

# 2.7 Solve Proportions Using Cross Products



**Before**

You solved proportions using the multiplication property of equality.

**Now**

You will solve proportions using cross products.

**Why?**

So you can find the height of a scale model, as in Ex. 39.

## Key Vocabulary

- cross product
- scale drawing
- scale model
- scale



**CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems.\*

In a proportion, a **cross product** is the product of the numerator of one ratio and the denominator of the other ratio. The following property involving cross products can be used to solve proportions.

## KEY CONCEPT

*For Your Notebook*

### Cross Products Property

**Words** The cross products of a proportion are equal.

**Example**  $\frac{3}{4} = \frac{6}{8}$   $4 \cdot 6 = 24$   
 $3 \cdot 8 = 24$

**Algebra** If  $\frac{a}{b} = \frac{c}{d}$  where  $b \neq 0$  and  $d \neq 0$ , then  $ad = bc$ .

The proportion  $\frac{3}{4} = \frac{6}{8}$  can be written as  $3:4 = 6:8$ . In this form, 4 and 6 are called the *means* of the proportion, and 3 and 8 are called the *extremes* of the proportion. This is why the cross products property is also called the *means-extremes property*.

## EXAMPLE 1 Use the cross products property

Solve the proportion  $\frac{8}{x} = \frac{6}{15}$ .

$$\frac{8}{x} = \frac{6}{15}$$

**Write original proportion.**

$$8 \cdot 15 = x \cdot 6$$

**Cross products property**

$$120 = 6x$$

**Simplify.**

$$20 = x$$

**Divide each side by 6.**

► The solution is 20. Check by substituting 20 for  $x$  in the original proportion.

**CHECK**  $\frac{8}{20} \stackrel{?}{=} \frac{6}{15}$

**Substitute 20 for  $x$ .**

$$8 \cdot 15 \stackrel{?}{=} 20 \cdot 6$$

**Cross products property**

$$120 = 120 \checkmark$$

**Simplify. Solution checks.**



## EXAMPLE 2 Standardized Test Practice

What is the value of  $x$  in the proportion  $\frac{4}{x} = \frac{8}{x-3}$ ?

- (A) -6      (B) -3      (C) 3      (D) 6

### Solution

#### ANOTHER WAY

Because 8 is twice 4, you can reason that  $x - 3$  must be twice  $x$ :

$$\begin{aligned}x - 3 &= 2x \\ -3 &= x\end{aligned}$$

$$\frac{4}{x} = \frac{8}{x-3}$$

Write original proportion.

$$4(x - 3) = x \cdot 8$$

Cross products property

$$4x - 12 = 8x$$

Simplify.

$$-12 = 4x$$

Subtract  $4x$  from each side.

$$-3 = x$$

Divide each side by 4.

▶ The value of  $x$  is  $-3$ . The correct answer is B. (A) (B) (C) (D)

## EXAMPLE 3 Write and solve a proportion

**SEALS** Each day, the seals at an aquarium are each fed 8 pounds of food for every 100 pounds of their body weight. A seal at the aquarium weighs 280 pounds. How much food should the seal be fed per day?

### Solution

**STEP 1** Write a proportion involving two ratios that compare the amount of food with the weight of the seal.

$$\frac{8}{100} = \frac{x}{280} \quad \begin{array}{l} \leftarrow \text{amount of food} \\ \leftarrow \text{weight of seal} \end{array}$$

**STEP 2** Solve the proportion.

$$\frac{8}{100} = \frac{x}{280}$$

Write proportion.

$$8 \cdot 280 = 100 \cdot x$$

Cross products property

$$2240 = 100x$$

Simplify.

$$22.4 = x$$

Divide each side by 100.

▶ A 280 pound seal should be fed 22.4 pounds of food per day.

#### ANOTHER WAY

You can also solve the proportion by multiplying each side of the equation by 280.



### GUIDED PRACTICE for Examples 1, 2, and 3

Solve the proportion. Check your solution.

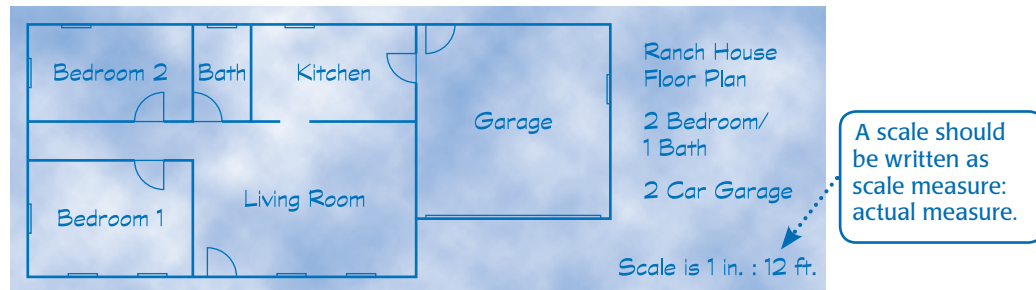
1.  $\frac{4}{a} = \frac{24}{30}$

2.  $\frac{3}{x} = \frac{2}{x-6}$

3.  $\frac{m}{5} = \frac{m-6}{4}$

4. **WHAT IF?** In Example 3, suppose the seal weighs 260 pounds. How much food should the seal be fed per day?

**SCALE DRAWINGS AND SCALE MODELS** The floor plan below is an example of a *scale drawing*. A **scale drawing** is a two-dimensional drawing of an object in which the dimensions of the drawing are in proportion to the dimensions of the object. A **scale model** is a three-dimensional model of an object in which the dimensions of the model are in proportion to the dimensions of the object.



The **scale** of a scale drawing or scale model relates the drawing's or model's dimensions and the actual dimensions. For example, the scale 1 in. : 12 ft on the floor plan means that 1 inch in the floor plan represents an actual distance of 12 feet.

**EXAMPLE 4** Use the scale on a map

**MAPS** Use a metric ruler and the map of Ohio to estimate the distance between Cleveland and Cincinnati.

**Solution**

From the map's scale, 1 centimeter represents 85 kilometers. On the map, the distance between Cleveland and Cincinnati is about 4.2 centimeters.

Write and solve a proportion to find the distance  $d$  between the cities.

$$\frac{1}{85} = \frac{4.2}{d} \quad \begin{array}{l} \leftarrow \text{centimeters} \\ \leftarrow \text{kilometers} \end{array}$$

$$1 \cdot d = 85 \cdot 4.2 \quad \text{Cross products property}$$

$$d = 357 \quad \text{Simplify.}$$

▶ The actual distance between Cleveland and Cincinnati is about 357 kilometers.



**GUIDED PRACTICE** for Example 4

5. Use a metric ruler and the map in Example 4 to estimate the distance (in kilometers) between Columbus and Cleveland.
6. **MODEL SHIPS** The ship model kits sold at a hobby store have a scale of 1 ft : 600 ft. A completed model of the *Queen Elizabeth II* is 1.6 feet long. Estimate the actual length of the *Queen Elizabeth II*.

## 2.7 EXERCISES

### HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**  
Exs. 13 and 39

★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 15, 16, 41, and 42

◆ = **MULTIPLE REPRESENTATIONS**  
Ex. 40

### SKILL PRACTICE

- VOCABULARY** Copy and complete: In a proportion, a(n)   ? is the product of the numerator of one ratio and the denominator of the other ratio.
- ★ **WRITING** A scale drawing has a scale of 1 cm : 3 m. *Explain* how the scale can be used to find the actual distance between objects in the drawing.

#### EXAMPLES

#### 1 and 2

for Exs. 3–18

#### SOLVING PROPORTIONS Solve the proportion. Check your solution.

- $\frac{2}{3} = \frac{4}{x}$
- $\frac{3}{y} = \frac{15}{35}$
- $\frac{13}{6} = \frac{52}{z}$
- $\frac{10}{45} = \frac{v}{27}$
- $\frac{5m}{6} = \frac{10}{12}$
- $\frac{3k}{27} = \frac{2}{3}$
- $\frac{-49}{7} = \frac{a+7}{6}$
- $\frac{6}{t+4} = \frac{42}{77}$
- $\frac{8}{12} = \frac{r}{r+1}$
- $\frac{n}{n-12} = \frac{9}{5}$
13.  $\frac{11}{w} = \frac{33}{w+24}$
- $\frac{18}{d+13} = \frac{6}{d-13}$

- ★ **MULTIPLE CHOICE** What is the value of  $h$  in the proportion  $\frac{15}{-2h} = \frac{5}{12}$ ?

(A) -36      (B) -18      (C) 18      (D) 36

- ★ **MULTIPLE CHOICE** What is the value of  $s$  in the proportion  $\frac{7}{s-14} = \frac{21}{s+18}$ ?

(A) -48      (B) -16      (C) 3      (D) 30

#### ERROR ANALYSIS Describe and correct the error in solving the proportion.

17.

$$\frac{4}{3} = \frac{16}{x}$$

$$4 \cdot 16 = 4 \cdot x$$

$$64 = 4x$$

$$16 = x$$

18.

$$\frac{18}{14} = \frac{b+2}{b}$$

$$18b = 14b + 2$$

$$4b = 2$$

$$b = 0.5$$

#### SOLVING PROPORTIONS Solve the proportion. Check your solution.

- $\frac{7}{3} = \frac{2x+5}{x}$
- $\frac{a}{9a-2} = \frac{1}{8}$
- $\frac{24}{5z+4} = \frac{4}{z-1}$
- $\frac{c-8}{-2} = \frac{11-4c}{11}$
- $\frac{k-8}{7+k} = \frac{-1}{5}$
- $\frac{2}{-3} = \frac{4v+4}{2v+14}$
- $\frac{m+1}{4} = \frac{3m+6}{7}$
- $\frac{6}{4+2w} = \frac{-2}{w-10}$
- $\frac{n+0.3}{n-3.2} = \frac{9}{2}$
- $\frac{-3}{11} = \frac{5-h}{h+1.4}$
- $\frac{4}{b-3.9} = \frac{2}{b+1}$
- $\frac{16.5+3t}{3} = \frac{0.9-t}{-5}$

31. **REASONING** The statements below justify the cross products property. Copy and complete the justification.

$\frac{a}{b} = \frac{c}{d}$	<b>Given</b>
$bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}$	a. <u>    ?</u>
$\frac{bd \cdot a}{b} = \frac{bd \cdot c}{d}$	b. <u>    ?</u>
$ad = cb$	c. <u>    ?</u>

32. **CHALLENGE** In the proportion  $\frac{5}{h} = \frac{k}{14}$ , what happens to the value of  $h$  as the value of  $k$  increases? *Explain.*

## PROBLEM SOLVING

**EXAMPLE 3**  
for Exs. 33–34

33. **RECIPES** A recipe that yields 12 buttermilk biscuits calls for 2 cups of flour. How much flour is needed to make 30 biscuits?
34. **DIGITAL PHOTOGRAPHS** It took 7.2 minutes to upload 8 digital photographs from your computer to a website. At this rate, how long will it take to upload 20 photographs?

**EXAMPLE 4**  
for Exs. 35–39

**MAPS** A map has a scale of 1 cm : 15 km. Use the given map distance to find the actual distance.

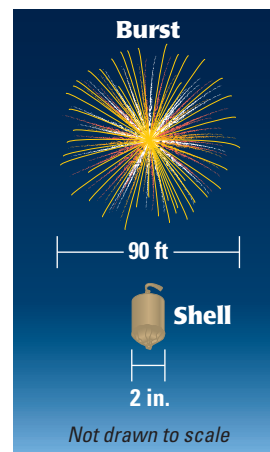
35. 6 cm                      36. 3.2 cm                      37. 0.5 cm                      38. 4.7 cm

39. **SCALE MODEL** An exhibit at Tobu World Square in Japan includes a scale model of the Empire State Building. The model was built using a scale of 1 m : 25 m. The height of the actual Empire State Building is 443.2 meters. What is the height of the model?

40. **MULTIPLE REPRESENTATIONS** The diameter of the burst of a firework is proportional to the diameter of the shell of the firework.

- a. **Writing a Proportion** Use the information in the diagram to find the burst diameter for a 4.75 inch shell.
- b. **Making a Table** Make a table of burst diameters for 2, 3, 4, 5, and 6 inch shells. Use the table to check your answer to part (a).

41. **★ SHORT RESPONSE** The ratio of the length of a soccer field to the width of the field is 3 : 2. A scale drawing of a soccer field has a scale of 1 in. : 20 yd. The length of the field in the drawing is 6 inches. What is the actual width of the field? *Explain* your reasoning.



42. **★ EXTENDED RESPONSE** A mole is a unit of measurement used in chemistry. The masses of one mole of three elements are in the table.

Element	Mass of 1 mole
Hydrogen	1.008 grams
Carbon	12.011 grams
Oxygen	15.999 grams

- a. A 100 gram sample of ascorbic acid contains 4.58 grams of hydrogen. To the nearest tenth, find the number of moles of hydrogen.
- b. A 100 gram sample of ascorbic acid contains 54.5 grams of oxygen. To the nearest tenth, find the number of moles of oxygen in the sample.
- c. The ratio of moles of hydrogen to moles of carbon in ascorbic acid is 4:3. How does this ratio compare with the ratio of moles of hydrogen to moles of oxygen in ascorbic acid? *Explain.*

43. **CHALLENGE** In one high school, there are 90 seniors, 142 juniors, 175 sophomores, and 218 freshmen. Ideally, in the apportionment of the 30 seats on the student council, the number of seats each class has is proportional to the number of class members. Assign a number of seats on the council to each class. *Explain* your reasoning.