

# 3.3 Graph Using Intercepts



**Before**

You graphed a linear equation using a table of values.

**Now**

You will graph a linear equation using intercepts.

**Why**

So you can find a submersible's location, as in Example 5.

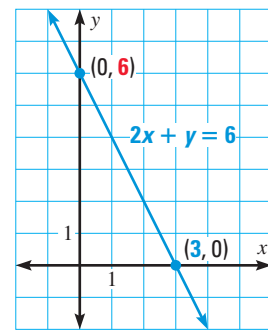
## Key Vocabulary

- **x-intercept**
- **y-intercept**

You can use the fact that two points determine a line to graph a linear equation. Two convenient points are the points where the graph crosses the axes.

An **x-intercept** of a graph is the  $x$ -coordinate of a point where the graph crosses the  $x$ -axis. A **y-intercept** of a graph is the  $y$ -coordinate of a point where the graph crosses the  $y$ -axis.

To find the  $x$ -intercept of the graph of a linear equation, find the value of  $x$  when  $y = 0$ . To find the  $y$ -intercept of the graph, find the value of  $y$  when  $x = 0$ .



**CC.9-12.F.IF.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima.\*

## EXAMPLE 1 Find the intercepts of the graph of an equation

Find the  $x$ -intercept and the  $y$ -intercept of the graph of  $2x + 7y = 28$ .

### Solution

To find the  $x$ -intercept, substitute 0 for  $y$  and solve for  $x$ .

$$2x + 7y = 28 \quad \text{Write original equation.}$$

$$2x + 7(0) = 28 \quad \text{Substitute 0 for } y.$$

$$x = \frac{28}{2} = 14 \quad \text{Solve for } x.$$

To find the  $y$ -intercept, substitute 0 for  $x$  and solve for  $y$ .

$$2x + 7y = 28 \quad \text{Write original equation.}$$

$$2(0) + 7y = 28 \quad \text{Substitute 0 for } x.$$

$$y = \frac{28}{7} = 4 \quad \text{Solve for } y.$$

► The  $x$ -intercept is 14. The  $y$ -intercept is 4.



## GUIDED PRACTICE for Example 1

Find the  $x$ -intercept and the  $y$ -intercept of the graph of the equation.

1.  $3x + 2y = 6$

2.  $4x - 2y = 10$

3.  $-3x + 5y = -15$

## EXAMPLE 2 Use intercepts to graph an equation

Graph the equation  $x + 2y = 4$ .

### Solution

**STEP 1** Find the intercepts.

$$x + 2y = 4$$

$$x + 2(0) = 4$$

$$x = 4 \leftarrow \text{x-intercept}$$

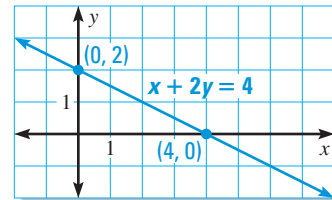
$$x + 2y = 4$$

$$0 + 2y = 4$$

$$y = 2 \leftarrow \text{y-intercept}$$

**STEP 2** Plot points. The x-intercept is 4, so plot the point  $(4, 0)$ . The y-intercept is 2, so plot the point  $(0, 2)$ . Draw a line through the points.

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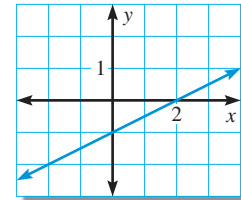


### CHECK A GRAPH

Be sure to check the graph by finding a third solution of the equation and checking to see that the corresponding point is on the graph.

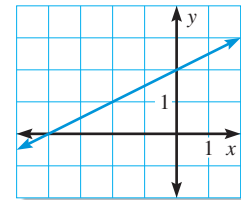
## EXAMPLE 3 Use a graph to find intercepts

The graph crosses the  $x$ -axis at  $(2, 0)$ . The  $x$ -intercept is 2. The graph crosses the  $y$ -axis at  $(0, -1)$ . The  $y$ -intercept is  $-1$ .



### GUIDED PRACTICE for Examples 2 and 3

- Graph  $6x + 7y = 42$ . Label the points where the line crosses the axes.
- Identify the  $x$ -intercept and the  $y$ -intercept of the graph shown at the right.



### KEY CONCEPT

### For Your Notebook

#### Relating Intercepts, Points, and Graphs

##### Intercepts

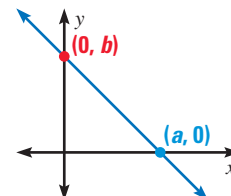
The  $x$  intercept of a graph is  $a$ .

The  $y$ -intercept of a graph is  $b$ .

##### Points

The graph crosses the  $x$ -axis at  $(a, 0)$ .

The graph crosses the  $y$ -axis at  $(0, b)$ .



## EXAMPLE 4 Solve a multi-step problem

**EVENT PLANNING** You are helping to plan an awards banquet for your school, and you need to rent tables to seat 180 people. Tables come in two sizes. Small tables seat 4 people, and large tables seat 6 people. This situation can be modeled by the equation

$$4x + 6y = 180$$

where  $x$  is the number of small tables and  $y$  is the number of large tables.

- Find the intercepts of the graph of the equation.
- Graph the equation.
- Give four possibilities for the number of each size table you could rent.

### Solution

**STEP 1** Find the intercepts.

$$4x + 6y = 180$$

$$4x + 6(0) = 180$$

$$x = 45 \leftarrow \text{x-intercept}$$

$$4x + 6y = 180$$

$$4(0) + 6y = 180$$

$$y = 30 \leftarrow \text{y-intercept}$$

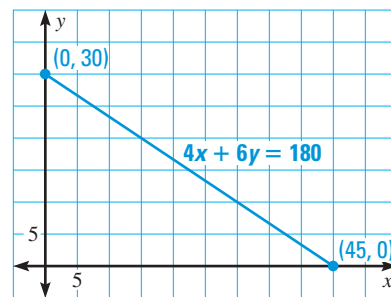
### DRAW A GRAPH

Although  $x$  and  $y$  represent whole numbers, it is convenient to draw an unbroken line segment that includes points whose coordinates are not whole numbers.

**STEP 2** Graph the equation.

The  $x$ -intercept is 45, so plot the point  $(45, 0)$ . The  $y$ -intercept is 30, so plot the point  $(0, 30)$ .

Since  $x$  and  $y$  both represent numbers of tables, neither  $x$  nor  $y$  can be negative. So, instead of drawing a line, draw the part of the line that is in Quadrant I.

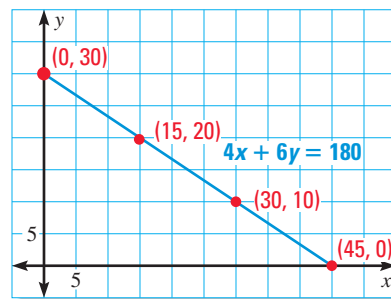


### FIND SOLUTIONS

Other points, such as  $(12, 22)$ , are also on the graph but are not as obvious as the points shown here because their coordinates are not multiples of 5.

**STEP 3** Find the number of tables. For this problem, only whole-number values of  $x$  and  $y$  make sense. You can see that the line passes through the points  $(0, 30)$ ,  $(15, 20)$ ,  $(30, 10)$ , and  $(45, 0)$ .

So, four possible combinations of tables that will seat 180 people are: 0 small and 30 large, 15 small and 20 large, 30 small and 10 large, and 45 small and 0 large.



### GUIDED PRACTICE for Example 4

6. **WHAT IF?** In Example 4, suppose the small tables cost \$9 to rent and the large tables cost \$14. Of the four possible combinations of tables given in the example, which rental is the least expensive? *Explain.*

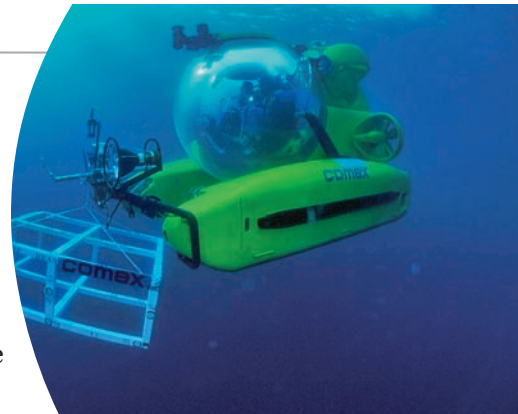
**EXAMPLE 5** Use a linear model

**SUBMERSIBLES** A submersible designed to explore the ocean floor is at an elevation of  $-13,000$  feet (13,000 feet below sea level). The submersible ascends to the surface at an average rate of 650 feet per minute. The elevation  $e$  (in feet) of the submersible is given by the function

$$e = 650t - 13,000$$

where  $t$  is the time (in minutes) since the submersible began to ascend.

- Find the intercepts of the graph of the function and state what the intercepts represent.
- Graph the function and identify its domain and range.

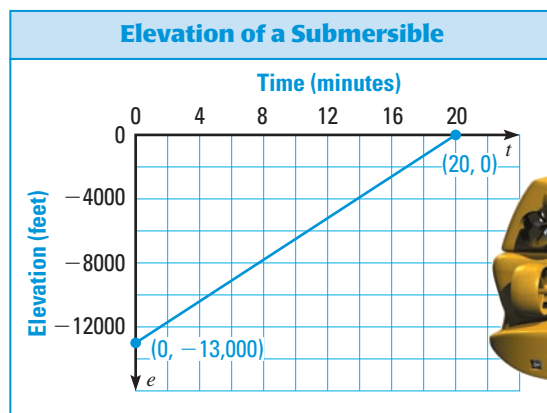
**Solution**

**STEP 1** Find the intercepts.

$$\begin{array}{l|l} 0 = 650t - 13,000 & e = 650(0) - 13,000 \\ 13,000 = 650t & e = -13,000 \leftarrow \text{e-intercept} \\ 20 = t \leftarrow \text{t-intercept} & \end{array}$$

The  $t$ -intercept represents the number of minutes the submersible takes to reach an elevation of 0 feet (sea level). The  $e$ -intercept represents the elevation of the submersible after 0 minutes (the time the ascent begins).

**STEP 2** Graph the function using the intercepts.



The submersible starts at an elevation of  $-13,000$  feet and ascends to an elevation of 0 feet. So, the range of the function is  $-13,000 \leq e \leq 0$ . From the graph, you can see that the domain of the function is  $0 \leq t \leq 20$ .

**NAME INTERCEPTS**

Because  $t$  is the independent variable, the horizontal axis is the  $t$ -axis, and you refer to the “ $t$ -intercept” of the graph of the function. Similarly, the vertical axis is the  $e$ -axis, and you refer to the “ $e$ -intercept.”

**GUIDED PRACTICE** for Example 5

7. **WHAT IF?** In Example 5, suppose the elevation of a second submersible is given by  $e = 500t - 10,000$ . Graph the function and identify its domain and range.

# 3.3 EXERCISES

## HOMWORK KEY

○ = See **WORKED-OUT SOLUTIONS**  
Exs. 21 and 47

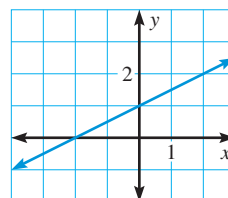
★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 37, 41, 49, and 50

◆ = **MULTIPLE REPRESENTATIONS**  
Ex. 44

### SKILL PRACTICE

- VOCABULARY** Copy and complete: The   ? of the graph of an equation is the value of  $x$  when  $y$  is zero.
- ★ **WRITING** What are the  $x$ -intercept and the  $y$ -intercept of the line passing through the points  $(0, 3)$  and  $(-4, 0)$ ? *Explain.*
- ERROR ANALYSIS** Describe and correct the error in finding the intercepts of the line shown.

The  $x$ -intercept is 1,  
and the  $y$ -intercept is  $-2$ .



**EXAMPLE 1**  
for Exs. 4–15

**FINDING INTERCEPTS** Find the  $x$ -intercept and the  $y$ -intercept of the graph of the equation.

- |                     |                          |                             |
|---------------------|--------------------------|-----------------------------|
| 4. $5x - y = 35$    | 5. $3x - 3y = 9$         | 6. $-3x + 9y = -18$         |
| 7. $4x + y = 4$     | 8. $2x + y = 10$         | 9. $2x - 8y = 24$           |
| 10. $3x + 0.5y = 6$ | 11. $0.2x + 3.2y = 12.8$ | 12. $y = 2x + 24$           |
| 13. $y = -14x + 7$  | 14. $y = -4.8x + 1.2$    | 15. $y = \frac{3}{5}x - 12$ |

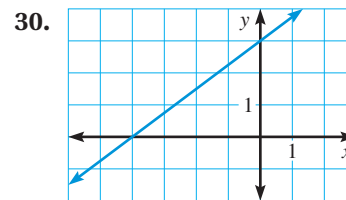
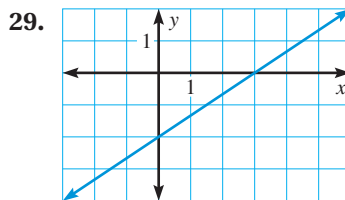
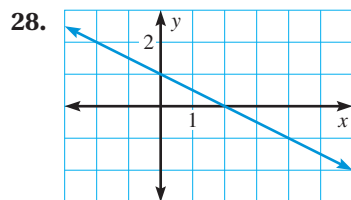
**EXAMPLE 2**  
for Exs. 16–27

**GRAPHING LINES** Graph the equation. Label the points where the line crosses the axes.

- |                     |                     |                                      |
|---------------------|---------------------|--------------------------------------|
| 16. $y = x + 3$     | 17. $y = x - 2$     | 18. $y = 4x - 8$                     |
| 19. $y = 5 + 10x$   | 20. $y = -2 + 8x$   | 21. $y = -4x + 3$                    |
| 22. $3x + y = 15$   | 23. $x - 4y = 18$   | 24. $8x - 5y = 80$                   |
| 25. $-2x + 5y = 15$ | 26. $0.5x + 3y = 9$ | 27. $y = \frac{1}{2}x + \frac{1}{4}$ |

**EXAMPLE 3**  
for Exs. 28–30

**USING GRAPHS TO FIND INTERCEPTS** Identify the  $x$ -intercept and the  $y$ -intercept of the graph.



**USING INTERCEPTS** Draw the line that has the given intercepts.

31.  $x$ -intercept: 3  
 $y$ -intercept: 5

32.  $x$ -intercept:  $-2$   
 $y$ -intercept: 4

33.  $x$ -intercept:  $-5$   
 $y$ -intercept: 6

34.  $x$ -intercept: 9  
 $y$ -intercept:  $-1$

35.  $x$ -intercept:  $-8$   
 $y$ -intercept:  $-11$

36.  $x$ -intercept:  $-2$   
 $y$ -intercept:  $-6$

37. **★ MULTIPLE CHOICE** The  $x$ -intercept of the graph of  $Ax + 5y = 20$  is 2.  
What is the value of  $A$ ?

Ⓐ 2

Ⓑ 5

Ⓒ 7.5

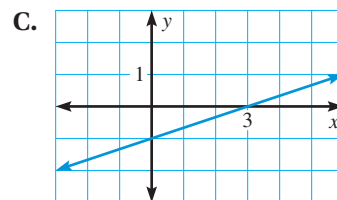
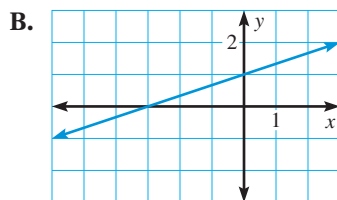
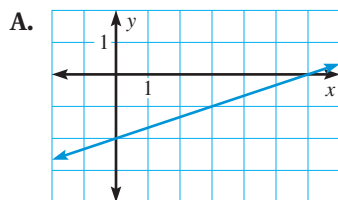
Ⓓ 10

**MATCHING EQUATIONS WITH GRAPHS** Match the equation with its graph.

38.  $2x - 6y = 6$

39.  $2x - 6y = -6$

40.  $2x - 6y = 12$

41. **★ WRITING** Is it possible for a line *not* to have an  $x$ -intercept?  
Is it possible for a line *not* to have a  $y$ -intercept? *Explain.*42. **REASONING** Consider the equation  $3x + 5y = k$ . What values could  $k$  have  
so that the  $x$ -intercept and the  $y$ -intercept of the equation's graph would  
both be integers? *Explain.*43. **CHALLENGE** If  $a \neq 0$ , find the intercepts of the graph of  $y = ax + b$  in  
terms of  $a$  and  $b$ .**PROBLEM SOLVING****EXAMPLES**

4 and 5

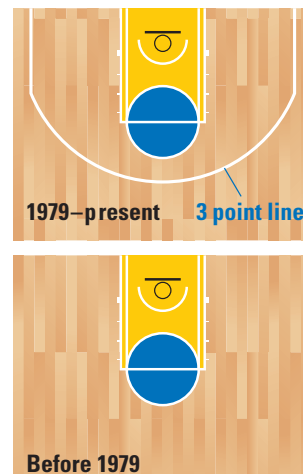
for Exs. 44–47

44. **◆ MULTIPLE REPRESENTATIONS** The perimeter of a rectangular park is  
72 feet. Let  $x$  be the park's width (in feet) and let  $y$  be its length (in feet).a. **Writing an Equation** Write an equation for the perimeter.b. **Drawing a Graph** Find the intercepts of the graph of the equation  
you wrote. Then graph the equation.45. **RECYCLING** In one state, small bottles have a refund value of \$.04 each,  
and large bottles have a refund value of \$.08 each. Your friend returns  
both small and large bottles and receives \$.56. This situation is given by  
 $4x + 8y = 56$  where  $x$  is the number of small bottles and  $y$  is the number  
of large bottles.

a. Find the intercepts of the graph of the equation. Graph the equation.

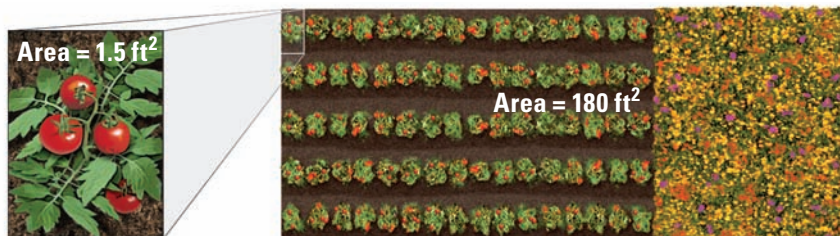
b. Give three possibilities for the number of each size bottle your friend  
could have returned.

46. **MULTI-STEP PROBLEM** Before 1979, there was no 3-point shot in professional basketball; players could score only 2-point field goals and 1-point free throws. In a game before 1979, a team scored a total of 128 points. This situation is given by the equation  $2x + y = 128$  where  $x$  is the possible number of field goals and  $y$  is the possible number of free throws.



- Find the intercepts of the graph of the equation. Graph the equation.
- What do the intercepts mean in this situation?
- What are three possible numbers of field goals and free throws the team could have scored?
- If the team made 24 free throws, how many field goals were made?

47. **COMMUNITY GARDENS** A family has a plot in a community garden. The family is going to plant vegetables, flowers, or both. The diagram shows the area used by one vegetable plant and the area of the entire plot. The area  $f$  (in square feet) of the plot left for flowers is given by  $f = 180 - 1.5v$  where  $v$  is the number of vegetable plants the family plants.



- Find the intercepts of the graph of the function and state what the intercepts represent.
  - Graph the function and identify its domain and range.
  - The family decides to plant 80 vegetable plants. How many square feet are left to plant flowers?
48. **CAR SHARING** A member of a car-sharing program can use a car for \$6 per hour and \$.50 per mile. The member uses the car for one day and is charged \$44. This situation is given by

$$6t + 0.5d = 44$$

where  $t$  is the time (in hours) the car is used and  $d$  is the distance (in miles) the car is driven. Give three examples of the number of hours the member could have used the car and the number of miles the member could have driven the car.

49. **★ SHORT RESPONSE** A humidifier is a device used to put moisture into the air by turning water to vapor. A humidifier has a tank that can hold 1.5 gallons of water. The humidifier can disperse the water at a rate of 0.12 gallon per hour. The amount of water  $w$  (in gallons) left in the humidifier after  $t$  hours of use is given by the function

$$w = 1.5 - 0.12t.$$

After how many hours of use will you have to refill the humidifier?  
*Explain* how you found your answer.

50. ★ **EXTENDED RESPONSE** You borrow \$180 from a friend who doesn't charge you interest. You work out a payment schedule in which you will make weekly payments to your friend. The balance  $B$  (in dollars) of the loan is given by the function  $B = 180 - pn$  where  $p$  is the weekly payment and  $n$  is the number of weeks you make payments.
- Interpret** Without finding the intercepts, state what they represent.
  - Graph** Graph the function if you make weekly payments of \$20.
  - Identify** Find the domain and range of the function in part (b). How long will it take to pay back your friend?
  - CHALLENGE** Suppose you make payments of \$20 for three weeks. Then you make payments of \$15 until you have paid your friend back. How does this affect the graph? How many payments do you make?

## QUIZ

**Plot the point in a coordinate plane. Describe the location of the point.**

1.  $(-7, 2)$

2.  $(0, -5)$

3.  $(2, -6)$

**Graph the equation.**

4.  $-4x - 2y = 12$

5.  $y = -5$

6.  $x = 6$

**Find the  $x$ -intercept and the  $y$ -intercept of the graph of the equation.**

7.  $y = x + 7$

8.  $y = x - 3$

9.  $y = -5x + 2$

10.  $x + 3y = 15$

11.  $3x - 6y = 36$

12.  $-2x - 5y = 22$

13. **SWIMMING POOLS** A public swimming pool that holds 45,000 gallons of water is going to be drained for maintenance at a rate of 100 gallons per minute. The amount of water  $w$  (in gallons) in the pool after  $t$  minutes is given by the function  $w = 45,000 - 100t$ . Graph the function. Identify its domain and range. How much water is in the pool after 60 minutes? How many minutes will it take to empty the pool?

