Unit 20: Random Variables

SUMMARY OF VIDEO

The video starts with the random phenomenon of a coin flip – more specifically, by examining what could happen if a fair coin is flipped four times. The sample space for this experiment is given below.

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, TTHH, THTH, HTTT, THTT, TTHT, TTTH, TTTT}

Each of these outcomes is equally likely. However, we are not interested in the actual outcomes, but rather on the number of heads in four flips. So, we'll define *x* as follows:

x = number of heads in four flips of a coin

We are now focusing on what statisticians call a random variable: the numerical outcome associated with the random phenomenon. The probability distribution of a random variable x tells us the values that the random variable can take on and the probabilities associated with each.

In our four coin tosses, the random variable *x* could equal 0, 1, 2, 3, or 4. It is a discrete random variable since it has a finite number of possible values. Although each of these values is possible, they are not equally likely as can be seen from the probability distribution in Table 20.1.

Value of *x*	0	1	2	3	4
Probability	0.0625	0.25	0.375	0.25	0.0625

Table 20.1. Probability distribution of x.

First, notice that the sum of all the probabilities is 1. Next, we need some notation – we use *p*(*x*) to denote the probability associated with a particular value of *x*. So, for instance, p(0) is the probability that the value of the random variable is 0. We represent this probability distribution graphically with the probability histogram in Figure 20.1.

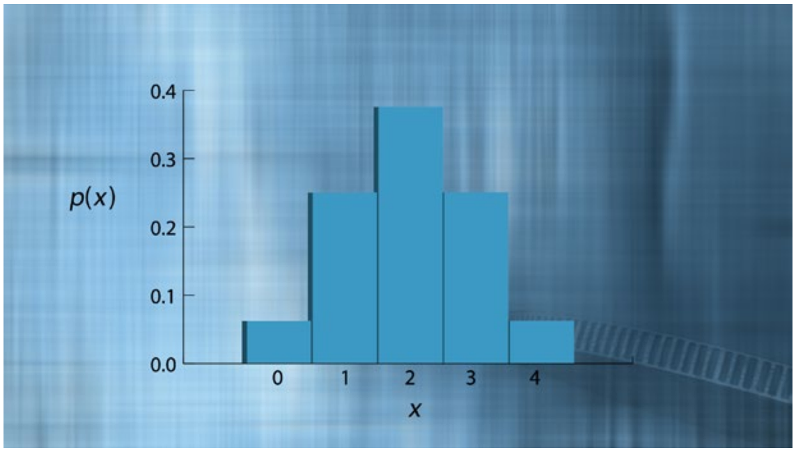

Figure 20.1. Probability histogram for x.

The horizontal axis shows the possible values of *x*, the bars have equal width, and the height of each bar represents the probability for that value. From the probability histogram, we can see that two is the most likely number of heads to come up in a string of four coin tosses. The histogram also tells us what we can expect from the data if we were to really run the experiment over and over again many times. However, instead of working with data, we can use the probability distribution.

The stakes are not very high when talking about coin tossing. But such calculations can be a matter of life and death when the events are critical equipment failures. On January 28, 1986, the space shuttle Challenger exploded shortly after liftoff. After the accident, President Ronald Reagan appointed a commission of experts to investigate its cause. Their eventual conclusion was that the accident was most likely caused by O-ring failure. O-rings sealed the field joints holding together the rocket boosters that would lift Challenger into orbit. The O-rings were supposed to contain hot, pressurized gases within the boosters. That morning, at least one failed to do so.

Could the disaster have been predicted? The first step in a probability analysis of field joint failure is to calculate the probability of failure in a single one of them. Under the Challenger flight conditions the probability of failure of a particular field joint was 0.023. That means that the probability of success of an individual field joint would be 0.977. However, there were six field joints. For the whole system to succeed, all six field joints had to succeed – in other words, zero failures. We form a probability distribution table for x = the number of failures in the six field joints.

x	0	1	2	3	4	5	6
$p(x)$	$p(0)$	p (1)	$p(2)$	$p(3)$	$p(4)$	$p(5)$	$p(6)$

Table 20.2. Probability distribution for x, the number of field joint failures.

We want to determine $p(0)$, the probability of zero failures. We will need to use the Multiplication Rule:

If A and B are independent events, then $P(A \text{ and } B) = P(A)P(B)$

Remember, the probability that each field joint would succeed was 0.977 and there are six field joints that all need to succeed – no failures allowed. We assume that the field joints are independent. Failure of one field joint should not affect the likelihood that another fails.

$p(0) = P(\text{all six field joints succeed})$

$= (0.977)\,(0.977)\,(0.977)\,(0.977)\,(0.977)\,(0.977)$

≈ 0.87 or around 87%

It is possible to complete all the other individual probabilities, but for now we will use the Complement Rule to calculate the likelihood of there being at least one field joint failure.

$P(\text{at least one failure}) = 1 - p(0) = 1 - 0.87 = 0.13$

So while the probability of an individual field joint failing is pretty low, the probability of at least one of the six failing is rather high, especially considering that astronauts' lives are at stake.

Over two hundred improvements were made to the next space shuttle after the Challenger disaster. NASA successfully launched shuttles almost 100 more times before retiring the space shuttle program in 2011. Of course a complex, state of the art technology like the shuttle system could never reduce the risk of failure to zero – and in fact, another disaster occurred in 2003 when the space shuttle Columbia disintegrated on re-entry to Earth's atmosphere. The Challenger and Columbia are tragic reminders of the risks of space exploration and the need for continued rigorous analysis.