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CHAPTER 2 Identifying the Domain of a Variable in a Formula

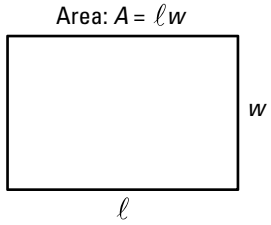
For most formulas modeling real-life situations, there are natural restrictions on the values that may be used for the variables in the formula. For a formula, the set of all values that may be meaningfully substituted for any variable in the formula is called the domain of that variable.

EXAMPLE 1 Identify the domain of a variable in a formula

Find the domain of each variable in the formula for the area of a rectangle, $A = \ell w$.

Solution:

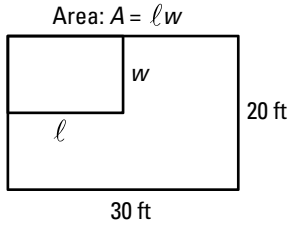
The formula $A = \ell w$ models the area A of a rectangle, where ℓ represents the length of the rectangle and w represents its width. Because neither the length nor the width of a rectangle can be either negative or zero, the values of ℓ and w cannot be either negative or zero. It is also true that the value of A cannot be negative or zero. Therefore, each of the variables, A, ℓ, and w in the formula has the set of all positive real numbers as its domain. ■



In real-life situations, given restrictions on the domain of certain variables in a formula will affect the domain of other variables in the formula.

EXAMPLE 2 Identify the domain of a variable in a formula

Suppose you have a rectangular yard that is 20 feet by 30 feet, and you want to create a rectangular garden within the yard as shown in the figure. Find the domain of each variable in the formula for the area of the garden, $A = \ell w$.



Solution:

As in Example 1, the values of ℓ and w cannot be either negative or zero. The length ℓ cannot be greater than 30 ft and the width w cannot be greater than 20 ft, since the garden must be within the yard. Therefore, the area of the garden A cannot be greater than (30 ft) • (20 ft) = 600 ft^2. The domain of ℓ is the set of positive real numbers less than or equal to 30, the domain of w is the set of positive real numbers less than or equal to 20, and the domain of A is the set of positive real numbers less than or equal to 600. ■

In the previous example, the domains of the variables were given as a range of real numbers. In some real-world situations, the domain of a variable can be further restricted to discrete values such as whole numbers or multiples of whole numbers.

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CHAPTER 2

Identifying the Domain of a Variable in a Formula *continued*

EXAMPLE 3

Identify the domain of a variable in a formula

The formula $C = 8.50x$ models the cost C in dollars for x adults to see a movie. Find the domain of the variable x.

Solution:

In this situation, the variable x represents a number of people. This number cannot be negative. It also cannot be irrational or a rational number that is not a whole number: $\sqrt{3}$ people and 4.5 people are both meaningless. The domain of the variable x in the formula $C = 8.50x$ is the set of whole numbers. ■

Before substituting a value for a variable in a formula, always ask whether the number is a meaningful value for the variable.

Practice

Find the domain of each variable in the formula.

1. Perimeter of a rectangle: $P = 2\ell + 2w$
2. Volume of a cube: $V = s^3$
3. Speed given distance and time: $s = \frac{d}{t}$
4. Density given mass and volume: $d = \frac{m}{v}$
5. Volume of a rectangular prism: $V = \ell wh$
6. Circumference of a circle: $c = \pi d$

Problem Solving

7. Suppose you have a square region that is 20 feet wide and 20 feet long and you wish to put a circular pool within that region. Find the domain of each variable in the formula for the area of the pool, $A = \pi r^2$.
8. Suppose you drive a car that gets 20 miles per gallon and the capacity of the tank is 14 gallons. The distance driven is given by the formula $d = 20g$. Find the domain of each variable in the formula assuming that the tank is not refilled.
9. The formula $R = 30x$ models the revenue R in dollars for selling x shirts. Find the domain of the variable x.
10. To use a gym for two weeks, you pay a sign-up fee of \$30 and \$5 for each day you use the gym. The formula $C = 30 + 5d$ models the cost C in dollars for going d days over the two week period. Find the domain of the variable d.
11. What are the possible values of the variable C in the formula $C = 8.50x$ from Example 3?
12. What are the possible values of the variable R in the formula $R = 30x$ from Exercise 9?
13. What are the possible values of the variable C in the formula $C = 30 + 5d$ from Exercise 10?