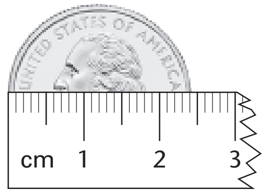
LESSON 2

Significant Digits

GOAL Determine the number of significant digits in a measurement and write results of computations involving measurements with the correct number of significant digits.

The diameter of a quarter to the nearest centimeter is 2 cm. The diameter of a quarter to the nearest millimeter is 24 mm. Notice that the second measurement is more precise and contains more digits than the first measurement. You can use the number of digits to describe the precision of a measurement.



Significant digits are the digits in a measurement that carry meaning contributing to the precision of the measurement. The table shows how to determine the number of significant digits in a measurement.

Determining Significant Digits			
Rule	**Example**	**Significant digits (bold)**	**Number of significant digits**
All nonzero digits	673.24	**673.24**	5
Zeros after the last nonzero digit and to the right of the decimal point	0.0030	0.00**30**	2
Zeros between significant digits	140.0	**140.0**	4

Zeros at the end of a whole number are usually assumed to be nonsignificant. For example, 350 kg has 2 significant digits, while 351 kg has 3 significant digits.

EXAMPLE 1 Identifying Significant Digits

Determine the number of significant digits in each measurement.

a. 290.01 g **b.** 0.8500 km **c.** 4000 mi

SOLUTION

a. The digits 2, 9, and 1 are nonzero digits, so they are significant digits. The zeros are between significant digits, so they are also significant digits.

290.01 g has 5 significant digits (**290.01** g).

b. The digits 8 and 5 are nonzero digits, so they are significant digits. The two zeros after the last nonzero digit are to the right of the decimal point and are significant digits.

0.8500 km has 4 significant digits (0.**8500** km).

c. The digit 4 is a nonzero digit, so it is a significant digit. The zeros at the end of a whole number are not significant.

4000 mi has 1 significant digit (**4**000 mi).

The zero in a measurement like 90 mm can be ambiguous. It is not clear whether the zero indicates that the measurement was made to the nearest centimeter (9 cm = 90 mm) or whether the measurement was made to the nearest millimeter (in which case the number of millimeters just happens to be a multiple of 10).

You can use scientific notation to avoid this ambiguity, since if a number is in scientific notation has a decimal part, any zeros to the right of the decimal point will be significant.

9×10^1 mm has 1 significant digit. 9.0×10^1 mm has 2 significant digits.

EXAMPLE 2 Identifying Significant Digits in Scientific Notation

Determine the number of significant digits in each measurement.

a. 3.1×10^8 yr

b. 7.00×10^3 mi

SOLUTION

a. The digits 3 and 1 are significant digits.

3.1×10^8 yr has 2 significant digits.

b. The digit 7 and the zeros to the right of the decimal point are all significant digits.

7.00×10^3 mi has 3 significant digits.

CHECK Examples 1 and 2

Determine the number of significant digits in each measurement.

1. 80,020 ft

2. 0.005 cm

3. 9.000×10^{15} km

4. Write the measurement 5400 mi in a way that shows that all the digits are significant digits.

When you perform calculations involving measurements, the number of significant digits that you write in your result depends on the number of significant digits in the given measurements. The table summarizes the rules.

Significant Digits in Calculations	
Operations	**Rule**
Addition Subtraction	Round the sum or difference to the same place as the last significant digit of the least precise measurement.
Multiplication Division	The product or quotient must have the same number of significant digits as the least precise measurement.

EXAMPLE 3 Calculating with Significant Digits

Perform the indicated operation. Write the answer with the correct number of significant digits.

a. 45.1 cm + 19.45 cm

b. $(7.25 \times 10^3 \text{ mi}) \times (8.2 \times 10^3 \text{ mi})$

SOLUTION

a. 45.1 cm + 19.45 cm = 64.55 cm

The least precise measurement is 45.1. Its last significant digit is in the tenths place. Round the sum to the tenths place.

The sum is 64.6 cm.

b. $(7.25 \times 10^3 \text{ mi}) \times (8.2 \times 10^3 \text{ mi}) = 5.945 \times 10^7 \text{ mi}^2$

7.25×10^3 has 3 significant digits; 8.2×10^3 has 2 significant digits. The product should have 2 significant digits.

The product is $5.9 \times 10^7 \text{ mi}^2$.

EXAMPLE 4 Applying Significant Digits

A student measures the length of a rectangular garden plot to the nearest 0.1 meter and finds that the length is 6.4 meters. Another student measures the width of the plot to the nearest meter and finds that the width is 2 meters. Use the correct number of significant digits to write the perimeter and area of the plot.

SOLUTION

The perimeter of the plot is 6.4 m + 2 m + 6.4 m + 2 m = 16.8 m.

The least precise measurement is 2 m. Its last significant digit is in the units place, so round the sum to the nearest whole number: 16.8 m ≈ 17 m.

The area of the plot is $6.4 \text{ m} \times 2 \text{ m} = 12.8 \text{ m}^2$.

The least precise measurement, 2 m, has 1 significant digit, so round the product to 1 significant digit: $12.8 \text{ m}^2 \approx 10 \text{ m}^2$.

The perimeter of the plot is 17 m. The area of the plot is 10 m^2.

CHECK Examples 3 and 4

Perform the indicated operation. Write the answer with the correct number of significant digits.

5. 21 ft − 16.4 ft

6. 265 mm × 312 mm

7. $5.1 \times 10^5 \text{ km} + 3.15 \times 10^5 \text{ km}$

8. A rectangular poster has an area of 852 in.^{2} and a length of 36 in. Use the correct number of significant digits to write the width of the poster.

EXERCISES

Determine the number of significant digits in the measurement.

1. 735 ft
2. 820 mi
3. 72.90 km
4. 0.05 in.
5. 2.000 gal
6. 9.030 kg
7. 8.0001 km
8. 5430 cm
9. 0.0080 kg
10. 9.00×10^3 mi
11. 5.001×10^6 ft
12. 4×10^8 yr
13. 1.050×10^9 kg
14. 2.0×10^1 yd
15. 1.0000×10^9 km

Perform the indicated operation. Write the answer with the correct number of significant digits.

16. 245 kg − 18.32 kg
17. 2.8 mi + 3.56 mi
18. 9.05 cm + 18 cm
19. 6.42 m × 7.51 m
20. 7.9 ft × 220 ft
21. 856 mL ÷ 45 mL
22. 5.1×10^4 mi + 1.345×10^4 mi
23. 8.1×10^3 yd − 3×10^3 yd
24. $(4.7 \times 10^3\text{ m}) \times (2.00 \times 10^5\text{ m})$
25. $(6.01 \times 10^8\text{ km}) \times (3.21 \times 10^3\text{ km})$

26. The Reflecting Pool is a rectangular body of water in front of the Lincoln Memorial in Washington, DC. A surveyor determines that the length of the pool to the nearest foot is 2029 ft and the width of the pool to the nearest foot is 167 ft. How should the surveyor report the perimeter and area of the pool using the correct number of significant digits?

27. A student measures the length of a cube to the nearest 0.1 cm and reports the length as 3.5 cm. Using the correct number of significant digits, how should the student report the area of one face of the cube? How should the student report the volume of the cube?

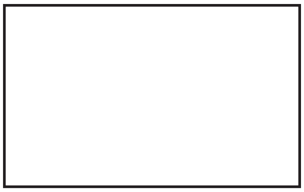
In Exercises 28–30, give an example of the measurement described.

28. A 5-digit measurement that has 3 significant digits

29. A measurement greater than 1000 cm that has 2 significant digits

30. A measurement less than 1 mm that has 4 significant digits

31. Use a metric ruler to measure the sides of the rectangle shown.



a. Write the length and width of the rectangle. Tell how many significant digits are in each measurement.

b. Find the area of the rectangle. Be sure to use the correct number of significant digits in your answer.

32. Brian drives 426 miles and uses 19.3 gallons of gas for the trip. Brian's calculator shows that $\frac{426}{19.3} \approx 22.07253886$, so he states that his car gets 22.07253886 miles to the gallon. Do you agree with Brian's statement? Why or why not?