## UNIT ACTIVITY: CHILDREN IN HOUSEHOLDS AND SCHOOL LUNCH

The U.S. government collects data on many variables having to do with households. In this activity, you will examine issues related to the number of children in households and participation in free school lunch programs.

1. Whether or not a household has children is a random phenomenon. From this random phenomenon we define random variable $\mu$ as follows:
$u=\left\{\begin{array}{l}0, \text { if no children in household } \\ 1, \text { if at least one child in household }\end{array}\right.$

The probability distribution of $\mu$ is given below. We have chosen to express the table with columns for the values of $\mu$ and corresponding values of $p(\mu)$. This will help with computation of mean and standard deviation later in this activity.

| $u$ | $p(u)$ |
| :---: | :---: |
| 0 | 0.586 |
| 1 | 0.432 |

Table 20.5. Probability distribution of $\mu$.
a. Draw a probability histogram for the probability distribution of $u$.
b. The balance point of the probability histogram is the mean, $\mu_{u}$, of the random variable $u$. Based on the probability histogram from (a), do you think the balance point of this distribution is less than 0.5 , at 0.5 , or above 0.5 ? Explain.
c. Next, we walk you through one method of calculating $\mu_{u}=\sum u \cdot p(u)$. Make a copy of Table 20.6. Enter the products of the entries from the first two columns into the third column. Then compute the sum of the entries in the third column to get $\mu_{u}$.

| $u$ | $p(u)$ | $(u)(p(u))$ |
| :---: | :---: | :---: |
| 0 | 0.586 |  |
| 1 | 0.432 |  |
| Sum $=$ |  |  |
|  |  |  |
|  |  |  |

Table 20.6. Method for calculating $\mu_{u}$.
d. You should have gotten $\mu_{u}=0.432$ for (c). (If not, go back and redo part (c).) Next, we walk you through one method of calculating the variance: $\sigma_{u}^{2}=\left(u-\mu_{u}\right)^{2} \cdot p(u)$. Then you can calculate the standard deviation by taking the square root of the variance. Make a copy of Table 20.7.

- In column three, enter the squared difference between the values of $\mu$ and $\mu_{u}$
- In column four, multiply the squared deviations by their corresponding probabilities.
- Sum column four to get $\sigma_{u}^{2}$.
- Take the square root of $\sigma_{u}^{2}$ to get the standard deviation.

| $u$ | $p(u)$ | $(u-0.432)^{\wedge} 2$ | $\left((u-0.432)^{\wedge} 2\right)(p(u))$ |
| :---: | :---: | :---: | :---: |
| 0 | 0.586 |  |  |
| 1 | 0.432 |  |  |
| Sum $=$ |  |  |  |

Table 20.7. Method for calculating $\sigma_{u}^{2}$.
2. In households with children, some of the school age children ate school lunches and others did not. Hence, we have another random phenomenon. Define a new random variable $v$ as follows:

$$
v=\left\{\begin{array}{l}
0, \text { if none of the children in household ate school lunch } \\
1, \text { if at least one child in household ate school lunch }
\end{array}\right.
$$

a. Table 20.8 contains data collected from a U.S. government survey on random variable $v$. Calculate the proportions for the outcomes of $v$ and enter them as estimates in the Probability column. Round probabilities to three decimals.

| $v$ | Frequency | Probability |
| :---: | :---: | :---: |
| 0 | 32,491 |  |
| 1 | 74,690 |  |
| Total |  |  |

Table 20.8. Data on random variable $v$.
b. Draw a diagram that represents all households broken down by whether or not there are children in the household, and then broken down further by whether or not at least one child in the household ate school lunch. Determine the proportion of all households that had at least one child who ate school lunch.
3. To ensure that children don't go hungry, some of the children who eat school lunches get free lunches. Define random variable $x$ as follows:
$x=\left\{\begin{array}{l}0, \text { if household in which children ate school lunch, none got it free } \\ 1, \text { if household in which children ate school lunch, at least one got it free }\end{array}\right.$
The probability distribution for $x$ is given in Table 20.9.

| $x$ | $p(x)$ |
| :---: | :---: |
| 0 | 0.432 |
| 1 | 0.568 |

Table 20.9. Probability distribution of $x$.
From question 2, we know that 74,690 households in the survey had a least one child who ate school lunch. Estimate the number of those households in which at least one child participated in the free lunch program.
4. Next, for those households with at least one child participating in the free lunch program, some households have multiple children participating in the program. Define random variable $y$ as follows:
$y=$ the number of children in the household participating in the free lunch program The probability distribution for $y$ is given in Table 20.10.

| $y$ | $p(y)$ |
| :---: | :---: |
| 1 | 0.394 |
| 2 | 0.337 |
| 3 | 0.176 |
| 4 | 0.063 |
| 5 | 0.019 |
| 6 | 0.007 |
| 7 | 0.002 |
| 8 | 0.001 |

Table 20.10. Probability distribution of $y$.
a. Draw a probability histogram representing the probability distribution of $y$.
b. Visually estimate the mean $\mu_{y}$ as the balance point of your histogram in (a). Does the balance point appear closer to 1,2 , or 3 ?
c. Calculate the mean of $y, \mu_{y}$, using the method outlined in 1 (c).
(Round your answer to three decimals.)
d. Calculate the standard deviation of $y, \sigma_{y}$, using the method outlined in question 1 (d).

