3.2 Graph Linear Equations

Before You plotted points in a coordinate plane.
Now You will graph linear equations in a coordinate plane.
Why? So you can find how meteorologists collect data, as in Ex. 40.

Key Vocabulary
- **standard form of a linear equation**
- **linear function**

An example of an equation in two variables is $2x + 5y = 8$. A **solution of an equation in two variables**, x and y, is an ordered pair (x, y) that produces a true statement when the values of x and y are substituted into the equation.

★ EXAMPLE 1 Standardized Test Practice

Which ordered pair is a solution of $3x - y = 7$?

Ⓐ $(3, 4)$ Ⓑ $(1, -4)$ Ⓒ $(5, -3)$ Ⓓ $(-1, -2)$

Solution

Check whether each ordered pair is a solution of the equation.

Test (3, 4):	$3x - y = 7$	Write original equation.
	$3(3) - 4 \stackrel{?}{=} 7$	Substitute 3 for x and 4 for y.
	$5 = 7$ ✗	Simplify.
Test (1, −4):	$3x - y = 7$	Write original equation.
	$3(1) - (-4) \stackrel{?}{=} 7$	Substitute 1 for x and −4 for y.
	$7 = 7$ ✓	Simplify.

So, $(3, 4)$ is *not* a solution, but $(1, -4)$ is a solution of $3x - y = 7$.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 1

1. Tell whether $\left(4, -\frac{1}{2}\right)$ is a solution of $x + 2y = 5$.

GRAPHS The **graph of an equation in two variables** is the set of points in a coordinate plane that represent all solutions of the equation. If the variables in an equation represent real numbers, one way to graph the equation is to make a table of values, plot enough points to recognize a pattern, and then connect the points. When making a table of values, choose convenient values of x that include negative values, zero, and positive values.

U.S. Navy photo

EXAMPLE 2 Graph an equation

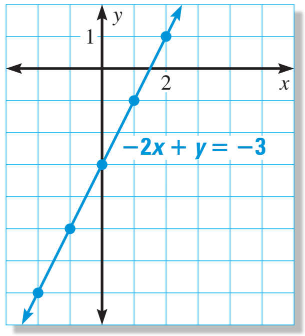
Graph the equation $-2x + y = -3$.

Solution

STEP 1 **Solve** the equation for y.

$$-2x + y = -3$$

$$y = 2x - 3$$



> **DRAW A GRAPH**
> If you continued to find solutions of the equation and plotted them, the line would fill in.

STEP 2 **Make** a table by choosing a few values for x and finding the values of y.

x	−2	−1	0	1	2
y	−7	−5	−3	−1	1

STEP 3 **Plot** the points. Notice that the points appear to lie on a line.

STEP 4 **Connect** the points by drawing a line through them. Use arrows to indicate that the graph goes on without end.

LINEAR EQUATIONS A **linear equation** is an equation whose graph is a line, such as the equation in Example 2. The **standard form** of a linear equation is

$$Ax + By = C$$

where A, B, and C are real numbers and A and B are not both zero.

Consider what happens when $A = 0$ or when $B = 0$. When $A = 0$, the equation becomes $By = C$, or $y = \frac{C}{B}$. Because $\frac{C}{B}$ is a constant, you can write $y = b$. Similarly, when $B = 0$, the equation becomes $Ax = C$, or $x = \frac{C}{A}$, and you can write $x = a$.

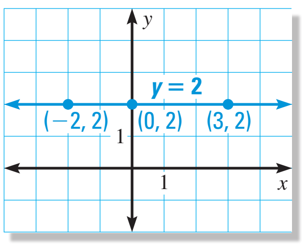
EXAMPLE 3 Graph $y = b$ and $x = a$

Graph (a) $y = 2$ and (b) $x = -1$.

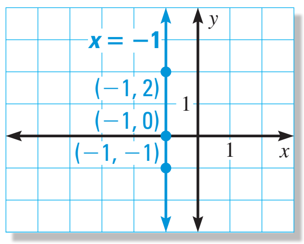
Solution

> **FIND A SOLUTION**
> The equations $y = 2$ and $0x + 1y = 2$ are equivalent. For any value of x, the ordered pair $(x, 2)$ is a solution of $y = 2$.

a. For every value of x, the value of y is 2. The graph of the equation $y = 2$ is a horizontal line 2 units above the x-axis.



b. For every value of y, the value of x is -1. The graph of the equation $x = -1$ is a vertical line 1 unit to the left of the y-axis.



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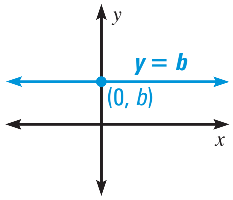
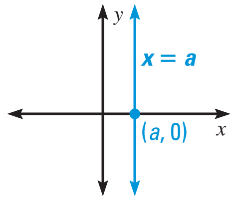
KEY CONCEPT *For Your Notebook*

Equations of Horizontal and Vertical Lines



The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.



The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

✓ GUIDED PRACTICE for Examples 2 and 3

Graph the equation.

2. $y + 3x = -2$ **3.** $y = 2.5$ **4.** $x = -4$

IDENTIFY A FUNCTION
The function $y = 2$ is a *constant function*. The graph of a constant function is a horizontal line.

LINEAR FUNCTIONS In Example 3, $y = 2$ is a function, while $x = -1$ is not a function. The equation $Ax + By = C$ represents a **linear function** provided $B \neq 0$ (that is, provided the graph of the equation is not a vertical line). If the domain of a linear function is not specified, it is understood to be all real numbers. The domain can be restricted, as shown in Example 4.

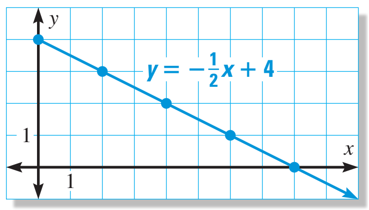
EXAMPLE 4 Graph a linear function

Graph the function $y = -\frac{1}{2}x + 4$ with domain $x \geq 0$. Then identify the range of the function.

Solution

STEP 1 **Make** a table.

x	0	2	4	6	8
y	4	3	2	1	0



STEP 2 **Plot** the points.

STEP 3 **Connect** the points with a ray because the domain is restricted.

STEP 4 **Identify** the range. From the graph, you can see that all points have a y-coordinate of 4 or less, so the range of the function is $y \leq 4$.

ANALYZE A FUNCTION
The function in Example 4 is called a *continuous* function. The graphs of continuous functions are unbroken.

✓ GUIDED PRACTICE for Example 4

5. Graph the function $y = -3x + 1$ with domain $x \leq 0$. Then identify the range of the function.

EXAMPLE 5 Solve a multi-step problem

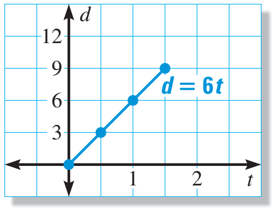
RUNNING The distance d (in miles) that a runner travels is given by the function $d = 6t$ where t is the time (in hours) spent running. The runner plans to go for a 1.5 hour run. Graph the function and identify its domain and range.

Solution

STEP 1 **Identify** whether the problem specifies the domain or the range. You know the amount of time the runner plans to spend running. Because time is the independent variable, the domain is specified in this problem. The domain of the function is $0 \le t \le 1.5$.

STEP 2 **Graph** the function. Make a table of values. Then plot and connect the points.

***t* (hours)**	0	0.5	1	1.5
***d* (miles)**	0	3	6	9



STEP 3 **Identify** the unspecified domain or range. From the table or graph, you can see that the range of the function is $0 \le d \le 9$.

ANALYZE GRAPHS
In Example 2, the domain is unrestricted, and the graph is a *line*. In Example 4, the domain is restricted to $x \ge 0$, and the graph is a *ray*. Here, the domain is restricted to $0 \le t \le 1.5$, and the graph is a *line segment*.

EXAMPLE 6 Solve a related problem

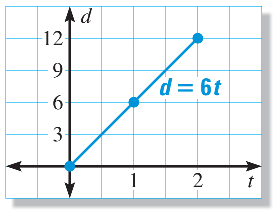
WHAT IF? Suppose the runner in Example 5 instead plans to run 12 miles. Graph the function and identify its domain and range.

Solution

STEP 1 **Identify** whether the problem specifies the domain or the range. You are given the distance that the runner plans to travel. Because distance is the dependent variable, the range is specified in this problem. The range of the function is $0 \le d \le 12$.

STEP 2 **Graph** the function. To make a table, you can substitute d-values (be sure to include 0 and 12) into the function $d = 6t$ and solve for t.

***t* (hours)**	0	1	2
***d* (miles)**	0	6	12



STEP 3 **Identify** the unspecified domain or range. From the table or graph, you can see that the domain of the function is $0 \le t \le 2$.

SOLVE FOR *t*
To find the time it takes the runner to run 12 miles, solve the equation $6t = 12$ to get $t = 2$.

✓ GUIDED PRACTICE for Examples 5 and 6

6. **GAS COSTS** For gas that costs \$2 per gallon, the equation $C = 2g$ gives the cost C (in dollars) of pumping g gallons of gas. You plan to pump \$10 worth of gas. Graph the function and identify its domain and range.

3.2 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 3, 11, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 10, 32, 33, 39, and 41

◆ = **MULTIPLE REPRESENTATIONS** Ex. 40

SKILL PRACTICE

1. VOCABULARY The equation $Ax + By = C$ represents a(n) _?_ provided $B \neq 0$.

2. ★ WRITING Is the equation $y = 6x + 4$ in standard form? *Explain.*

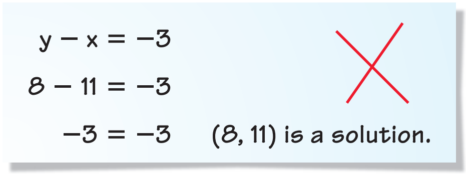
EXAMPLE 1 for Exs. 3–10

CHECKING SOLUTIONS **Tell whether the ordered pair is a solution of the equation.**

3. $2y + x = 4$; $(-2, 3)$ **4.** $3x - 2y = -5$; $(-1, 1)$ **5.** $x = 9$; $(9, 6)$

6. $y = -7$; $(-7, 0)$ **7.** $-7x - 4y = 1$; $(-3, -5)$ **8.** $-5y - 6x = 0$; $(-6, 5)$

9. ERROR ANALYSIS *Describe* and correct the error in determining whether (8, 11) is a solution of $y - x = -3$.



10. ★ MULTIPLE CHOICE Which ordered pair is a solution of $6x + 3y = 18$?

Ⓐ $(-2, -10)$ Ⓑ $(-2, 10)$ Ⓒ $(2, 10)$ Ⓓ $(10, -2)$

EXAMPLES 2 and 3 for Exs. 11–25

GRAPHING EQUATIONS **Graph the equation.**

11. $y + x = 2$ **12.** $y - 2x = 5$ **13.** $y - 3x = 0$ **14.** $y + 4x = 1$

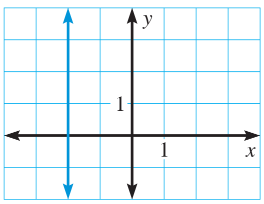
15. $2y - 6x = 10$ **16.** $3y + 4x = 12$ **17.** $x - 2y = 3$ **18.** $3x + 2y = 8$

19. $x = 0$ **20.** $y = 0$ **21.** $y = -4$ **22.** $x = 2$

MATCHING EQUATIONS WITH GRAPHS **Match the equation with its graph.**

23. $y - x = 0$ **24.** $x = -2$ **25.** $y = -1$

A.



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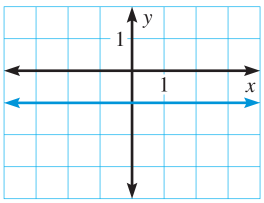
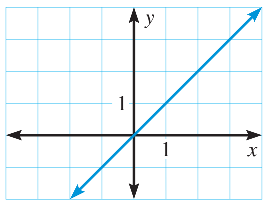


C.



EXAMPLE 4 for Exs. 26–31

GRAPHING FUNCTIONS **Graph the function with the given domain. Then identify the range of the function.**

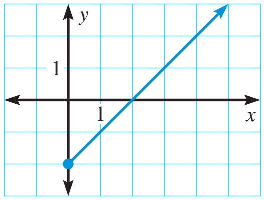
26. $y = 3x - 2$; domain: $x \geq 0$ **27.** $y = -5x + 3$; domain: $x \leq 0$

28. $y = 4$; domain: $x \leq 5$ **29.** $y = -6$; domain: $x \geq 5$

30. $y = 2x + 3$; domain: $-4 \leq x \leq 0$ **31.** $y = -x - 1$; domain: $-1 \leq x \leq 3$

32. ★ OPEN-ENDED Graph $x - y = 3$ and $2x - 2y = 6$. *Explain* why the equations look different but have the same graph. Find another equation that looks different from the two given equations but has the same graph.

33. ★ **MULTIPLE CHOICE** Which statement is true for the function whose graph is shown?



Ⓐ The domain is unrestricted.

Ⓑ The domain is $x \le -2$.

Ⓒ The range is $y \le -2$.

Ⓓ The range is $y \ge -2$.

34. **CHALLENGE** If $(3, n)$ is a solution of $Ax + 3y = 6$ and $(n, 5)$ is a solution of $5x + y = 20$, what is the value of A?

PROBLEM SOLVING

EXAMPLES 5 and 6 for Exs. 35–39

35. **BAKING** The weight w (in pounds) of a loaf of bread that a recipe yields is given by the function $w = \frac{1}{2}f$ where f is the number of cups of flour used. You have 4 cups of flour. Graph the function and identify its domain and range. What is the weight of the largest loaf of bread you can make?

36. **TRAVEL** After visiting relatives who live 200 miles away, your family drives home at an average speed of 50 miles per hour. Your distance d (in miles) from home is given by $d = 200 - 50t$ where t is the time (in hours) spent driving. Graph the function and identify its domain and range. What is your distance from home after driving for 1.5 hours?

37. **EARTH SCIENCE** The temperature T (in degrees Celsius) of Earth's crust can be modeled by the function $T = 20 + 25d$ where d is the distance (in kilometers) from the surface.

a. A scientist studies organisms in the first 4 kilometers of Earth's crust. Graph the function and identify its domain and range. What is the temperature at the deepest part of the section of crust?

b. Suppose the scientist studies organisms in a section of the crust where the temperature is between 20°C and 95°C. Graph the function and identify its domain and range. How many kilometers deep is the section of crust?

38. **MULTI-STEP PROBLEM** A fashion designer orders fabric that costs \$30 per yard. The designer wants the fabric to be dyed, which costs \$100. The total cost C (in dollars) of the fabric is given by the function

$$C = 30f + 100$$

where f is the number of yards of fabric.

a. The designer orders 3 yards of fabric. How much does the fabric cost? *Explain.*

b. Suppose the designer can spend \$500 on fabric. How many yards of fabric can the designer buy? *Explain.*

○ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE** = **MULTIPLE REPRESENTATIONS**

39. ★ **SHORT RESPONSE** An emergency cell phone charger requires you to turn a small crank in order to create the energy needed to recharge the phone's battery. If you turn the crank 120 times per minute, the total number r of revolutions that you turn the crank is given by

$$r = 120t$$

where t is the time (in minutes) spent turning the crank.

a. Graph the function and identify its domain and range.

b. Identify the domain and range if you stop turning the crank after 4 minutes. *Explain* how this affects the appearance of the graph.

40. **MULTIPLE REPRESENTATIONS** The National Weather Service releases weather balloons twice daily at over 90 locations in the United States in order to collect data for meteorologists. The height h (in feet) of a balloon is a function of the time t (in seconds) after the balloon is released, as shown.

a. **Making a Table** Make a table showing the height of a balloon after t seconds for $t = 0$ through $t = 10$.

b. **Drawing a Graph** A balloon bursts after a flight of about 7200 seconds. Graph the function and identify the domain and range.

41. ★ **EXTENDED RESPONSE** Students can pay for lunch at a school in one of two ways. Students can either make a payment of $30 per month or they can buy lunch daily for $2.50 per lunch.

a. **Graph** Graph the function $y = 30$ to represent the monthly payment plan. Using the same coordinate plane, graph the function $y = 2.5x$ to represent the daily payment plan.

b. **CHALLENGE** What are the coordinates of the point that is a solution of both functions? What does that point mean in this situation?

c. **CHALLENGE** A student eats an average of 15 school lunches per month. How should the student pay, daily or monthly? *Explain*.