3.4 Find Slope and Rate of Change

Key Vocabulary
• slope
• rate of change

COMMON CORE

CC.9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

READING

Read x_1 as "x sub one."
Think "x-coordinate of the first point."
Read y_1 as "y sub one."
Think "y-coordinate of the first point."

The **slope** of a nonvertical line is the ratio of the vertical change (the *rise*) to the horizontal change (the *run*) between any two points on the line. The slope of a line is represented by the letter m.

KEY CONCEPT *For Your Notebook*
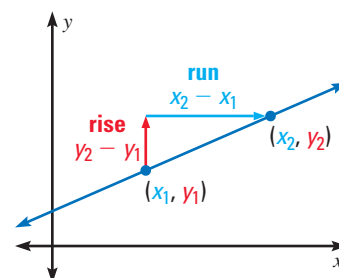
Finding the Slope of a Line

Words

The slope m of the nonvertical line passing through the two points (x_1, y_1) and (x_2, y_2) is the ratio of the rise (change in y) to the run (change in x).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

Symbols

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Graph

EXAMPLE 1 **Find a positive slope**

Find the slope of the line shown.

Let $(x_1, y_1) = (-4, 2)$ and $(x_2, y_2) = (2, 6)$.
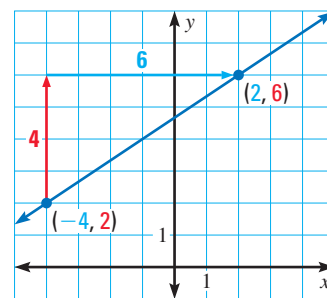
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **Write formula for slope.**

$= \dfrac{6 - 2}{2 - (-4)}$ **Substitute.**

$= \dfrac{4}{6} = \dfrac{2}{3}$ **Simplify.**

AVOID ERRORS

Be sure to keep the x- and y-coordinates in the same order in both the numerator and denominator when calculating slope.

The line rises from left to right. The slope is positive.

✓ **GUIDED PRACTICE** for Example 1

Find the slope of the line that passes through the points.

1. $(5, 2)$ and $(4, -1)$

2. $(-2, 3)$ and $(4, 6)$

3. $\left(\dfrac{9}{2}, 5\right)$ and $\left(\dfrac{1}{2}, -3\right)$

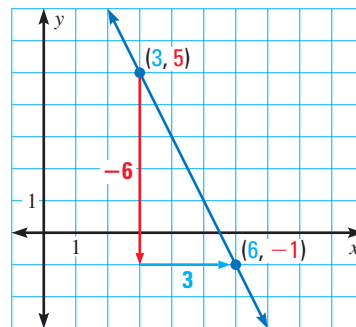© Kevin Fleming/Corbis

EXAMPLE 2 Find a negative slope

Find the slope of the line shown.

Let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (6, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ **Write formula for slope.**

$$= \frac{-1 - 5}{6 - 3}$$ **Substitute.**

$$= \frac{-6}{3} = -2$$ **Simplify.**

The line falls from left to right.
The slope is negative.
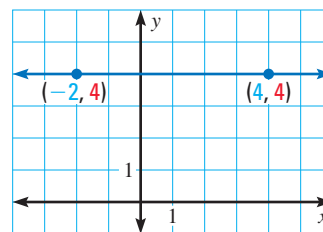
EXAMPLE 3 Find the slope of a horizontal line

Find the slope of the line shown.

Let $(x_1, y_1) = (-2, 4)$ and $(x_2, y_2) = (4, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ **Write formula for slope.**

$$= \frac{4 - 4}{4 - (-2)}$$ **Substitute.**

$$= \frac{0}{6} = 0$$ **Simplify.**

The line is horizontal.
The slope is zero.

EXAMPLE 4 Find the slope of a vertical line

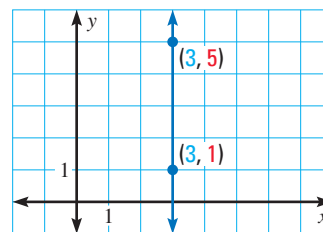**Find the slope of the line shown.**

Let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (3, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ **Write formula for slope.**

$$= \frac{1 - 5}{3 - 3}$$ **Substitute.**

$$= \frac{-4}{0}$$ **Division by zero is undefined.**

The line is vertical.
The slope is undefined.

▶ Because division by zero is undefined, the slope of a vertical line is undefined.

✓ **GUIDED PRACTICE** for Examples 2, 3, and 4

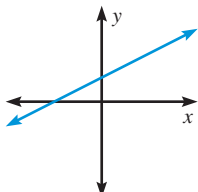**Find the slope of the line that passes through the points.**

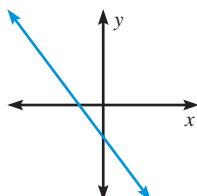**4.** (5, 2) and (5, −2)   **5.** (0, 4) and (−3, 4)   **6.** (0, 6) and (5, −4)
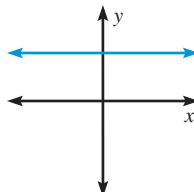
Classification of Lines by Slope

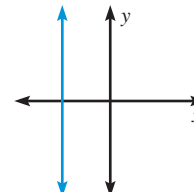A line with positive slope ($m > 0$) *rises* from left to right.

A line with negative slope ($m < 0$) *falls* from left to right.

A line with zero slope ($m = 0$) is *horizontal.*

A line with undefined slope is *vertical.*

RATE OF CHANGE A **rate of change** compares a change in one quantity to a change in another quantity. For example, if you are paid $60 for working 5 hours, then your hourly wage is $12 per hour, a rate of change that describes how your pay increases with respect to time spent working.

EXAMPLE 5 Find a rate of change

INTERNET CAFE The table shows the cost of using a computer at an Internet cafe for a given amount of time. Find the rate of change in cost with respect to time.

Time (hours)	2	4	6
Cost (dollars)	7	14	21

Solution

ANALYZE UNITS
Because the cost is in dollars and time is in hours, the rate of change in cost with respect to time is expressed in dollars per hour.

$$\text{Rate of change} = \frac{\text{change in cost}}{\text{change in time}}$$

$$= \frac{14 - 7}{4 - 2} = \frac{7}{2} = 3.5$$

▶ The rate of change in cost is $3.50 per hour.

✓ **GUIDED PRACTICE** for Example 5

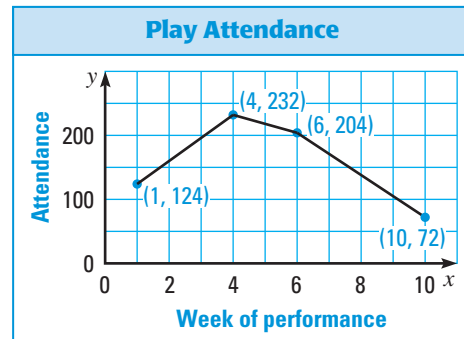7. **EXERCISE** The table shows the distance a person walks for exercise. Find the rate of change in distance with respect to time.

Time (minutes)	Distance (miles)
30	1.5
60	3
90	4.5

SLOPE AND RATE OF CHANGE You can interpret the slope of a line as a rate of change. When given graphs of real-world data, you can compare rates of change by comparing slopes of lines.

EXAMPLE 6   Use a graph to find and compare rates of change

COMMUNITY THEATER A community theater performed a play each Saturday evening for 10 consecutive weeks. The graph shows the attendance for the performances in weeks 1, 4, 6, and 10. Describe the rates of change in attendance with respect to time.

Play Attendance

y axis: Attendance, with points (1, 124), (4, 232), (6, 204), (10, 72). *x* axis: Week of performance, marked 0, 2, 4, 6, 8, 10.

Solution

Find the rates of change using the slope formula.

Weeks 1–4: $\dfrac{232 - 124}{4 - 1} = \dfrac{108}{3} = 36$ people per week

INTERPRET RATE OF CHANGE
A negative rate of change indicates a decrease.

Weeks 4–6: $\dfrac{204 - 232}{6 - 4} = \dfrac{-28}{2} = -14$ people per week

**Weeks 6–10:** $\dfrac{72 - 204}{10 - 6} = \dfrac{-132}{4} = -33$ people per week

▶ Attendance increased during the early weeks of performing the play. Then attendance decreased, slowly at first, then more rapidly.

EXAMPLE 7   Interpret a graph

COMMUTING TO SCHOOL A student commutes from home to school by walking and by riding a bus. Describe the student's commute in words.

y axis: Distance (miles). *x* axis: Time (minutes).
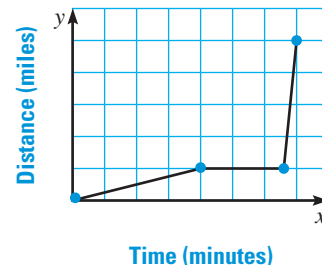
Solution

The first segment of the graph is not very steep, so the student is not traveling very far with respect to time. The student must be walking. The second segment has a zero slope, so the student must not be moving. He or she is waiting for the bus. The last segment is steep, so the student is traveling far with respect to time. The student must be riding the bus.

Animated Algebra at my.hrw.com

✓ **GUIDED PRACTICE**   for Examples 6 and 7

8. WHAT IF? How would the answer to Example 6 change if you knew that attendance was 70 people in week 12?

9. WHAT IF? Using the graph in Example 7, draw a graph that represents the student's commute from school to home.

3.4 **EXERCISES**

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The __?__ of a nonvertical line is the ratio of the vertical change to the horizontal change between any two points on the line.

2. ★ **WRITING** Without calculating the slope, how can you tell that the slope of the line that passes through the points $(-5, -3)$ and $(2, 4)$ is positive?

3. **ERROR ANALYSIS** *Describe* and correct the error in calculating the slope of the line passing through the points $(5, 3)$ and $(2, 6)$.
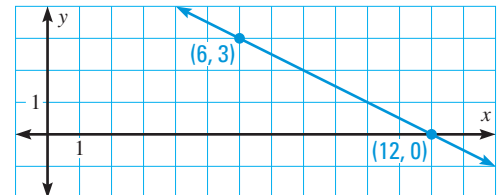
$$m = \frac{6 - 3}{5 - 2} = \frac{3}{3} = 1 \quad ✗$$

EXAMPLES 1, 2, 3, and 4 for Exs. 4–18

FINDING SLOPE **Tell whether the slope of the line is *positive, negative, zero,* or *undefined*. Then find the slope if it exists.**

4.

5.

6.
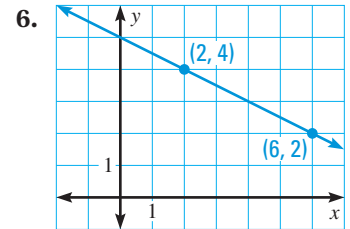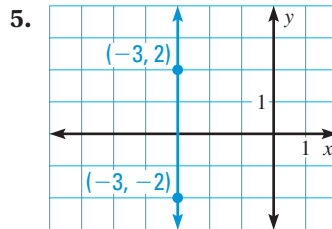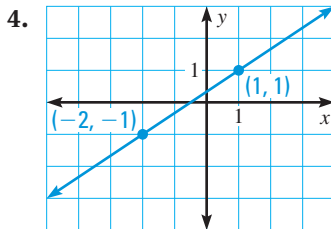
7. **ERROR ANALYSIS** *Describe* and correct the error in calculating the slope of the line shown.

$$m = \frac{12 - 6}{0 - 3} = \frac{6}{-3} = -2 \quad ✗$$

FINDING SLOPE **Find the slope of the line that passes through the points.**

8. $(-2, -1)$ and $(4, 5)$

9. $(-3, -2)$ and $(-3, 6)$

10. $(5, -3)$ and $(-5, -3)$

11. $(1, 3)$ and $(3, -2)$

12. $(-3, 4)$ and $(4, 1)$

13. $(1, -3)$ and $(7, 3)$

14. $(0, 0)$ and $(0, -6)$

15. $(-9, 1)$ and $(1, 1)$

16. $(-10, -2)$ and $(-8, 8)$

17. ★ **MULTIPLE CHOICE** The slope of the line that passes through the points $(-2, -3)$ and $(8, -3)$ is __?__.

Ⓐ positive   Ⓑ negative   Ⓒ zero   Ⓓ undefined

18. ★ **MULTIPLE CHOICE** What is the slope of the line that passes through the points $(7, -9)$ and $(-13, -6)$?

Ⓐ $-\frac{3}{20}$ Ⓑ $\frac{3}{20}$ Ⓒ $\frac{3}{4}$ Ⓓ $\frac{5}{2}$

EXAMPLE 5
for Exs. 19–20

19. MOVIE RENTALS The table shows the number of days you keep a rented movie before returning it and the total cost of renting the movie. Find the rate of change in cost with respect to time and interpret its meaning.

Time (days)	4	5	6	7
Total cost (dollars)	6.00	8.25	10.50	12.75

20. AMUSEMENT PARK The table shows the amount of time spent at an amusement park and the admission fee the park charges. Find the rate of change in the fee with respect to time spent at the park and interpret its meaning.
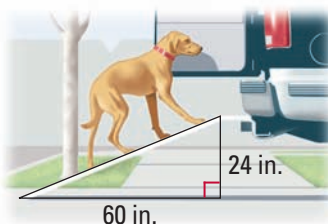
Time (hours)	4	5	6
Admission fee (dollars)	34.99	34.99	34.99

FINDING SLOPE Find the slope of the object. Round to the nearest tenth.

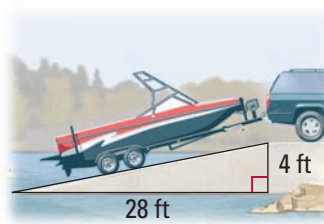**21.** Skateboard ramp

15 in.
54 in.

22. Pet ramp

24 in.
60 in.

23. Boat ramp

4 ft
28 ft
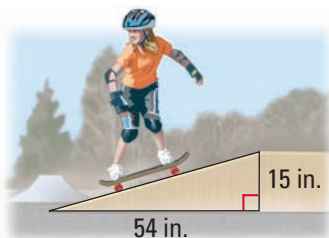
In Exercises 24–32, use the example below to find the value of x or y so that the line passing through the given points has the given slope.

EXAMPLE **Find a coordinate given the slope of a line**

Find the value of x so that the line that passes through the points $(2, 3)$ and $(x, 9)$ has a slope of $\frac{3}{2}$.

Solution

Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (x, 9)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Write formula for slope.}$$

$$\frac{3}{2} = \frac{9 - 3}{x - 2} \qquad \text{Substitute values.}$$

$$3(x - 2) = 2(9 - 3) \qquad \text{Cross products property}$$

$$3x - 6 = 12 \qquad \text{Simplify.}$$

$$x = 6 \qquad \text{Solve for } x.$$

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24. $(x, 4), (6, -1); m = \frac{5}{6}$

25. $(0, y), (-2, 1); m = -8$

26. $(8, 1), (x, 7); m = -\frac{1}{2}$

27. $(5, 4), (-5, y); m = \frac{3}{5}$

28. $(-9, y), (0, -3); m = -\frac{7}{9}$

29. $(x, 9), (-1, 19); m = 5$

30. $(9, 3), (-6, 7y); m = 3$

31. $(-3, y + 1), (0, 4); m = 6$

32. $\left(\frac{x}{2}, 7\right), (-10, 15); m = 4$

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**

33. REASONING The point $(-1, 8)$ is on a line that has a slope of -3. Is the point $(4, -7)$ on the same line? *Explain* your reasoning.

34. ★ WRITING Is a line with undefined slope the graph of a function? *Explain*.

35. CHALLENGE Given two points (x_1, y_1) and (x_2, y_2) such that $x_1 \neq x_2$, show that $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$. What does this result tell you about calculating the slope of a line?

PROBLEM SOLVING

36. OCEANOGRAPHY Ocean water levels are measured hourly at a monitoring station. The table shows the water level (in meters) on one particular morning. *Describe* the rates of change in water levels throughout the morning.

Hours since 12:00 A.M.	1	3	8	10	12
Water level (meters)	2	1.4	0.5	1	1.8

37. MULTI-STEP PROBLEM Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph shows the temperatures in a kiln while firing a piece of pottery (after the kiln is preheated to 250°F).

a. Determine the time interval during which the temperature in the kiln showed the greatest rate of change.

**b.** Determine the time interval during which the temperature in the kiln showed the least rate of change.
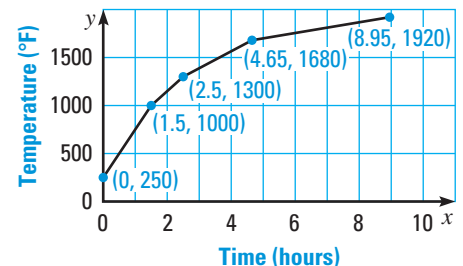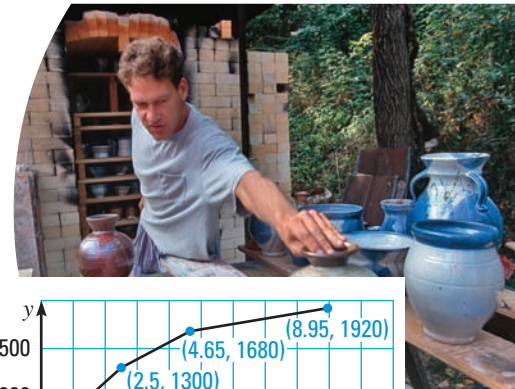
**38. FLYING** The graph shows the altitude of a plane during 4 hours of a flight. Give a verbal description of the flight.
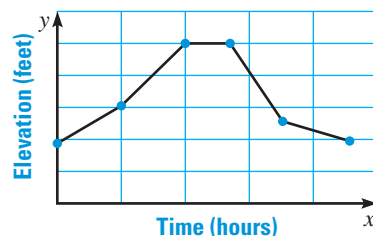
39. HIKING The graph shows the elevation of a hiker walking on a mountain trail. Give a verbal description of the hike.

3.4   Find Slope and Rate of Change   **179**
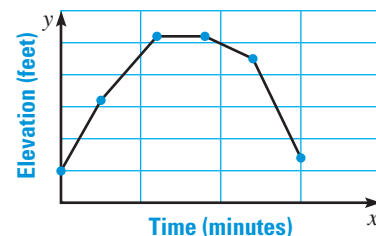
40. ★ **EXTENDED RESPONSE** The graph shows the number (in thousands) of undergraduate students who majored in biological science, engineering, or liberal arts in the United States from 1990 to 2000.

a. During which two-year period did the number of engineering students decrease the most? Estimate the rate of change for this time period.

b. During which two-year period did the number of liberal arts students increase the most? Estimate the rate of change for this time period.

c. How did the total number of students majoring in biological science, engineering, and liberal arts change in the 10 year period? *Explain* your thinking.

Undergraduate Students
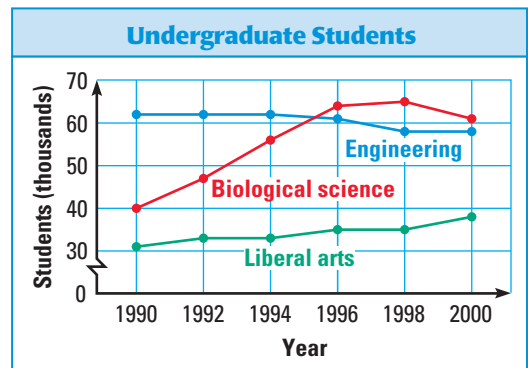
41. **CHALLENGE** Imagine the containers below being filled with water at a constant rate. Sketch a graph that shows the water level for each container during the time it takes to fill the container with water.

a.

b.

c.