multi-step problem**

ELEVATORS The elevator that takes passengers from the lobby of the John Hancock Center in Chicago to the observation level travels 150 feet in 5 seconds. The observation level is located on the 94th floor, at 1029 feet above the ground. Find the time it takes the elevator to travel from the lobby to the observation level.

Solution

STEP 1 **Write** a proportion involving two ratios that compare the amount of time the elevator has ascended with the distance traveled.

$$\frac{5}{150} = \frac{x}{1029} \xleftarrow{\text{seconds}}$$
$$\xleftarrow{\text{feet}}$$

STEP 2 **Solve** the proportion.

$\dfrac{5}{150} = \dfrac{x}{1029}$	**Write proportion.**
$1029 \cdot \dfrac{5}{150} = 1029 \cdot \dfrac{x}{1029}$	**Multiply each side by 1029.**
$\dfrac{5145}{150} = x$	**Simplify.**
$34.3 = x$	**Use a calculator.**

▶ The elevator travels from the lobby to the observation level in 34.3 seconds.

CHECK You can use a table to check the reasonableness of your answer.

GENERATE TABLE
As the amount of time increases by 5 seconds, the distance traveled increases by 150 feet.

Time (sec)	5	10	15	20	25	30	35
Distance traveled (ft)	150	300	450	600	750	900	1050

The solution, 34.3 seconds, is slightly less than 35 seconds, and 1029 feet is slightly less than 1050 feet. So, the solution is reasonable.
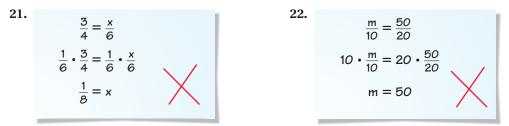
✓ **GUIDED PRACTICE** **for Example 3**

6. **WHAT IF?** In Example 3, suppose the elevator travels 125 feet in 5 seconds. Find the time it will take for the elevator to travel from the lobby to the observation level.

7. **ASTRONOMY** When two full moons appear in the same month, the second full moon is called a blue moon. On average, 2 blue moons occur every 5 years. Find the number of blue moons that are likely to occur in the next 25 years.

2.6 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A proportion is an equation that states that two ___?___ are equivalent.

2. ★ **WRITING** Write a ratio of two quantities in three different ways.

SIMPLIFYING RATIOS Tell whether the ratio is in simplest form. If not, write it in simplest form.

3. 14 to 18

4. $5 : 13$

5. $\dfrac{24}{25}$

6. 28 to 32

EXAMPLE 2
for Exs. 7–22

SOLVING PROPORTIONS Solve the proportion. Check your solution.

7. $\dfrac{2}{5} = \dfrac{x}{3}$

8. $\dfrac{4}{1} = \dfrac{z}{16}$

9. $\dfrac{c}{8} = \dfrac{11}{4}$

10. $\dfrac{36}{12} = \dfrac{x}{2}$

11. $\dfrac{16}{7} = \dfrac{m}{21}$

12. $\dfrac{k}{9} = \dfrac{10}{18}$

13. $\dfrac{5}{8} = \dfrac{t}{24}$

14. $\dfrac{d}{5} = \dfrac{80}{100}$

15. $\dfrac{v}{20} = \dfrac{8}{4}$

16. $\dfrac{r}{60} = \dfrac{40}{50}$

(17.) $\dfrac{16}{48} = \dfrac{n}{36}$

18. $\dfrac{49}{98} = \dfrac{s}{112}$

19. ★ **MULTIPLE CHOICE** What is the value of x in the proportion $\dfrac{8}{5} = \dfrac{x}{20}$?

Ⓐ 2   Ⓑ 23   Ⓒ 32   Ⓓ 40

20. ★ **MULTIPLE CHOICE** What is the value of z in the proportion $\dfrac{z}{15} = \dfrac{28}{35}$?

Ⓐ 8   Ⓑ 12   Ⓒ 18.75   Ⓓ 425

ERROR ANALYSIS *Describe* and correct the error in solving the proportion.

21.
$$\frac{3}{4} = \frac{x}{6}$$
$$\frac{1}{6} \cdot \frac{3}{4} = \frac{1}{6} \cdot \frac{x}{6}$$
$$\frac{1}{8} = x$$
✗

22.
$$\frac{m}{10} = \frac{50}{20}$$
$$10 \cdot \frac{m}{10} = 20 \cdot \frac{50}{20}$$
$$m = 50$$
✗

WRITING AND SOLVING PROPORTIONS Write the sentence as a proportion. Then solve the proportion.

23. 3 is to 8 as x is to 32.

24. 5 is to 7 as a is to 49.

25. x is to 4 as 8 is to 16.

26. y is to 20 as 9 is to 5.

27. b is to 10 as 7 is to 2.

28. 4 is to 12 as n is to 3.

29. 12 is to 18 as d is to 27.

30. t is to 21 as 40 is to 28.

SOLVING PROPORTIONS Solve the proportion. Check your solution.

31. $\dfrac{b}{0.5} = \dfrac{9}{2.5}$

32. $\dfrac{1.1}{1.2} = \dfrac{n}{3.6}$

33. $\dfrac{2.1}{7.7} = \dfrac{v}{8.8}$

34. $\dfrac{36}{54} = \dfrac{2x}{6}$

35. $\dfrac{3a}{4} = \dfrac{36}{12}$

36. $\dfrac{10h}{108} = \dfrac{5}{9}$

37. $\dfrac{6r}{10} = \dfrac{36}{15}$

38. $\dfrac{12}{42} = \dfrac{4w}{56}$

39. $\dfrac{m+3}{8} = \dfrac{40}{64}$

40. $\dfrac{5}{13} = \dfrac{k-4}{39}$

41. $\dfrac{7}{112} = \dfrac{c-3}{8}$

42. $\dfrac{6+n}{60} = \dfrac{15}{90}$

43. ★ **SHORT RESPONSE** Is it possible to write a proportion using the numbers 3, 4, 6, and 8? *Explain* your reasoning.

44. CHALLENGE If $\dfrac{a}{b} = \dfrac{c}{d}$ for nonzero numbers a, b, c, and d, is it also true

that $\dfrac{a}{c} = \dfrac{b}{d}$? *Explain*.

PROBLEM SOLVING

45. GOVERNMENT There are 435 representatives in the U.S. House of Representatives. Of the 435 representatives, 6 are from Kentucky. Find the ratio of the number of representatives from Kentucky to the total number of representatives.

46. CONTEST Of the 30 champions of the National Spelling Bee from 1974 to 2003, 16 are boys. Find the ratio of the number of champions who are girls to the number who are boys.

PIZZA SALES The table shows the number of pizzas sold at a pizzeria during a week. Use the information to find the specified ratio.

47. Small pizzas to large pizzas

48. Medium pizzas to large pizzas

49. Large pizzas to all pizzas

Size	Small	Medium	Large
Pizzas	96	144	240

50. READING A student can read 7 pages of a book in 10 minutes. How many pages of the book can the student read in 30 minutes?

51. SOCCER In the first 4 games of the season, a soccer team scored a total of 10 goals. If this trend continues, how many goals will the team score in the 18 remaining games of the season?

52. ◆◆ MULTIPLE REPRESENTATIONS A movie is filmed so that the ratio of the length to the width of the image on the screen is 1.85 : 1.

a. Writing a Proportion Write and solve a proportion to find the length of the image on the screen when the width of the image is 38 feet.

b. Making a Table Make a table that shows the length of an image when the width of the image is 20, 25, 30, 35, and 40 feet. Use your table to check the reasonableness of your answer to part (a).

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**   ◆◆ = **MULTIPLE REPRESENTATIONS**

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53. MULTI-STEP PROBLEM One day, the ratio of skiers to snowboarders on the mountain at a ski resort was 13 : 10. The resort sold a total of 253 lift tickets during the day.

    a. Find the ratio of snowboarders on the mountain to all of the skiers and snowboarders on the mountain.

    b. Use the ratio from part (a) to find the number of lift tickets sold to snowboarders during the day.

    c. During the same day, the ratio of snowboarders who rented snowboards to snowboarders that have their own snowboards is 4 : 7. Find the number of snowboarders who rented a snowboard.

54. ★ EXTENDED RESPONSE You and a friend are waiting in separate lines to purchase concert tickets.

    a. Interpret Every 10 minutes, the cashier at the head of your line helps 3 people. There are 11 people in line in front of you. Write a proportion that can be used to determine how long you will have to wait to purchase tickets.

    b. Interpret Every 5 minutes, the cashier at the head of your friend's line helps 2 people. There are 14 people in line in front of your friend. Write a proportion that can be used to determine how long your friend will have to wait to purchase tickets.

    c. Compare Will you or your friend be able to purchase concert tickets first? *Explain*.

55. CHALLENGE A car traveling 50 miles per hour goes 15 miles farther in the same amount of time as a car traveling 30 miles per hour. Find the distance that each car travels.