

**CHAPTER
3****Distinguishing Between Discrete and Continuous Variables**

The usual reason for a function $y = f(x)$ to be discrete rather than continuous is that one or both of the variables x and y are discrete variables. We are used to thinking of variables as ranging over the real numbers, but sometimes a variable can only take on discrete values, spaced out along the real number line.

The main use of discrete variables is for quantities of objects that must be present either as a whole or not at all. Whether this applies to a given type of object can depend on the context.

EXAMPLE 1 Distinguish between discrete and continuous variables

Would each of the named quantities be best represented by a continuous variable or a discrete one?

- a. gallons of milk in a family refrigerator b. gallons of milk in a store refrigerator

Solution:

- a. Someone checking on how much milk a family has in its refrigerator will count a half-gallon container, or a partly-full gallon container, as a fraction of a gallon. This context calls for a continuous variable.
- b. Someone counting milk containers at a store (e.g., for inventory) will use whole numbers only. Gallon, half-gallon, and quart containers will be counted separately. This context calls for a discrete variable to represent gallons of milk. ■

Normally, discrete variables take on integer values. Sometimes, however, they can represent quantities that are not necessarily integer-valued.

EXAMPLE 2 Identify discrete values

What are the discrete possible values in each case?

- a. The price, in dollars, of an item on a store shelf.
- b. Number of people that show up for a meeting
- c. Number of right answers on a multiple-choice test
- d. Miles traveled by a car, as indicated by the odometer

Solution:

- a. Prices must be multiples of \$0.01, so \$0.01, \$0.02, \$0.03, . . .
- b. People are counted using the numbers 0, 1, 2, 3, . . .
- c. Correct answers are counted using the numbers 0, 1, 2, 3, . . .
- d. An odometer indicates miles in increments of 0.1 mile, so 0.0, 0.1, 0.2, 0.3, . . . ■

Distinguishing Between Discrete and Continuous Variables *continued*

In applied problems, we sometimes pretend that discrete variables are continuous, and then we round our results to a nearby discrete value.

EXAMPLE 3 Round to a discrete value

The population of a small town, t years after 2000, is given by $P(t) = 34e^{0.08t}$. What is the projected 2009 population? (Note: e is an irrational number with a value of approximately 2.72. Use the e^x key on your graphing calculator for this example.)

Solution:

The year 2009 corresponds to $t = 9$, and $P(9) = 34e^{0.08 \cdot 9} \approx 69.85$. Rounded to the nearest whole number, the population in 2009 is projected to be 70 people. ■

Practice

State whether each of the following quantities is a discrete or a continuous variable.

1. Number of pages in a book
2. Amount of liquid in a glass
3. Gas prices (which typically end with “9/10”)
4. Olympic finishing times in the men’s 10k Olympic biathlon (which are measured to the nearest tenth-second).
5. Distance traveled by an airplane
6. Oven temperature in recipes (which are usually in increments of 25°F)
7. Number of apples on a tree
8. **Writing** Suppose we make it a practice to write the value of any variable as a decimal rounded to three decimal places. Does this mean all the variables we are working with are discrete variables? Why or why not?