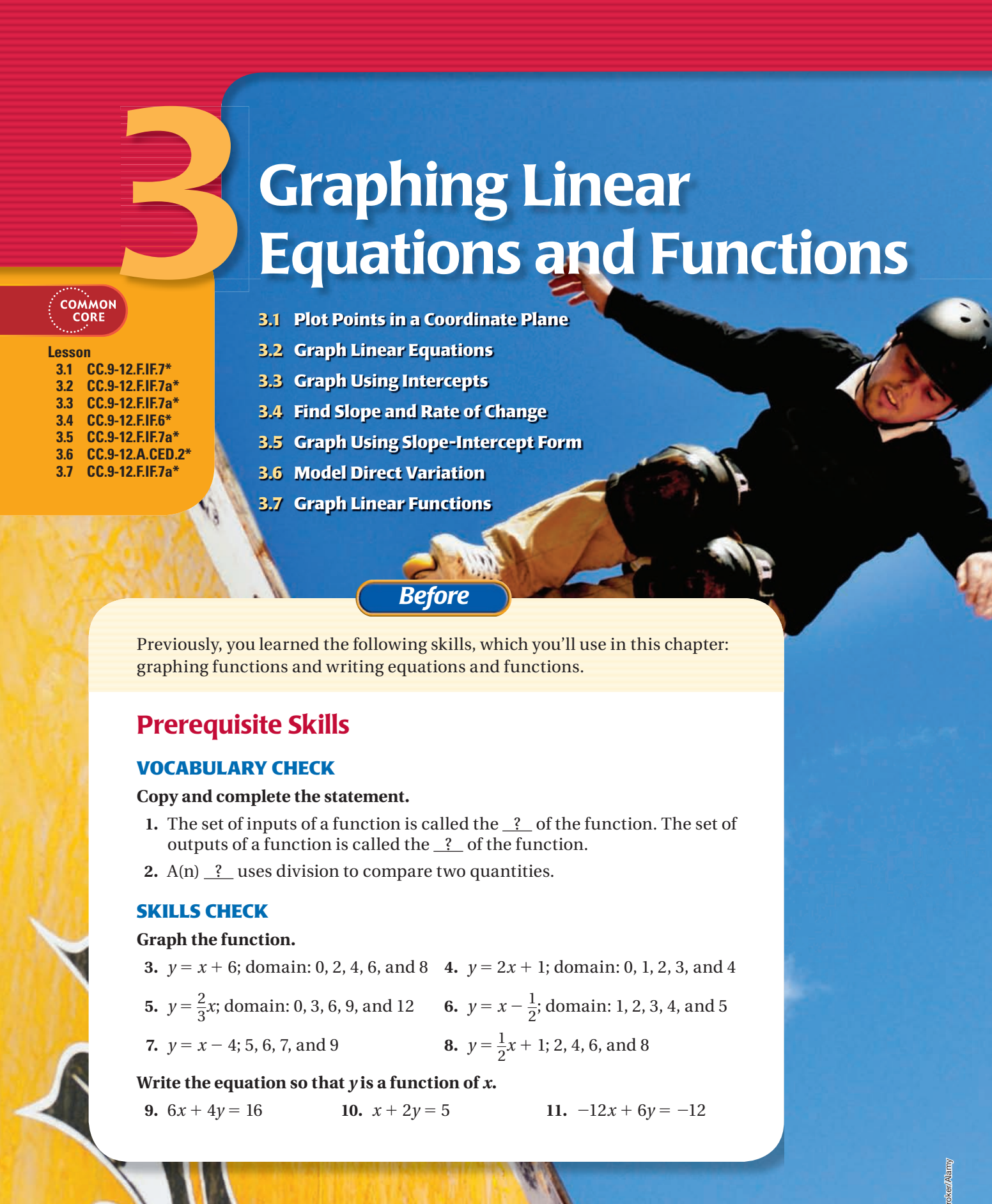
# 3 Graphing Linear Equations and Functions

COMMON CORE

| Lesson | |
|---|---|
| 3.1 | CC.9-12.F.IF.7* |
| 3.2 | CC.9-12.F.IF.7a* |
| 3.3 | CC.9-12.F.IF.7a* |
| 3.4 | CC.9-12.F.IF.6* |
| 3.5 | CC.9-12.F.IF.7a* |
| 3.6 | CC.9-12.A.CED.2* |
| 3.7 | CC.9-12.F.IF.7a* |





## Before

Previously, you learned the following skills, which you'll use in this chapter: graphing functions and writing equations and functions.

## Prerequisite Skills

### VOCABULARY CHECK

**Copy and complete the statement.**

1. The set of inputs of a function is called the __?__ of the function. The set of outputs of a function is called the __?__ of the function.
2. A(n) __?__ uses division to compare two quantities.

### SKILLS CHECK

**Graph the function.**

3. $y = x + 6$; domain: 0, 2, 4, 6, and 8
4. $y = 2x + 1$; domain: 0, 1, 2, 3, and 4
5. $y = \frac{2}{3}x$; domain: 0, 3, 6, 9, and 12
6. $y = x - \frac{1}{2}$; domain: 1, 2, 3, 4, and 5
7. $y = x - 4$; 5, 6, 7, and 9
8. $y = \frac{1}{2}x + 1$; 2, 4, 6, and 8

**Write the equation so that $y$ is a function of $x$.**

9. $6x + 4y = 16$
10. $x + 2y = 5$
11. $-12x + 6y = -12$

## Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

## Big Ideas

1. **Graphing linear equations and functions using a variety of methods**
2. **Recognizing how changes in linear equations and functions affect their graphs**
3. **Using graphs of linear equations and functions to solve real-world problems**

### Key Vocabulary

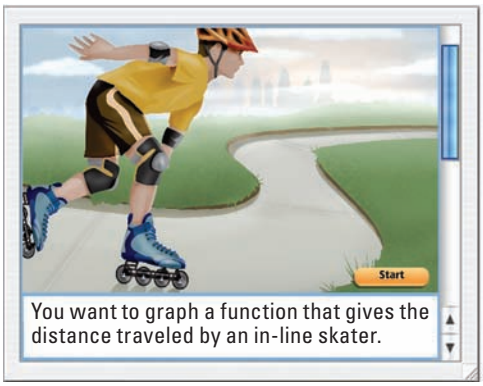
- quadrant
- standard form of a linear equation
- linear function
- *x*-intercept
- *y*-intercept
- slope
- rate of change
- slope-intercept form
- parallel
- direct variation
- constant of variation
- function notation
- family of functions
- parent linear function

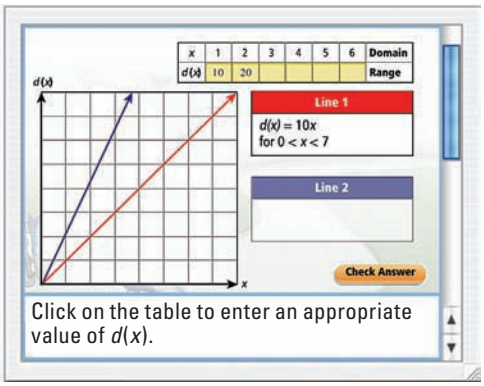
## Why?

You can graph linear functions to solve problems involving distance. For example, you can graph a linear function to find the time it takes and in-line skater to travel a particular distance at a particular speed.

### Animated Algebra

The animation illustrated below helps you answer a question from this chapter: How can you graph a function that models the distance an in-line skater travels over time?




Animated Algebra at my.hrw.com

# 3.1 Plot Points in a Coordinate Plane

**Before** You graphed numbers on a number line.

**Now** You will identify and plot points in a coordinate plane.

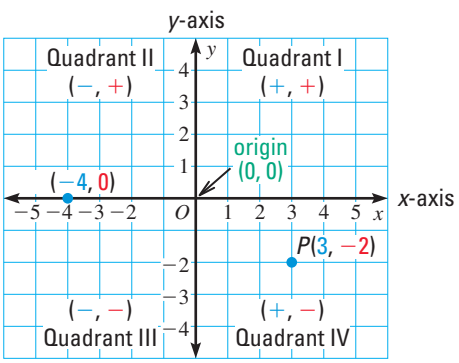
**Why?** So you can interpret photos of Earth taken from space, as in Ex. 36.

**Key Vocabulary**
- **quadrants**
- **coordinate plane**
- **ordered pair**

You have used a coordinate plane to graph ordered pairs whose coordinates were nonnegative. If you extend the $x$-axis and $y$-axis to include negative values, you divide the coordinate plane into four regions called **quadrants**, labeled I, II, III, and IV as shown.

Points in Quadrant I have two positive coordinates. Points in the other three quadrants have at least one negative coordinate.

For example, point $P$ is in Quadrant IV and has an $x$-coordinate of 3 and a $y$-coordinate of $-2$. A point on an axis, such as point $Q$, is not considered to be in any of the four quadrants.




**READING**
The $x$-coordinate of a point is sometimes called the *abscissa*. The $y$-coordinate of a point is sometimes called the *ordinate*.

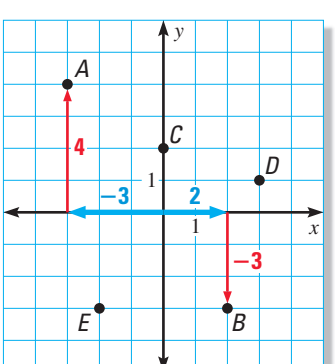
COMMON CORE

**CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

## EXAMPLE 1 Name points in a coordinate plane

**Give the coordinates of the point.**

**a.** $A$ **b.** $B$




**Solution**

**a.** Point $A$ is 3 units to the left of the origin and 4 units up. So, the $x$-coordinate is $-3$, and the $y$-coordinate is 4. The coordinates are $(-3, 4)$.

**b.** Point $B$ is 2 units to the right of the origin and 3 units down. So, the $x$-coordinate is 2, and the $y$-coordinate is $-3$. The coordinates are $(2, -3)$.

## GUIDED PRACTICE for Example 1

1. Use the coordinate plane in Example 1 to give the coordinates of points $C$, $D$, and $E$.
2. What is the $y$-coordinate of any point on the $x$-axis?

## EXAMPLE 2 Plot points in a coordinate plane

**Plot the point in a coordinate plane. Describe the location of the point.**

**a.** $A(-4, 4)$ **b.** $B(3, -2)$ **c.** $C(0, -4)$

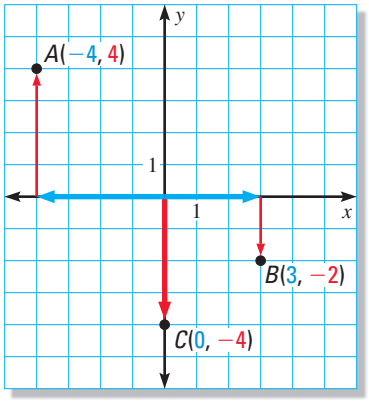
**Solution**

**a.** Begin at the origin. First move 4 units to the left, then 4 units up. Point $A$ is in Quadrant II.

**b.** Begin at the origin. First move 3 units to the right, then 2 units down. Point $B$ is in Quadrant IV.

**c.** Begin at the origin and move 4 units down. Point $C$ is on the $y$-axis.

Animated Algebra at my.hrw.com




## EXAMPLE 3 Graph a function

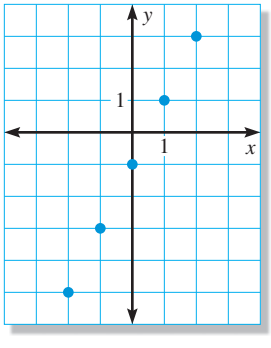
**Graph the function $y = 2x - 1$ with domain $-2, -1, 0, 1$, and 2. Then identify the range of the function.**

**Solution**

**STEP 1** **Make** a table by substituting the domain values into the function.

| $x$ | $y = 2x - 1$ |
|---|---|
| $-2$ | $y = 2(-2) - 1 = -5$ |
| $-1$ | $y = 2(-1) - 1 = -3$ |
| $0$ | $y = 2(0) - 1 = -1$ |
| $1$ | $y = 2(1) - 1 = 1$ |
| $2$ | $y = 2(2) - 1 = 3$ |

**STEP 2** **List** the ordered pairs: $(-2, -5)$, $(-1, -3)$, $(0, -1)$, $(1, 1)$, $(2, 3)$. Then graph the function.




**STEP 3** **Identify** the range. The range consists of the $y$-values from the table: $-5, -3, -1, 1$, and 3.

**ANALYZE A FUNCTION**

The function in Example 3 is called a *discrete* function. The graphs of discrete functions consist of isolated points.

## GUIDED PRACTICE for Examples 2 and 3

**Plot the point in a coordinate plane. *Describe* the location of the point.**

**3.** $A(2, 5)$ **4.** $B(-1, 0)$ **5.** $C(-2, -1)$ **6.** $D(-5, 3)$

**7.** Graph the function $y = -\frac{1}{3}x + 2$ with domain $-6, -3, 0, 3$, and 6. Then identify the range of the function.

## EXAMPLE 4 Graph a function represented by a table

**VOTING** In 1920 the ratification of the 19th amendment to the United States Constitution gave women the right to vote. The table shows the number (to the nearest million) of votes cast in presidential elections both before and since women were able to vote.

Presidential campaign button

−4 means 4 years before 1920, or 1916.

0 represents the year 1920.

| Years before or since 1920 | −12 | −8 | −4 | 0 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|---|
| Votes (millions) | 15 | 15 | 19 | 27 | 29 | 37 | 40 |

**a.** Explain how you know that the table represents a function.

**b.** Graph the function represented by the table.

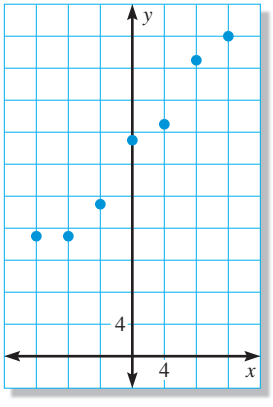
**c.** Describe any trend in the number of votes cast.

### Solution

**a.** The table represents a function because each input has exactly one output.

**b.** To graph the function, let $x$ be the number of years before or since 1920. Let $y$ be the number of votes cast (in millions).

The graph of the function is shown.




**c.** In the three election years before 1920, the number of votes cast was less than 20 million. In 1920, the number of votes cast was greater than 20 million. The number of votes cast continued to increase in the three election years since 1920.

## GUIDED PRACTICE for Example 4

**8. VOTING** The presidential election in 1972 was the first election in which 18-year-olds were allowed to vote. The table shows the number (to the nearest million) of votes cast in presidential elections both before and since 1972.

| Years before or since 1972 | −12 | −8 | −4 | 0 | 4 | 8 | 12 |
|---|---|---|---|---|---|---|---|
| Votes (millions) | 69 | 71 | 73 | 78 | 82 | 87 | 93 |

**a.** *Explain* how you know the graph represents a function.

**b.** Graph the function represented by the table.

**c.** *Describe* any trend in the number of votes cast.

# 3.1 EXERCISES

**HOMEWORK KEY**

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 15, 25, and 37
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 23, 33, and 41
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 40

## SKILL PRACTICE

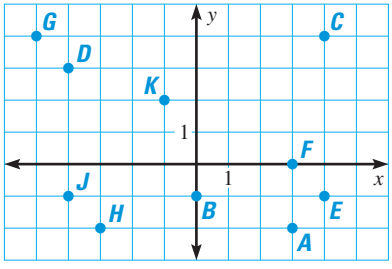
1. **VOCABULARY** What is the $x$-coordinate of the point $(5, -3)$? What is the $y$-coordinate?

2. ★ **WRITING** One of the coordinates of a point is negative while the other is positive. Can you determine the quadrant in which the point lies? *Explain*.

**EXAMPLE 1** for Exs. 3–13

**NAMING POINTS** **Give the coordinates of the point.**

3. $A$
4. $B$
5. $C$
6. $D$
7. $E$
8. $F$
9. $G$
10. $H$
11. $J$
12. $K$




13. ★ **MULTIPLE CHOICE** A point is located 3 units to the left of the origin and 6 units up. What are the coordinates of the point?

Ⓐ $(3, 6)$ Ⓑ $(-3, 6)$ Ⓒ $(6, 3)$ Ⓓ $(6, -3)$

**EXAMPLE 2** for Exs. 14–22

**PLOTTING POINTS** **Plot the point in a coordinate plane. *Describe* the location of the point.**

14. $P(5, 5)$
15. $Q(-1, 5)$
16. $R(-3, 0)$
17. $S(0, 0)$
18. $T(-3, -4)$
19. $U(0, 6)$
20. $V(1.5, 4)$
21. $W(3, -2.5)$

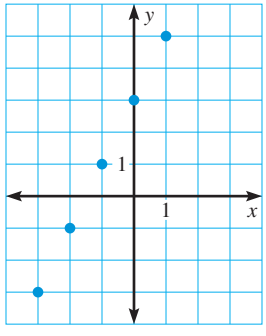
22. **ERROR ANALYSIS** *Describe* and correct the error in describing the location of the point $W(6, -6)$.

> Point $W(6, -6)$ is 6 units to the left of the origin and 6 units up.

**EXAMPLE 3** for Exs. 23–27

23. ★ **MULTIPLE CHOICE** Which number is in the range of the function whose graph is shown?

Ⓐ $-2$ Ⓑ $-1$ Ⓒ $0$ Ⓓ $2$



**GRAPHING FUNCTIONS** **Graph the function with the given domain. Then identify the range of the function.**

**24.** $y = -x + 1$; domain: $-2, -1, 0, 1, 2$

**25.** $y = 2x - 5$; domain: $-2, -1, 0, 1, 2$

**26.** $y = -\frac{2}{3}x - 1$; domain: $-6, -3, 0, 3, 6$

**27.** $y = \frac{1}{2}x + 1$; domain: $-6, -4, -2, 0, 2$

**28.** **GEOMETRY** Plot the points $W(-4, -2)$, $X(-4, 4)$, $Y(4, 4)$, and $Z(4, -2)$ in a coordinate plane. Connect the points in order. Connect point $Z$ to point $W$. Identify the resulting figure. Find its perimeter and area.

**REASONING** **Without plotting the point, tell whether it is in Quadrant I, II, III, or IV. *Explain* your reasoning.**

**29.** $(4, -11)$ **30.** $(40, -40)$ **31.** $(-18, 15)$ **32.** $(-32, -22)$

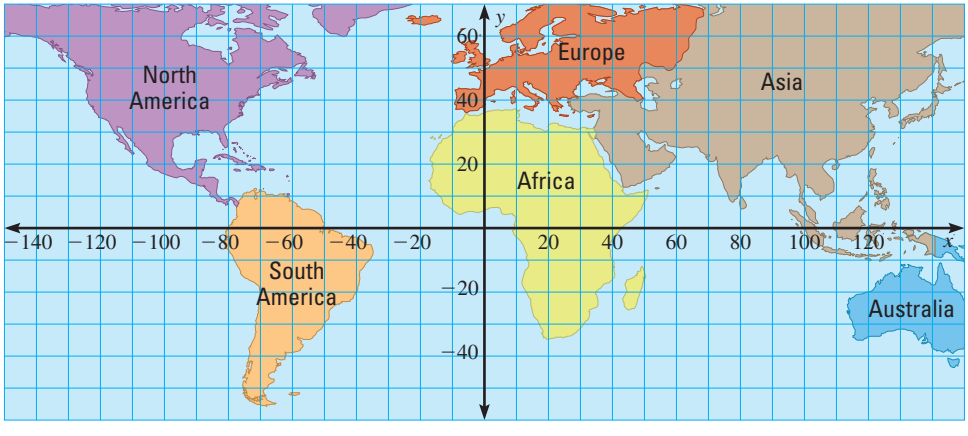
**33.** ★ **WRITING** *Explain* how can you tell by looking at the coordinates of a point whether the point is on the $x$-axis or on the $y$-axis.

**34.** **REASONING** Plot the point $J(-4, 3)$ in a coordinate plane. Plot three additional points in the same coordinate plane so that each of the four points lies in a different quadrant and the figure formed by connecting the points is a square. *Explain* how you located the points.

**35.** **CHALLENGE** Suppose the point $(a, b)$ lies in Quadrant IV. *Describe* the location of the following points: $(b, a)$, $(2a, -2b)$, and $(-b, -a)$. *Explain* your reasoning.

## PROBLEM SOLVING

**36.** **ASTRONAUT PHOTOGRAPHY** Astronauts use a coordinate system to describe the locations of objects they photograph from space. The $x$-axis is the equator, 0° latitude. The $y$-axis is the prime meridian, 0° longitude. The names and coordinates of some lakes photographed from space are given. Use the map to determine on which continent each lake is located.




**a.** Lake Kulundinskoye: (80, 53)

**b.** Lake Champlain: (−73, 45)

**c.** Lake Van: (43, 39)

**d.** Lake Viedma: (−73, −50)

**e.** Lake Saint Clair: (−83, 43)

**f.** Starnberger Lake: (12, 48)



○ = See WORKED-OUT SOLUTIONS in Student Resources

★ = STANDARDIZED TEST PRACTICE

= MULTIPLE REPRESENTATIONS

**EXAMPLE 4**
for Exs. 37–39

**37. RECORD TEMPERATURES** The table shows the record low temperatures (in degrees Fahrenheit) for Odessa, Texas, for each day in the first week of February. *Explain* how you know the table represents a function. Graph the data from the table.

| Day in February | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Record low (degrees Fahrenheit) | −8 | −11 | 10 | 8 | 10 | 9 | 11 |

**38. STOCK VALUE** The table shows the change in value (in dollars) of a stock over five days.

| Day | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Change in value (dollars) | −0.30 | 0.10 | 0.15 | 0.35 | 0.11 |

**a.** *Explain* how you know the table represents a function. Graph the data from the table.

**b.** *Describe* any trend in the change in value of the stock.

**39. MULTI-STEP PROBLEM** The difference between what the federal government collects and what it spends during a fiscal year is called the federal surplus or deficit. The table shows the federal surplus or deficit (in billions of dollars) in the 1990s. (A negative number represents a deficit.)

| Years since 1990 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Surplus or deficit (billions) | −221 | −269 | −290 | −255 | −203 | −164 | −108 | −22 | 69 | 126 |

**a.** Graph the function represented by the table.

**b.** What conclusions can you make from the graph?

**40. MULTIPLE REPRESENTATIONS** Low-density lipoproteins (LDL) transport cholesterol in the bloodstream throughout the body. A high LDL number is associated with an increased risk of cardiovascular disease. A patient's LDL number in 1999 was 189 milligrams per deciliter (mg/dL). To lower that number, the patient went on a diet. The annual LDL numbers for the patient in years after 1999 are 169, 154, 145, 139, and 136

| Years since 1999 | 1 | 2 | ? | ? | ? |
|---|---|---|---|---|---|
| Annual changes in LDL (mg/dL) | −20 | −15 | ? | ? | ? |

**a. Making a Table** Use the given information to copy and complete the table that shows the annual change in the patient's LDL number since 1999.

**b. Drawing a Graph** Graph the ordered pairs from the table.

**c. Describing in Words** Based on the graph, what can you conclude about the diet's effectiveness in lowering the patient's LDL number?

**41.** ★ **EXTENDED RESPONSE** In a scientific study, researchers asked men to report their heights and weights. Then the researchers measured the actual heights and weights of the men. The data for six men are shown in the table. One row of the table represents the data for one man.

| Height (inches) | | | Weight (pounds) | | |
|---|---|---|---|---|---|
| **Reported** | **Measured** | **Difference** | **Reported** | **Measured** | **Difference** |
| 70 | 68 | $70 - 68 = 2$ | 154 | 146 | $154 - 146 = 8$ |
| 70 | 67.5 | ? | 141 | 143 | ? |
| 78.5 | 77.5 | ? | 165 | 168 | ? |
| 68 | 69 | ? | 146 | 143 | ? |
| 71 | 72 | ? | 220 | 223 | ? |
| 70 | 70 | ? | 176 | 176 | ? |

**a. Calculate** Copy and complete the table.

**b. Graph** For each participant, write an ordered pair $(x, y)$ where $x$ is the difference of the reported and measured heights and $y$ is the difference of the reported and measured weights. Then plot the ordered pairs in a coordinate plane.

**c. CHALLENGE** What does the origin represent in this situation?

**d. CHALLENGE** Which quadrant has the greatest number of points? *Explain* what it means for a point to be in that quadrant.