3.5 Graph Using Slope-Intercept Form

Before You found slopes and graphed equations using intercepts.

Now You will graph linear equations using slope-intercept form.

Why? So you can model a worker's earnings, as in Ex. 43.

Key Vocabulary
- **slope-intercept form**
- **parallel**

In the activity, you saw how the slope and y-intercept of the graph of a linear equation in the form $y = mx + b$ are related to the equation.

KEY CONCEPT *For Your Notebook*

Finding the Slope and y-Intercept of a Line

Words

A linear equation of the form $y = mx + b$ is written in **slope-intercept form** where m is the slope and b is the y-intercept of the equation's graph.

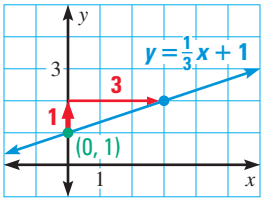
Symbols

$y = mx + b$

slope: m; y-intercept: b

$y = \frac{1}{3}x + 1$

Graph



EXAMPLE 1 Identify slope and y-intercept

Identify the slope and y-intercept of the line with the given equation.

a. $y = 3x + 4$ **b.** $3x + y = 2$

Solution

a. The equation is in the form $y = mx + b$. So, the slope of the line is 3, and the y-intercept is 4.

b. Rewrite the equation in slope-intercept form by solving for y.

$3x + y = 2$ **Write original equation.**

$y = -3x + 2$ **Subtract $3x$ from each side.**

▶ The line has a slope of -3 and a y-intercept of 2.

REWRITE EQUATIONS
When you rewrite a linear equation in slope-intercept form, you are expressing y as a function of x.

GUIDED PRACTICE for Example 1

Identify the slope and y-intercept of the line with the given equation.

1. $y = 5x - 3$ **2.** $3x - 3y = 12$ **3.** $x + 4y = 6$

EXAMPLE 2 Graph an equation using slope-intercept form

Graph the equation $2x + y = 3$.

Solution

STEP 1 **Rewrite** the equation in slope-intercept form.

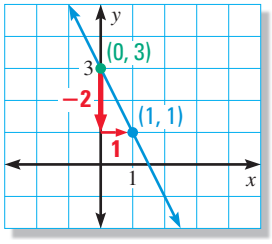
$y = -2x + 3$

STEP 2 **Identify** the slope and the y-intercept.

$m = -2$ and $b = 3$

STEP 3 **Plot** the point that corresponds to the y-intercept, $(0, 3)$.

STEP 4 **Use** the slope to locate a second point on the line. Draw a line through the two points.



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CHECK REASONABLENESS
To check the line drawn in Example 2, substitute the coordinates of the second point into the original equation. You should get a true statement.

MODELING In real-world problems that can be modeled by linear equations, the y-intercept is often an initial value, and the slope is a rate of change.

EXAMPLE 3 Change slopes of lines

ESCALATORS To get from one floor to another at a library, you can take either the stairs or the escalator. You can climb stairs at a rate of 1.75 feet per second, and the escalator rises at a rate of 2 feet per second. You have to travel a vertical distance of 28 feet. The equations model the vertical distance d (in feet) you have left to travel after t seconds.

Stairs: $d = -1.75t + 28$ **Escalator:** $d = -2t + 28$

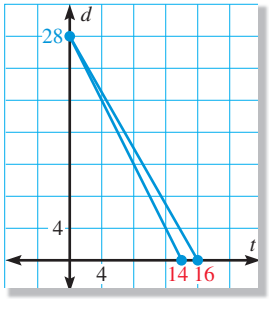
a. Graph the equations in the same coordinate plane.

b. How much time do you save by taking the escalator?

Solution

a. Draw the graph of $d = -1.75t + 28$ using the fact that the d-intercept is 28 and the slope is -1.75. Similarly, draw the graph of $d = -2t + 28$. The graphs make sense only in the first quadrant.

b. The equation $d = -1.75t + 28$ has a t-intercept of **16**. The equation $d = -2t + 28$ has a t-intercept of **14**. So, you save $16 - 14 = 2$ seconds by taking the escalator.



GUIDED PRACTICE for Examples 2 and 3

4. Graph the equation $y = -2x + 5$.

5. **WHAT IF?** In Example 3, suppose a person can climb stairs at a rate of 1.4 feet per second. How much time does taking the escalator save?

EXAMPLE 4 Change intercepts of lines

TELEVISION A company produced two 30 second commercials, one for $300,000 and the second for $400,000. Each airing of either commercial on a particular station costs $150,000. The cost C (in thousands of dollars) to produce the first commercial and air it n times is given by $C = 150n + 300$. The cost to produce the second and air it n times is given by $C = 150n + 400$.

a. Graph both equations in the same coordinate plane.

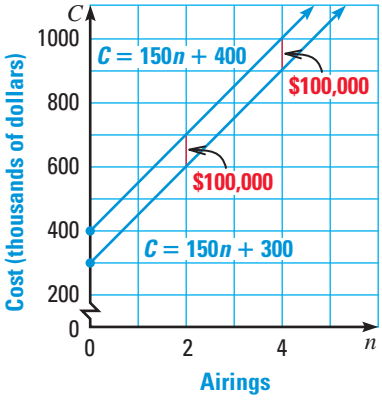
b. Based on the graphs, what is the difference of the costs to produce each commercial and air it 2 times? 4 times? What do you notice about the differences of the costs?

Solution

a. The graphs of the equations are shown.

b. You can see that the vertical distance between the lines is $100,000 when $n = 2$ and $n = 4$.

The difference of the costs is $100,000 no matter how many times the commercials are aired.



PARALLEL LINES Two lines in the same plane are **parallel** if they do not intersect. Because slope gives the rate at which a line rises or falls, two nonvertical lines with the same slope are parallel.

EXAMPLE 5 Identify parallel lines

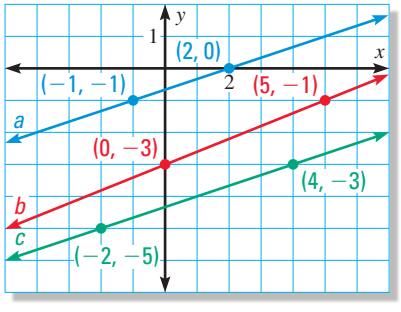
Determine which of the lines are parallel.

Find the slope of each line.

Line *a*: $m = \frac{-1 - 0}{-1 - 2} = \frac{-1}{-3} = \frac{1}{3}$

Line *b*: $m = \frac{-3 - (-1)}{0 - 5} = \frac{-2}{-5} = \frac{2}{5}$

Line *c*: $m = \frac{-5 - (-3)}{-2 - 4} = \frac{-2}{-6} = \frac{1}{3}$



▶ Line a and line c have the same slope, so they are parallel.

GUIDED PRACTICE for Examples 4 and 5

6. WHAT IF? In Example 4, suppose that the cost of producing and airing a third commercial is given by $C = 150n + 200$. Graph the equation. Find the difference of the costs of the second commercial and the third.

7. Determine which lines are parallel: line a through $(-1, 2)$ and $(3, 4)$; line b through $(2, 2)$ and $(5, 8)$; line c through $(-9, -2)$ and $(-3, 1)$.

3.5 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 11, 21, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 10, 36, 42, and 44

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Two lines in the same plane are __?__ if they do not intersect.

2. ★ **WRITING** What is the slope-intercept form of a linear equation? *Explain* why this form is called slope-intercept form.

EXAMPLE 1 for Exs. 3–16

SLOPE AND *y*-INTERCEPT **Identify the slope and *y*-intercept of the line with the given equation.**

3. $y = 2x + 1$
4. $y = -x$
5. $y = 6 - 3x$
6. $y = -7 + 5x$
7. $y = \frac{2}{3}x - 1$
8. $y = -\frac{1}{4}x + 8$

9. ★ **MULTIPLE CHOICE** What is the slope of the line with the equation $y = -18x - 9$?

   (A) -18 (B) -9 (C) 9 (D) 18

10. ★ **MULTIPLE CHOICE** What is the *y*-intercept of the line with the equation $x - 3y = -12$?

    (A) -12 (B) -4 (C) 4 (D) 12

REWRITING EQUATIONS **Rewrite the equation in slope-intercept form. Then identify the slope and the *y*-intercept of the line.**

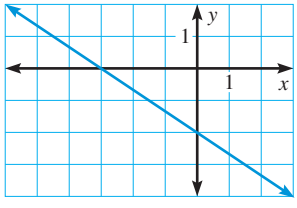
11. $4x + y = 1$
12. $x - y = 6$
13. $6x - 3y = -9$
14. $-12x - 4y = 2$
15. $2x + 5y = -10$
16. $-x - 10y = 20$

EXAMPLE 2 for Exs. 17–29

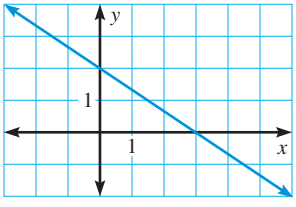
MATCHING EQUATIONS WITH GRAPHS **Match the equation with its graph.**

17. $2x + 3y = 6$
18. $2x + 3y = -6$
19. $2x - 3y = 6$

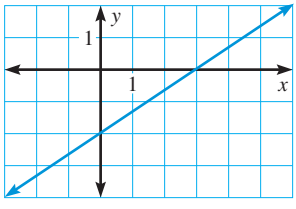
A.



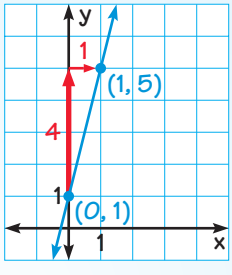
B.



C.



20. **ERROR ANALYSIS** *Describe* and correct the error in graphing the equation $y = 4x - 1$.



GRAPHING EQUATIONS **Graph the equation.**

21. $y = -6x + 1$ **22.** $y = 3x + 2$ **23.** $y = -x + 7$

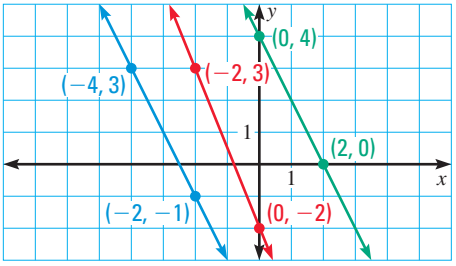
24. $y = \frac{2}{3}x$ **25.** $y = \frac{1}{4}x - 5$ **26.** $y = -\frac{5}{2}x + 2$

27. $7x - 2y = -11$ **28.** $-8x - 2y = 32$ **29.** $-x - 0.5y = 2.5$

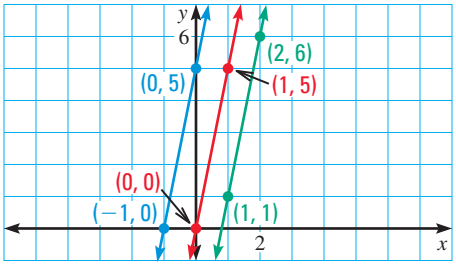
EXAMPLE 5 for Exs. 30–35

PARALLEL LINES **Determine which lines are parallel.**

30.



31.



PARALLEL LINES **Tell whether the graphs of the two equations are parallel lines. *Explain* your reasoning.**

32. $y = 5x - 7$, $5x + y = 7$ **33.** $y = 3x + 2$, $-7 + 3x = y$

34. $y = -0.5x$, $x + 2y = 18$ **35.** $4x + y = 3$, $x + 4y = 3$

36. ★ **OPEN-ENDED** Write the equation of a line that is parallel to $6x + y = 24$. *Explain* your reasoning.

REASONING **Find the value of k so that the lines through the given points are parallel.**

37. Line 1: $(-4, -2)$ and $(0, 0)$
Line 2: $(2, 7)$ and $(k, 5)$

38. Line 1: $(-1, 9)$ and $(-6, -6)$
Line 2: $(-7, k)$ and $(0, -2)$

39. **CHALLENGE** To show that the slope of a line is constant, let (x_1, y_1) and (x_2, y_2) be any two points on the line $y = mx + b$. Use the equation of the line to express y_1 in terms of x_1 and y_2 in terms of x_2. Then show that the slope between the two points is m.

PROBLEM SOLVING

EXAMPLES 3 and 4 for Exs. 40–44

40. **HOCKEY** Your family spends \$60 on tickets to a hockey game and \$4 per hour for parking. The total cost C (in dollars) is given by $C = 60 + 4t$ where t is the time (in hours) your family's car is parked.

a. Graph the equation.

b. Suppose the parking fee is raised to \$5.50 per hour so that the total cost of tickets and parking for t hours is $C = 60 + 5.5t$. Graph the equation in the same coordinate plane as the equation in part (a).

c. How much more does it cost to go to a game for 4 hours after the parking fee is raised?

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

41. SPEED LIMITS In 1995 Pennsylvania changed its maximum speed limit on rural interstate highways, as shown below. The diagram also shows the distance d (in miles) a person could travel driving at the maximum speed limit for t hours both before and after 1995.

a. Graph both equations in the same coordinate plane.

b. Use the graphs to find the difference of the distances a person could drive in 3 hours before and after the speed limit was changed.

42. ★ SHORT RESPONSE A service station charges \$40 per hour for labor plus the cost of parts to repair a car. Parts can either be ordered from the car dealership for \$250 or from a warehouse for \$200. The equations below give the total repair cost C (in dollars) for a repair that takes t hours using parts from the dealership or from the warehouse.

Dealership: $C = 40t + 250$ **Warehouse:** $C = 40t + 200$

a. Graph both equations in the same coordinate plane.

b. Use the graphs to find the difference of the costs if the repair takes 3 hours. What if the repair takes 4 hours? What do you notice about the differences of the costs? *Explain*.

43. FACTORY SHIFTS Welders at a factory can work one of two shifts. Welders on the first shift earn \$12 per hour while workers on the second shift earn \$14 per hour. The total amount a (in dollars) a first-shift worker earns is given by $a = 12t$ where t is the time (in hours) worked. The total amount a second-shift worker earns is given by $a = 14t$.

a. Graph both equations in the same coordinate plane. What do the slopes and the a-intercepts of the graphs mean in this situation?

b. How much more money does a welder earn for a 40 hour week if he or she works the second shift rather than the first shift?

44. ★ EXTENDED RESPONSE An artist is renting a booth at an art show. A small booth costs \$350 to rent. The artist plans to sell framed pictures for \$50 each. The profit P (in dollars) the artist makes after selling p pictures is given by $P = 50p - 350$.

a. Graph the equation.

b. If the artist decides to rent a larger booth for \$500, the profit is given by $P = 50p - 500$. Graph this equation on the same coordinate plane you used in part (a).

c. The artist can display 80 pictures in the small booth and 120 in the larger booth. If the artist is able to sell all of the pictures, which booth should the artist rent? *Explain*.

45. CHALLENGE To use a rock climbing wall at a college, a person who does not attend the college has to pay a $5 certification fee plus $3 per visit. The total cost C (in dollars) for a person who does not attend the college is given by $C = 3v + 5$ where v is the number of visits to the rock climbing wall. A student at the college pays only an $8 certification fee, so the total cost for a student is given by $C = 8$.

a. Graph both equations in the same coordinate plane. At what point do the lines intersect? What does the point of intersection represent?

b. When will a nonstudent pay more than a student? When will a student pay more than a nonstudent? *Explain*.

QUIZ

Find the slope of the line that passes through the points.

1. (3, −11) and (0, 4)

2. (2, 1) and (8, 4)

3. (−4, −1) and (−1, −1)

Identify the slope and y-intercept of the line with the given equation.

4. $y = -x + 9$

5. $2x + 9y = -18$

6. $-x + 6y = 21$

Graph the equation.

7. $y = -2x + 11$

8. $y = \frac{5}{3}x - 8$

9. $-3x - 4y = -12$

10. RED OAKS Red oak trees grow at a rate of about 2 feet per year. You buy and plant two red oak trees, one that is 6 feet tall and one that is 8 feet tall. The height h (in feet) of the shorter tree can be modeled by $h = 2t + 6$ where t is the time (in years) since you planted the tree. The height of the taller tree can be modeled by $h = 2t + 8$.

a. Graph both equations in the same coordinate plane.

b. Use the graphs to find the difference of the heights of the trees 5 years after you plant them. What is the difference after 10 years? What do you notice about the difference of the heights of the two trees?