

CHAPTER
3**Interpreting the Slope and y -Intercept**

When a linear equation is in slope-intercept form, $y = mx + b$, the graphical interpretation of m and b is that b indicates where the line crosses the y -axis, while m indicates the extent to which the line is tilted away from the horizontal.

However, we can also give an algebraic interpretation for m and b , based on the fact that the expression $mx + b$ contains two terms, one of which, b , is a *fixed term*, because its value doesn't change when x changes. The other term, mx , is a *variable term*, whose value varies with x in a way controlled by the value of m .

EXAMPLE 1 Interpret the slope and y -intercept

A large pot of water is boiling on a stove. As the water boils away, the amount left in the pot decreases. The number of gallons of water, y , remaining in the pot after x minutes, is given by $y = -\frac{3}{80}x + \frac{3}{4}$. Explain the meaning of the quantities $-\frac{3}{80}$ and $\frac{3}{4}$ in the equation.

Solution:

The quantity $\frac{3}{4}$ represents the value of y when $x = 0$. This is the initial quantity of water: the pot started out with $\frac{3}{4}$ gallons. When x increases by 1 minute, y decreases by $\frac{3}{80}$ gallons. In other words, $-\frac{3}{80}$ represents the change in the amount of water during one minute of boiling. The minus sign means that as time goes on, the amount of water decreases. ■

In Example 1, where x represented time, $b = \frac{3}{4}$ was the *initial value* and $m = -\frac{3}{80}$ was the *rate of change* over time, in gallons per minute.

The quantity mx represents the *total change* that has occurred at time x .

Of course, the variable x does not always represent time. It can instead, for instance, represent the output volume of a manufacturing process.

EXAMPLE 2 Interpret the slope and y -intercept

A factory produces fiber optic cable. The cost in dollars, y , of one day's worth of production is given by $y = 0.4x + 30,000$, where x is the number of meters of cable produced. Explain the meaning of the quantities 0.4 and 30,000.

Solution:

The quantity 30,000 represents the factory's *fixed cost* for one day of production. This would include the mortgage or rental cost of inhabiting the building and any other costs that do not depend on how many meters of cable are produced.

The quantity $0.4x$, on the other hand, represents the *variable cost*, which goes up or down depending on whether more or less cable is produced in a given day. The variable cost includes the cost of raw materials used during production. The quantity 0.4 can be interpreted as the cost of producing 1 meter of cable, given that the factory is already up and operating for the day. ■

Interpreting the Slope and y-Intercept *continued***Practice**

Writing For each formula in the following application contexts, explain the meaning of the constant term and of the coefficient in the variable term.

1. The population of a town, x years after 2003, is given by $P(x) = 15.6x + 1842$.
2. A student's score, y , on a test with 30 questions is calculated and recorded as $y = 40 + 2x$, where x is the number of questions answered correctly.
3. A plumber who retrofits old houses with new copper pipe charges $C(x) = 800 + 3.3x$ dollars for a house with x square feet of living area.
4. In a certain place, the air temperature (in degrees Celsius) over the course of the morning is a function of time according to the equation $T(t) = -10 + 0.15t$, where t is minutes after 6:00 A.M.

For each situation, give an appropriate linear equation in y-intercept form.

5. A water tank initially containing 150 liters of water has more water pumped in, at the rate of 7 liters per minute.
6. Each of four roommates contributes \$450 per month toward the rent payment (which covers utilities) and also pays $\frac{1}{4}$ of the phone bill.
7. A car traveling at a constant speed of 55 miles per hour begins to accelerate, gaining 2 miles per hour in speed every second.
8. The cost of renting a car for one day is 48 dollars plus 15 cents per mile.
9. **Explain** What does it mean when a linear equation does not have a constant term—for instance, the equation $v(t) = 9.81t$, where v is the speed of a falling object in meters per second, and t is number of seconds after the object was released?