3.6 Model Direct Variation

Before You wrote and graphed linear equations.

Now You will write and graph direct variation equations.

Why? So you can model distance traveled, as in Ex. 40.

Key Vocabulary
- direct variation
- constant of variation

Two variables x and y show **direct variation** provided $y = ax$ and $a \neq 0$. The nonzero number a is called the **constant of variation**, and y is said to *vary directly* with x.

The equation $y = 5x$ is an example of direct variation, and the constant of variation is 5. The equation $y = x + 5$ is *not* an example of direct variation.

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

EXAMPLE 1 Identify direct variation equations

Tell whether the equation represents direct variation. If so, identify the constant of variation.

a. $2x - 3y = 0$ **b.** $-x + y = 4$

Solution

To tell whether an equation represents direct variation, try to rewrite the equation in the form $y = ax$.

a. $2x - 3y = 0$ Write original equation.

$-3y = -2x$ Subtract $2x$ from each side.

$y = \frac{2}{3}x$ Simplify.

▶ Because the equation $2x - 3y = 0$ can be rewritten in the form $y = ax$, it represents direct variation. The constant of variation is $\frac{2}{3}$.

b. $-x + y = 4$ Write original equation.

$y = x + 4$ Add x to each side.

▶ Because the equation $-x + y = 4$ cannot be rewritten in the form $y = ax$, it does not represent direct variation.

✓ GUIDED PRACTICE for Example 1

Tell whether the equation represents direct variation. If so, identify the constant of variation.

1. $-x + y = 1$ **2.** $2x + y = 0$ **3.** $4x - 5y = 0$

DIRECT VARIATION GRAPHS Notice that a direct variation equation, $y = ax$, is a linear equation in slope-intercept form, $y = mx + b$, with $m = a$ and $b = 0$. The graph of a direct variation equation is a line with a slope of a and a y-intercept of 0. So, the line passes through the origin.

EXAMPLE 2 Graph direct variation equations

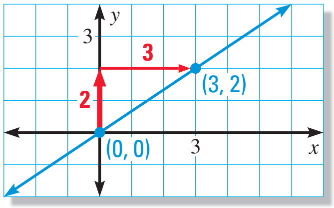
Graph the direct variation equation.

a. $y = \frac{2}{3}x$

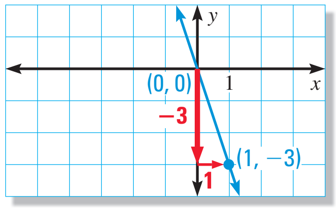
b. $y = -3x$

Solution

a. Plot a point at the origin. The slope is equal to the constant of variation, or $\frac{2}{3}$. Find and plot a second point, then draw a line through the points.



b. Plot a point at the origin. The slope is equal to the constant of variation, or -3. Find and plot a second point, then draw a line through the points.

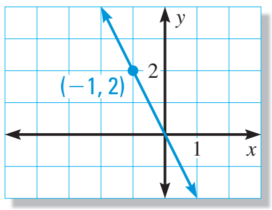


DRAW A GRAPH
If the constant of variation is positive, the graph of $y = ax$ passes through Quadrants I and III. If the constant of variation is negative, the graph of $y = ax$ passes through Quadrants II and IV.

Animated Algebra at my.hrw.com

EXAMPLE 3 Write and use a direct variation equation

The graph of a direct variation equation is shown.



a. Write the direct variation equation.

b. Find the value of y when $x = 30$.

Solution

a. Because y varies directly with x, the equation has the form $y = ax$. Use the fact that $y = 2$ when $x = -1$ to find a.

$y = ax$ — Write direct variation equation.

$2 = a(-1)$ — Substitute.

$-2 = a$ — Solve for a.

▶ A direct variation equation that relates x and y is $y = -2x$.

b. When $x = 30$, $y = -2(30) = -60$.

✓ GUIDED PRACTICE for Examples 2 and 3

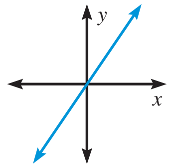
4. Graph the direct variation equation $y = 2x$.

5. The graph of a direct variation equation passes through the point (4, 6). Write the direct variation equation and find the value of y when $x = 24$.

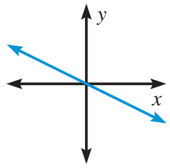
KEY CONCEPT — *For Your Notebook*

Properties of Graphs of Direct Variation Equations

- The graph of a direct variation equation is a line through the origin.
- The slope of the graph of $y = ax$ is a.



$a > 0$



$a < 0$

EXAMPLE 4 Solve a multi-step problem

ANOTHER WAY
For alternative methods for solving Example 4, see the **Problem Solving Workshop**.

SALTWATER AQUARIUM The number s of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number w of gallons of water in the tank. A pet shop owner recommends adding 100 tablespoons of sea salt to a 20 gallon tank.

- Write a direct variation equation that relates w and s.
- How many tablespoons of salt should be added to a 30 gallon saltwater fish tank?

Solution

STEP 1 **Write** a direct variation equation. Because s varies directly with w, you can use the equation $s = aw$. Also use the fact that $s = 100$ when $w = 20$.

$s = aw$ **Write direct variation equation.**

$100 = a(20)$ **Substitute.**

$5 = a$ **Solve for *a*.**

RECOGNIZE RATE OF CHANGE
The value of a in Example 4 is a rate of change: 5 tablespoons of sea salt per gallon of water.

▶ A direct variation equation that relates w and s is $s = 5w$.

STEP 2 **Find** the number of tablespoons of salt that should be added to a 30 gallon saltwater fish tank. Use your direct variation equation from Step 1.

$s = 5w$ **Write direct variation equation.**

$s = 5(30)$ **Substitute 30 for *w*.**

$s = 150$ **Simplify.**

▶ You should add 150 tablespoons of salt to a 30 gallon fish tank.

✓ GUIDED PRACTICE for Example 4

6. **WHAT IF?** In Example 4, suppose the fish tank is a 25 gallon tank. How many tablespoons of salt should be added to the tank?

RATIOS The direct variation equation $y = ax$ can be rewritten as $\frac{y}{x} = a$ for $x \neq 0$. So, in a direct variation, the ratio of y to x is constant for all nonzero data pairs (x, y).

EXAMPLE 5 Use a direct variation model

ONLINE MUSIC The table shows the cost C of downloading s songs at an Internet music site.

Number of songs, s	Cost, C (dollars)
3	2.97
5	4.95
7	6.93

a. Explain why C varies directly with s.

b. Write a direct variation equation that relates s and C.

CHECK RATIOS
For real-world data, the ratios may not be exactly equal. You may still be able to use a direct variation model when the ratios are approximately equal.

Solution

a. To explain why C varies directly with s, compare the ratios $\frac{C}{s}$ for all data pairs (s, C): $\frac{2.97}{3} = \frac{4.95}{5} = \frac{6.93}{7} = 0.99$.

Because the ratios all equal 0.99, C varies directly with s.

b. A direct variation equation is $C = 0.99s$.

✓ GUIDED PRACTICE for Example 5

7. **WHAT IF?** In Example 5, suppose the website charges a total of \$1.99 for the first 5 songs you download and \$.99 for each song after the first 5. Is it reasonable to use a direct variation model for this situation? *Explain.*

3.6 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 7, 21, and 43

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 28, 38, 43, 44, and 46

◆ = **MULTIPLE REPRESENTATIONS** Ex. 45

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Two variables x and y show __?__ provided $y = ax$ and $a \neq 0$.

2. ★ **WRITING** A line has a slope of -3 and a y-intercept of 4. Is the equation of the line a direct variation equation? *Explain.*

EXAMPLE 1
for Exs. 3–10

IDENTIFYING DIRECT VARIATION EQUATIONS **Tell whether the equation represents direct variation. If so, identify the constant of variation.**

3. $y = x$

4. $y = 5x - 1$

5. $2x + y = 3$

6. $x - 3y = 0$

7. $8x + 2y = 0$

8. $2.4x + 6 = 1.2y$

9. ★ **MULTIPLE CHOICE** Which equation is a direct variation equation?

(A) $y = 7 - 3x$ (B) $3x - 7y = 1$ (C) $3x - 7y = 0$ (D) $3y = 7x - 1$

10. **ERROR ANALYSIS** *Describe* and correct the error in identifying the constant of variation for the direct variation equation $-5x + 3y = 0$.

$-5x + 3y = 0$
$3y = 5x$
The constant of variation is 5.

EXAMPLE 2 for Exs. 11–22

GRAPHING EQUATIONS **Graph the direct variation equation.**

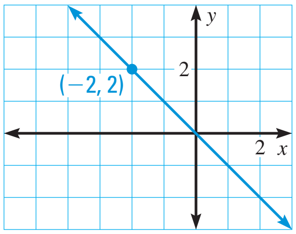
11. $y = x$ 12. $y = 3x$ 13. $y = -4x$ 14. $y = 5x$

15. $y = \frac{4}{3}x$ 16. $y = \frac{1}{2}x$ 17. $y = -\frac{1}{3}x$ 18. $y = -\frac{3}{2}x$

19. $12y = -24x$ 20. $10y = 25x$ 21. $4x + y = 0$ 22. $y - 1.25x = 0$

EXAMPLE 3 for Exs. 23–25

WRITING EQUATIONS **The graph of a direct variation equation is shown. Write the direct variation equation. Then find the value of y when $x = 8$.**

23.


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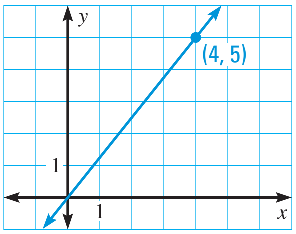

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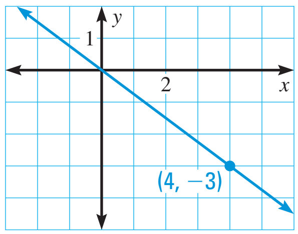


IDENTIFYING DIRECT VARIATION EQUATIONS **Tell whether the table represents direct variation. If so, write the direct variation equation.**

26.

x	1	2	3	4	6
y	5	10	15	20	30

27.

x	−3	−1	1	3	5
y	−2	0	2	4	6

28. ★ **WRITING** A student says that a direct variation equation can be used to model the data in the table. *Explain* why the student is mistaken.

x	2	4	8	16
y	1	2	4	6

WRITING EQUATIONS **Given that y varies directly with x, use the specified values to write a direct variation equation that relates x and y.**

29. $x = 3, y = 9$ 30. $x = 2, y = 26$ 31. $x = 14, y = 7$

32. $x = 15, y = -5$ 33. $x = -2, y = -2$ 34. $x = -18, y = -4$

35. $x = \frac{1}{4}, y = 1$ 36. $x = -6, y = 15$ 37. $x = -5.2, y = 1.4$

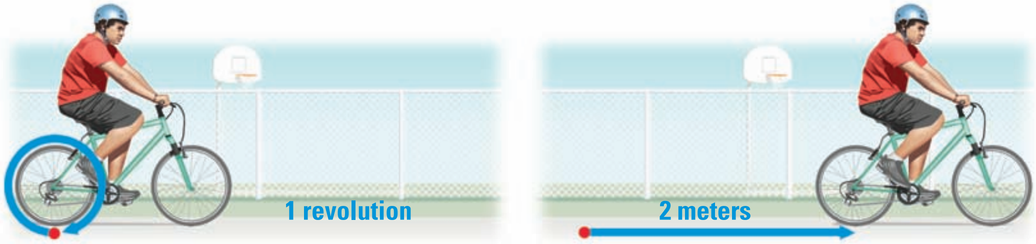
38. ★ **WRITING** If y varies directly with x, does x vary directly with y? If so, what is the relationship between the constants of variation? *Explain.*

39. **CHALLENGE** The slope of a line is $-\frac{1}{3}$, and the point $(-6, 2)$ lies on the line. Use the formula for the slope of a line to determine if the equation of the line is a direct variation equation.

PROBLEM SOLVING

EXAMPLE 4
for Exs. 40–42

40. **BICYCLES** The distance d (in meters) you travel on a bicycle varies directly with the number r of revolutions that the rear tire completes. You travel about 2 meters on a mountain bike for every revolution of the tire.



a. Write a direct variation equation that relates r and d.

b. How many meters do you travel in 1500 tire revolutions?

41. **VACATION TIME** At one company, the amount of vacation v (in hours) an employee earns varies directly with the amount of time t (in weeks) he or she works. An employee who works 2 weeks earns 3 hours of vacation.

a. Write a direct variation equation that relates t and v.

b. How many hours of vacation time does an employee earn in 8 weeks?

42. **LANDSCAPING** Landscapers plan to spread a layer of stone on a path. The number s of bags of stone needed depends on the depth d (in inches) of the layer. They need 10 bags to spread a layer of stone that is 2 inches deep. Write a direct variation equation that relates d and s. Then find the number of bags needed to spread a layer that is 3 inches deep.

EXAMPLE 5
for Exs. 43–44

43. ★ **SHORT RESPONSE** At a recycling center, computers and computer accessories can be recycled for a fee f based on weight w, as shown in the table.

Weight, w (pounds)	Fee, f (dollars)
10	2.50
15	3.75
30	7.50

a. *Explain* why f varies directly with w.

b. Write a direct variation equation that relates w and f, and identify the rate of change that the constant of variation represents. Find the total recycling fee for an 18 pound computer and a 10 pound printer.

44. ★ **SHORT RESPONSE** You can buy gold chain by the inch. The table shows the price of gold chain for various lengths.

Length, ℓ (inches)	7	9	16	18
Price, p (dollars)	8.75	11.25	20.00	22.50

a. *Explain* why p varies directly with ℓ.

b. Write a direct variation equation that relates ℓ and p, and identify the rate of change that the constant of variation represents. If you have $30, what is the longest chain that you can buy?

○ = See WORKED-OUT SOLUTIONS in Student Resources
★ = STANDARDIZED TEST PRACTICE
◆ = MULTIPLE REPRESENTATIONS

45. **MULTIPLE REPRESENTATIONS** The total cost of riding the subway to and from school every day is $1.50.

a. Making a Table Make a table that shows the number d of school days and the total cost C (in dollars) for trips to and from school for some values of d. Assume you travel to school once each school day and home from school once each school day.

b. Drawing a Graph Graph the ordered pairs from the table and draw a ray through them.

c. Writing an Equation Write an equation of the graph from part (b). Is it a direct variation equation? *Explain*. If there are 22 school days in one month, what will it cost to ride the subway to and from school for that month?

46. ★ **EXTENDED RESPONSE** The table shows the average number of field goals attempted t and the average number of field goals made m per game for all NCAA Division I women's basketball teams for 9 consecutive seasons.

Attempted field goals, t	61.8	61.9	61.8	60.8	59.5	59.0	58.9	59.2	58.4
Field goals made, m	25.7	25.6	25.6	25.2	24.5	24.6	24.5	24.3	24.0

a. Write Why is it reasonable to use a direct variation model for this situation? Write a direct variation equation that relates t and m. Find the constant of variation to the nearest tenth.

b. Estimate The highest average number of attempted field goals in one season was 66.2. Estimate the number of field goals made that season.

c. Explain If the average number of field goals made was increasing rather than decreasing and the number of attempted field goals continued to decrease, would the data show direct variation? *Explain*.

47. **CHALLENGE** In Exercise 40, you found an equation showing that the distance traveled on a bike varies directly with the number of revolutions that the rear tire completes. The number r of tire revolutions varies directly with the number p of pedal revolutions. In a particular gear, you travel about 1.3 meters for every 5 revolutions of the pedals. Show that distance traveled varies directly with pedal revolutions.

PROBLEM SOLVING WORKSHOP
LESSON 3.6

Using ALTERNATIVE METHODS

Another Way to Solve Example 4

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

MULTIPLE REPRESENTATIONS In Example 4, you saw how to solve the problem about how much salt to add to a saltwater fish tank by writing and using a direct variation equation. You can also solve the problem using a graph or a proportion.

PROBLEM

SALTWATER AQUARIUM The number *s* of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number *w* of gallons of water in the tank. A pet shop owner recommends adding 100 tablespoons of sea salt to a 20 gallon tank. How many tablespoons of salt should be added to a 30 gallon saltwater fish tank?

METHOD 1

Using a Graph An alternative approach is to use a graph.

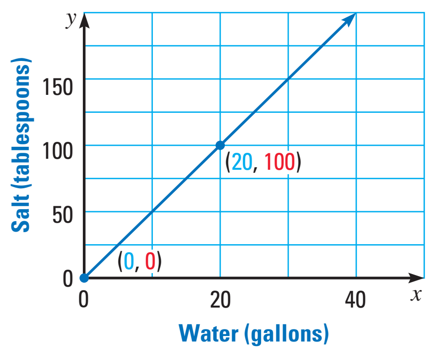
STEP 1 **Read** the problem. It tells you an amount of salt for a certain size fish tank. You can also assume that if a fishtank has no water, then no salt needs to be added. Write ordered pairs for this information.

Number of gallons → (20, 100) ← Tablespoons of salt

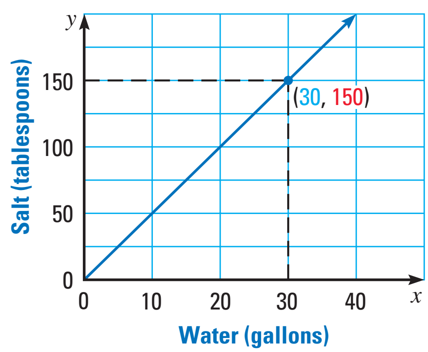
(0, 0)

STEP 2 **Graph** the ordered pairs. Draw a line through the points.

The coordinates of points on the line give the amounts of salt that should be added to fish tanks of various sizes.



STEP 3 **Find** the point on the graph that has an *x*-coordinate of 30. The *y*-coordinate of this point is 150, so 150 tablespoons of salt should be added to a 30 gallon tank.



METHOD 2

Writing a Proportion Another alternative approach is to write and solve a proportion.

STEP 1 **Write** a proportion involving two ratios that each compare the amount of water (in gallons) to the amount of salt (in tablespoons).

$$\frac{20}{100} = \frac{30}{s} \begin{array}{l} \leftarrow \text{amount of water (gallons)} \\ \leftarrow \text{amount of salt (tablespoons)} \end{array}$$

STEP 2 **Solve** the proportion.

$\frac{20}{100} = \frac{30}{s}$	**Write proportion.**
$20s = 100 \cdot 30$	**Cross products property**
$20s = 3000$	**Simplify.**
$s = 150$	**Divide each side by 20.**

▶ You should add 150 tablespoons of salt to a 30 gallon tank.

CHECK Check your answer by writing each ratio in simplest form.

$\frac{20}{100} = \frac{1}{5}$ and $\frac{30}{150} = \frac{1}{5}$

Because each ratio simplifies to $\frac{1}{5}$, the answer is correct.

PRACTICE

1. **WHAT IF?** Suppose the fish tank in the problem above is a 22 gallon tank. How many tablespoons of salt should be added to the tank? *Describe* which method you used to solve this problem.

2. **ADVERTISING** A local newspaper charges by the word for printing classified ads. A 14 word ad costs \$5.88. How much would a 21 word ad cost? Solve this problem using two different methods.

3. **REASONING** In Exercise 2, how can you quickly determine the cost of a 7 word ad? *Explain* how you could use the cost of a 7 word ad to solve the problem.

4. **NUTRITION** A company sells fruit smoothies in two sizes of bottles: 6 fluid ounces and 10 fluid ounces. You know that a 6 ounce bottle contains 96 milligrams of sodium. How many milligrams of sodium does a 10 ounce bottle contain?

5. **ERROR ANALYSIS** A student solved the problem in Exercise 4 as shown. *Describe* and correct the error made.

   Let x = the number of milligrams of sodium in a 10 ounce bottle.

   $$\frac{6}{x} = \frac{10}{96}$$
   $$576 = 10x$$
   $$57.6 = x$$

6. **SLEEPING** You find an online calculator that calculates the number of calories you burn while sleeping. The results for various sleeping times are shown. About how many more calories would you burn by sleeping for 9.5 hours than for 8 hours? Choose any method for solving the problem.

Hours of sleep	6.5	7	8.5	9
Calories burned	390	420	510	540