# 3.7 Graph Linear Functions

**Before** You graphed linear equations and functions.
**Now** You will use function notation.
**Why?** So you can model an animal population, as in Example 3.

**Key Vocabulary**
- function notation
- family of functions
- parent linear function

You have seen linear functions written in the form $y = mx + b$. By naming a function $f$, you can write it using **function notation**.

$$f(x) = mx + b \qquad \text{Function notation}$$

The symbol $f(x)$ is another name for $y$ and is read as "the value of $f$ at $x$," or simply as "$f$ of $x$." It does *not* mean $f$ times $x$. You can use letters other than $f$, such as $g$ or $h$, to name functions.

COMMON CORE

**CC.9-12.F.IF.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima.*

## EXAMPLE 1 Standardized Test Practice

**What is the value of the function $f(x) = 3x - 15$ when $x = -3$?**

Ⓐ $-24$ Ⓑ $-6$ Ⓒ $-2$ Ⓓ $8$

**Solution**

$f(x) = 3x - 15$ **Write original function.**

$f(-3) = 3(-3) - 15$ **Substitute $-3$ for $x$.**

$= -24$ **Simplify.**

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

## ✓ GUIDED PRACTICE for Example 1

1. Evaluate the function $h(x) = -7x$ when $x = 7$.

## EXAMPLE 2 Find an x-value

**For the function $f(x) = 2x - 10$, find the value of $x$ so that $f(x) = 6$.**

$f(x) = 2x - 10$ **Write original function.**

$6 = 2x - 10$ **Substitute 6 for $f(x)$.**

$8 = x$ **Solve for $x$.**

▶ When $x = 8$, $f(x) = 6$.

Lisa Dearing/Alamy

**DOMAIN AND RANGE** The domain of a function consists of the values of $x$ for which the function is defined. The range consists of the values of $f(x)$ where $x$ is in the domain of $f$. The graph of a function $f$ is the set of all points $(x, f(x))$.

## EXAMPLE 3 Graph a function

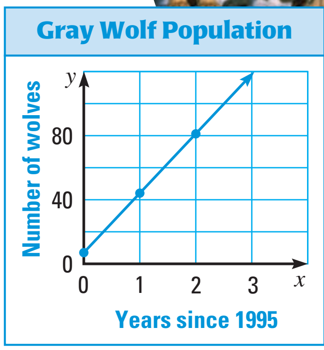
**INTERPRET MODELS**
The rate of change in the wolf population actually varied over time. The model simplifies the situation by assuming a steady rate of change.

**GRAY WOLF** The gray wolf population in central Idaho was monitored over several years for a project aimed at boosting the number of wolves. The number of wolves can be modeled by the function $f(x) = 37x + 7$ where $x$ is the number of years since 1995. Graph the function and identify its domain and range.

### Solution

To graph the function, make a table.

| $x$ | $f(x)$ |
|---|---|
| 0 | $37(0) + 7 = 7$ |
| 1 | $37(1) + 7 = 44$ |
| 2 | $37(2) + 7 = 81$ |




The domain of the function is $x \geq 0$. From the graph or table, you can see that the range of the function is $f(x) \geq 7$.

## ✓ GUIDED PRACTICE for Examples 2 and 3

**2. WOLF POPULATION** Use the model from Example 3 to find the value of $x$ so that $f(x) = 155$. *Explain* what the solution means in this situation.

**IDENTIFY PARAMETERS**
Particular members of the family of linear functions are determined by the values of *m* and *b*, called *parameters*, in the general form $y = mx + b$.

**FAMILIES OF FUNCTIONS** A **family of functions** is a group of functions with similar characteristics. For example, functions that have the form $f(x) = mx + b$ constitute the family of *linear* functions.

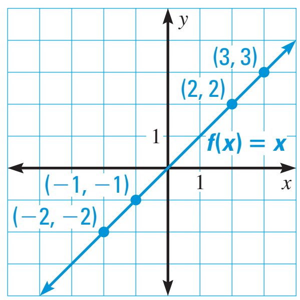
**KEY CONCEPT** *For Your Notebook*

### Parent Function for Linear Functions

The most basic linear function in the family of all linear functions, called the **parent linear function**, is:

$$f(x) = x$$

The graph of the parent linear function is shown.




**READING**
The parent linear function is also called the *identity function.*

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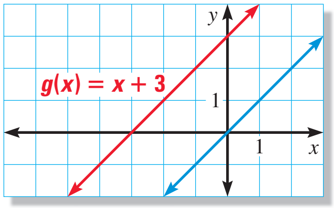
## EXAMPLE 4 Compare graphs with the graph $f(x) = x$

**Graph the function. Compare the graph with the graph of $f(x) = x$.**

**a.** $g(x) = x + 3$ **b.** $h(x) = 2x$

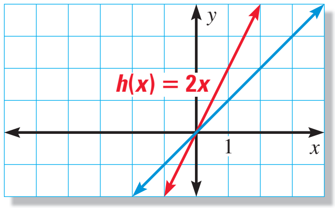
### Solution

**a.**




Because the graphs of ***g*** and ***f*** have the same slope, $m = 1$, the lines are parallel. Also, the $y$-intercept of the graph of $g$ is 3 more than the $y$-intercept of the graph of $f$.

**b.**




Because the slope of the graph of ***h*** is greater than the slope of the graph of ***f***, the graph of $h$ rises faster from left to right. The $y$-intercept for both graphs is 0, so both lines pass through the origin.

## ✓ GUIDED PRACTICE for Example 4

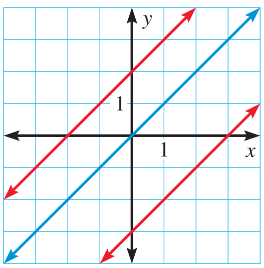
**3.** Graph $h(x) = -3x$. Compare the graph with the graph of $f(x) = x$.

## CONCEPT SUMMARY *For Your Notebook*

### Comparing Graphs of Linear Functions with the Graph of $f(x) = x$

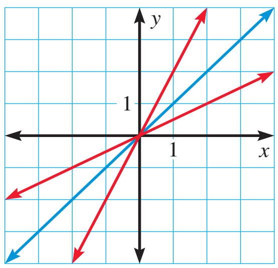
Changing $m$ or $b$ in the general linear function $g(x) = mx + b$ creates families of linear functions whose graphs are related to the graph of $f(x) = x$.

**$g(x) = x + b$**



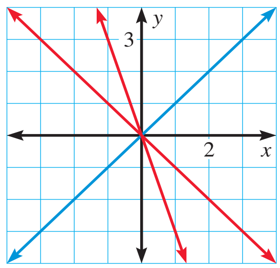

- The graphs have the same slope, but different $y$-intercepts.
- Graphs of this family are vertical translations of the graph of $f(x) = x$.

**$g(x) = mx$ where $m > 0$**




- The graphs have different (positive) slopes, but the same $y$-intercept.
- Graphs of this family are vertical stretches or shrinks of the graph of $f(x) = x$.

**$g(x) = mx$ where $m < 0$**



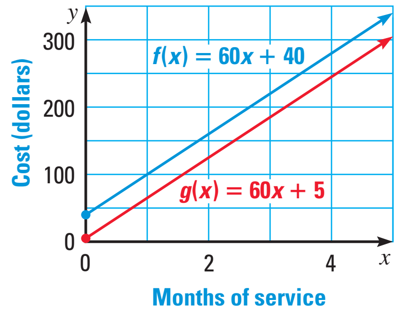

- The graphs have different (negative) slopes, but the same $y$-intercept.
- Graphs of this family are vertical stretches or shrinks with reflections in the $x$-axis of the graph of $f(x) = x$.

## EXAMPLE 5 Graph real-world functions

**CABLE** A cable company charges new customers \$40 for installation and \$60 per month for its service. The cost to the customer is given by the function $f(x) = 60x + 40$ where $x$ is the number of months of service. To attract new customers, the cable company reduces the installation fee to \$5. A function for the cost with the reduced installation fee is $g(x) = 60x + 5$. Graph both functions. How is the graph of $g$ related to the graph of $f$?

**Solution**

The graphs of both functions are shown. Both functions have a slope of 60, so they are parallel. The $y$-intercept of the graph of $g$ is 35 less than the graph of $f$. So, the graph of $g$ is a vertical translation of the graph of $f$.




**REVIEW TRANSFORMATIONS**
For help with transformations, see pp. SR12–SR13.

## ✓ GUIDED PRACTICE for Example 5

4. **WHAT IF?** In Example 5, suppose the monthly fee is \$70 so that the cost to the customer is given by $h(x) = 70x + 40$. Graph $f$ and $h$ in the same coordinate plane. How is the graph of $h$ related to the graph of $f$?

# 3.7 EXERCISES

**HOMEWORK KEY**
○ = See **WORKED-OUT SOLUTIONS** Exs. 3, 17, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 22, 35, 36, 44, and 45

## SKILL PRACTICE

1. **VOCABULARY** When you write the function $y = 3x + 12$ as $f(x) = 3x + 12$, you are using __?__.

2. ★ **WRITING** Would the functions $f(x) = -9x + 12$, $g(x) = -9x - 2$, and $h(x) = -9x$ be considered a family of functions? *Explain.*

**EXAMPLE 1** for Exs. 3–13

**EVALUATING FUNCTIONS** **Evaluate the function when $x = -2$, 0, and 3.**

3. $f(x) = 12x + 1$
4. $g(x) = -3x + 5$
5. $p(x) = -8x - 2$
6. $h(x) = 2.25x$
7. $m(x) = -6.5x$
8. $f(x) = -0.75x - 1$
9. $s(x) = \frac{2}{5}x + 3$
10. $d(x) = -\frac{3}{2}x + 5$
11. $h(x) = \frac{3}{4}x - 6$

12. **ERROR ANALYSIS** *Describe* and correct the error in evaluating the function $g(x) = -5x + 3$ when $x = -3$.

$g(-3) = -5(-3) + 3$
$-3g = 18$
$g = -6$

13. ★ **MULTIPLE CHOICE** Given $f(x) = -6.8x + 5$, what is the value of $f(-2)$?

Ⓐ $-18.6$ Ⓑ $-8.6$ Ⓒ $8.6$ Ⓓ $18.6$

**EXAMPLE 2**
for Exs. 14–22

**FINDING *X*-VALUES** **Find the value of $x$ so that the function has the given value.**

14. $f(x) = 6x + 9;\ 3$

15. $g(x) = -x + 5;\ 2$

16. $h(x) = -7x + 12;\ -9$

17. $j(x) = 4x + 11;\ -13$

18. $m(x) = 9x - 5;\ -2$

19. $n(x) = -2x - 21;\ -6$

20. $p(x) = -12x - 36;\ -3$

21. $q(x) = 8x - 32;\ -4$

22. ★ **MULTIPLE CHOICE** What value of $x$ makes $f(x) = 5$ if $f(x) = -2x + 25$?

Ⓐ $-15$ Ⓑ $-10$ Ⓒ $10$ Ⓓ $15$

**EXAMPLE 4**
for Exs. 23–34

**TRANSFORMATIONS OF LINEAR FUNCTIONS** **Graph the function. Compare the graph with the graph of $f(x) = x$.**

23. $g(x) = x + 5$

24. $h(x) = 6 + x$

25. $q(x) = x - 1$

26. $m(x) = x - 6$

27. $d(x) = x + 7$

28. $t(x) = x - 3$

29. $r(x) = 4x$

30. $w(x) = 5x$

31. $h(x) = -3x$

32. $k(x) = -6x$

33. $g(x) = \frac{1}{3}x$

34. $m(x) = -\frac{7}{2}x$

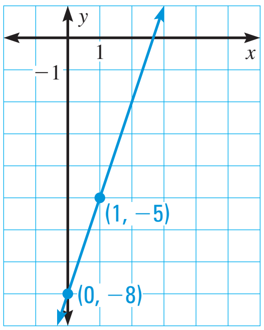
35. ★ **MULTIPLE CHOICE** The graph of which function is shown?

Ⓐ $f(x) = 3x + 8$

Ⓑ $f(x) = 3x - 8$

Ⓒ $f(x) = 8x + 3$

Ⓓ $f(x) = 8x - 3$




36. ★ **OPEN-ENDED** In this exercise you will compare the graphs of linear functions when their slopes and $y$-intercepts are changed.

**a.** Choose a linear function of the form $f(x) = mx + b$ where $m \neq 0$. Then graph the function.

**b.** Using the same $m$ and $b$ values as in part (a), graph the function $g(x) = 2mx + b$. How are the slope and $y$-intercept of the graph of $g$ related to the slope and $y$-intercept of the graph of $f$?

**c.** Using the same $m$ and $b$ values as in part (a), graph the function $h(x) = mx + (b - 3)$. How are the slope and $y$-intercept of the graph of $h$ related to the slope and $y$-intercept of the graph of $f$?

37. **REASONING** How is the graph of $g(x) = 1$ related to the graph of $h(x) = -1$?

38. **CHALLENGE** Suppose that $f(x) = 4x + 7$ and $g(x) = 2x$. What is a rule for $g(f(x))$? What is a rule for $f(g(x))$?



◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

## PROBLEM SOLVING

EXAMPLE 3
for Exs. 39–41

39. **MOVIE TICKETS** The average price of a movie ticket in the United States from 1980 to 2000 can be modeled by the function $f(x) = 0.10x + 2.75$ where $x$ is the number of years since 1980.

**a.** Graph the function and identify its domain and range.

**b.** Find the value of $x$ so that $f(x) = 4.55$. *Explain* what the solution means in this situation.

40. **DVD PLAYERS** The number (in thousands) of DVD players sold in the United States from 1998 to 2003 can be modeled by $f(x) = 4250x + 330$ where $x$ is the number of years since 1998.

**a.** Graph the function and identify its domain and range.

**b.** Find the value of $x$ so that $f(x) = 13{,}080$. *Explain* what the solution means in this situation.

41. **IN-LINE SKATING** An in-line skater's average speed is 10 miles per hour. The distance traveled after skating for $x$ hours is given by the function $d(x) = 10x$. Graph the function and identify its domain and range. How long did it take the skater to travel 15 miles? *Explain.*

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EXAMPLE 5
for Exs. 42–43

42. **HOME SECURITY** A home security company charges new customers $155 for the installation of security equipment and a monthly fee of $40. To attract more customers, the company reduces its installation fee to $75. The functions below give the total cost for $x$ months of service:

**Regular fee:** $f(x) = 40x + 155$ **Reduced fee:** $g(x) = 40x + 75$

Graph both functions. How is the graph of $g$ related to the graph of $f$?

43. **THEATERS** A ticket for a play at a theater costs $16. The revenue (in dollars) generated from the sale of $x$ tickets is given by $s(x) = 16x$. The theater managers raise the cost of tickets to $20. The revenue generated from the sale of $x$ tickets at that price is given by $r(x) = 20x$. Graph both functions. How is the graph of $r$ related to the graph of $s$?

44. ★ **EXTENDED RESPONSE** The cost of supplies, such as mustard and napkins, a pretzel vendor needs for one day is $75. Each pretzel costs the vendor $.50 to make. The total daily cost to the vendor is given by $C(x) = 0.5x + 75$ where $x$ is the number of pretzels the vendor makes.

**a. Graph** Graph the cost function.

**b. Graph** The vendor sells each pretzel for $3. The revenue is given by $R(x) = 3x$ where $x$ is the number of pretzels sold. Graph the function.

**c. Explain** The vendor's profit is the difference of the revenue and the cost. *Explain* how you could use the graphs to find the vendor's profit for any given number of pretzels made and sold.

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**45.** ★ **EXTENDED RESPONSE** The number of hours of daylight in Austin, Texas, during the month of March can be modeled by the function $\ell(x) = 0.03x + 11.5$ where $x$ is the day of the month.

**a.** **Graph** Graph the function and identify its domain and range.

**b.** **Graph** The number of hours of darkness can be modeled by the function $d(x) = 24 - \ell(x)$. Graph the function on the same coordinate plane as you used in part (a). Identify its domain and range.

**c.** **CHALLENGE** *Explain* how you could have obtained the graph of $d$ from the graph of $\ell$ using translations and reflections.

**d.** **CHALLENGE** What does the point where the graphs intersect mean in terms of the number of hours of daylight and darkness?

## QUIZ

**Given that $y$ varies directly with $x$, use the specified values to write a direct variation equation that relates $x$ and $y$.**

**1.** $x = 5, y = 10$

**2.** $x = 4, y = 6$

**3.** $x = 2, y = -16$

**Evaluate the function.**

**4.** $g(x) = 6x - 5$ when $x = 4$

**5.** $h(x) = 14x + 7$ when $x = 2$

**6.** $j(x) = 0.2x + 12.2$ when $x = 244$

**7.** $k(x) = \frac{5}{6}x + \frac{1}{3}$ when $x = 4$

**Graph the function. Compare the graph to the graph of $f(x) = x$.**

**8.** $g(x) = -4x$

**9.** $h(x) = x - 2$

**10.** **HOURLY WAGE** The table shows the number of hours that you worked for each of three weeks and the amount that you were paid. What is your hourly wage?

| Hours | 12 | 16 | 14 |
|---|---|---|---|
| Pay (dollars) | 84 | 112 | 98 |



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