LESSON 32

Modeling Data with Functions

GOAL Model data using quadratic, exponential, or power functions; use models to make predictions.

When a scatter plot shows that data do not fit a linear pattern, you may be able to use a nonlinear function to model the data.

Three Types of Nonlinear Functions

Type of function	Form
quadratic	$y = ax^2 + bx + c$, where $a \neq 0$
exponential	$y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$
power	$y = ax^b$, where $a \neq 0$

EXAMPLE 1 Finding a Quadratic Model

The table below gives the stopping distance y (in feet) for a car traveling on a dry road at various speeds x (in miles per hour).

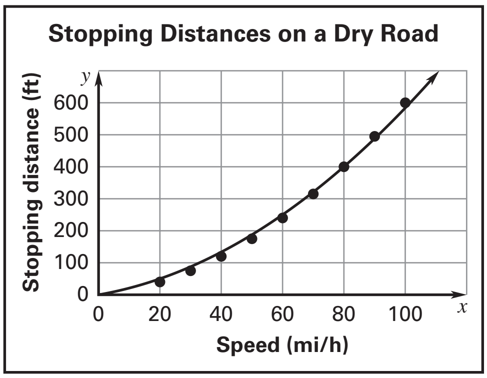
Speed, x (mi/h)	20	30	40	50	60	70	80	90	100
Stopping distance, y (ft)	40	75	120	175	240	315	400	495	600

a. Find an equation of the quadratic function that models the data.

b. Predict the stopping distance for a car traveling at 65 miles per hour.

SOLUTION

a. Plot the data pairs (x, y) in a coordinate plane. Then draw the parabola or part of a parabola that you think best fits the data.



Estimate the coordinates of three points on the parabola, such as (0, 0), (60, 250), and (80, 400).

Substitute the coordinates of the points into the model $y = ax^2 + bx + c$ to obtain a system of three linear equations.

$$c = 0$$
$$3600a + 60b + c = 250$$
$$6400a + 80b + c = 400$$

Solve the linear system. The solution is $a \approx 0.0417$, $b \approx 1.67$, and $c = 0$.

A quadratic model for the data is $y = 0.0417x^2 + 1.67x$.

b. To predict the stopping distance for a car traveling at 65 miles per hour, substitute 65 for x in the quadratic model.

$$y = 0.0417(65)^2 + 1.67(65) = 284.7325$$

The stopping distance is about 285 feet.

CHECK **Example 1**

1. Find an equation of the quadratic function that models the data.

x	−6	−2	0	1	3	5
y	20	4	1	2	6	17

Exponential Functions You can tell if an exponential function is a good model for a set of data (x, y) by graphing the set of data $(x, \ln y)$. If the new set of data fits a linear pattern, then an exponential function is a good model for the original set of data.

EXAMPLE 2 Finding an Exponential Model

The table below gives the value y (in dollars) of a boat x years after it was bought.

x	0	1	2	3	4	5
y	12,000	9600	7700	6100	4900	3900

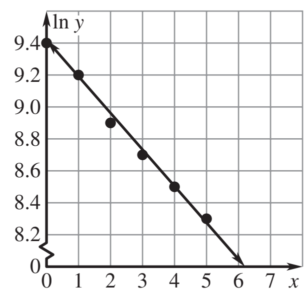
a. Find an equation of the exponential function that models the data.

b. Predict the value of the boat 7 years after it was bought.

SOLUTION

a. First use a calculator to create a new set of data $(x, \ln y)$. The values of $\ln y$ are rounded to the nearest tenth.

x	0	1	2	3	4	5
ln *y*	9.4	9.2	8.9	8.7	8.5	8.3



Then plot the new points as shown at the right. Sketch a line that appears to best fit the data. The points lie close to the line, so an exponential model is a good fit for the original data.

Choose two points that are on the line, such as (0.5, 9.3) and (4, 8.5). Use these points to find an equation of the line. Then solve for y.

$\ln y = -0.229x+9.41$ *Equation of line*

$y = e^{-0.229x + 9.41}$ *Exponentiate each side using base* e.

$y = e^{9.41}(e^{-0.229})^x$ *Use properties of exponents.*

$y \approx 12{,}200(0.795)^x$ *Simplify.*

An exponential model for the data is $y = 12{,}200(0.795)^x$.

b. To predict the value of the boat 7 years after it was bought, substitute 7 for x in the exponential model.

$y = 12{,}200(0.795)^7 \approx 2450$

You can predict that the value of the boat will be about $2450.

Power Functions You can tell if a power function is a good model for a set of data (x, y) by graphing the set of data $(\ln x, \ln y)$. If the new set of data fits a linear pattern, then a power function is a good model for the original set of data.

EXAMPLE 3 Finding a Power Model

The table below gives the population rank x and the estimated population y (in thousands) for the 9 most populated cities in the United States.

x	1	2	3	4	5	6	7	8	9
y	8053	3903	2940	2088	1520	1457	1292	1271	1224

a. Find an equation of the power function that models the data.

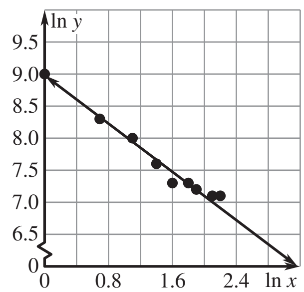
b. Predict the population of the city in the United States that has a population rank of 12.

SOLUTION

a. First use a calculator to create a new set of data $(\ln x, \ln y)$:

$\ln x$	0	0.69	1.1	1.4	1.6	1.8	1.9	2.1	2.2
$\ln y$	9.0	8.3	8.0	7.6	7.3	7.3	7.2	7.1	7.1

Then plot the new points as shown at the right. Sketch a line that appears to best fit the data. The points lie close to the line, so a power model is a good fit for the original data.



Choose two points that are on the line, such as (0.4, 8.6) and (2.1, 7). Use these points to find an equation of the line. Then solve for y.

$\ln y = -0.941 \ln x + 8.98$ — *Equation of line*

$\ln y = \ln x^{-0.941} + 8.98$ — *Power property of logarithms*

$\ln y = \ln x^{-0.941} + \ln e^{8.98}$ — *$\ln e^x = \log_e e^x = x$*

$\ln y \approx \ln x^{-0.941} + \ln 7940$ — *Simplify.*

$\ln y \approx \ln (7940x^{-0.941})$ — *Product property of logarithms*

$y \approx 7940x^{-0.941}$ — *$\ln x = \ln y$ if and only if $x = y$.*

A power model for the data is $y = 7940x^{-0.941}$.

b. To predict the population of the city in the United States that has a population rank of 12, substitute 12 for x in the power model.

$$y = 7940(12)^{-0.941} \approx 766$$

You can predict that the population of the city is about 766,000.

CHECK **Examples 2 and 3**

Find an equation of the given type of function that models the data.

2. Exponential function

x	−4	−2	0	2	4
y	−60	−16	−5	−1	−0.25

3. Power function

x	1	2	3	4	5	6
y	6.3	4.4	3.5	3.4	2.7	2.3

EXAMPLE 4 Using a Graphing Calculator to Find a Model

Use a graphing calculator to find a model for the data.

x	1	2	3	4	5	6	7	8
y	150	43	21	12	8	6	5	3

SOLUTION

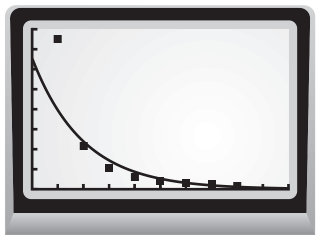
Step 1 Enter the x-values into L1 and the y-values into L2. Then graph the data in a scatter plot, as shown at the right.

It is difficult to tell whether the data fits an exponential model or a power model.

Step 2 Try finding an exponential model by entering the following keystrokes:

STAT ▶ 0 ENTER

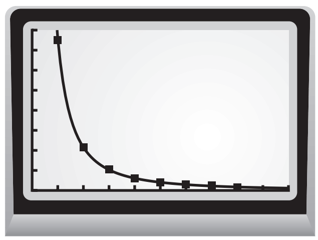
An exponential model for the data is $y = 131(0.604)^x$. The model is shown graphed with the data at the right.



Step 3 Now try finding a power model by entering the following keystrokes:

STAT ▶ A ENTER

A power model for the data is $y = 152x^{-1.82}$. The model is shown graphed with the data at the right.



Because the power model passes near more of the data points, $y = 152x^{-1.82}$ is the best-fitting model for the data.

CHECK **Example 4**

4. The table shows a town's population y for various years x since 1990. Use a graphing calculator to find a model for the data. Then predict the town's population in 2006.

x	0	2	4	6	8	10	12
y	3500	4050	4700	5300	6050	6900	7950

EXERCISES

Make a scatter plot of the data. Then find an equation of the quadratic function that models the data.

1.

x	−6	−4	−2	0	2	4	6	8
y	11	9	5	2	1	1	3	6

2.

x	−6	−4	−2	0	2	4	6	8
y	−38	−20	−4	3	−1	−16	−40	−61

Make a scatter plot of the data. Then find an equation of the exponential function that models the data.

3.

x	−4	−2	0	2	4	6	8
y	0.2	1	4	18	60	240	1000

4.

x	−3	−2	−1	0	1	2	3	4
y	14	10	8	6	4.5	3.2	2.5	2

Make a scatter plot of the data. Then find an equation of the power function that models the data.

5.

x	2	4	6	8	10	12	14	16
y	10	12	14	15	16	17	17.8	18.5

6.

x	1	3	5	7	9	11	13	15
y	1	7	21	43	75	117	150	202

In Exercises 7–10, use the table below, which gives the corn yield *y* (in bushels per acre) that results from various nitrogen fertilizer rates *x* (in pounds per acre).

Fertilizer rate, *x* (lb/acre)	90	100	110	120	130	140	150
Corn yield, *y* (bushels/acre)	85	90	95	100	100	95	88

7. Make a scatter plot of the data.

8. Find an equation of the quadratic function that models the data.

9. Use the quadratic model to predict the corn yield for a nitrogen fertilizer rate of 145 pounds per acre.

10. What amount of nitrogen fertilizer do you think would maximize the corn yield? Explain your reasoning.

In Exercises 11–13, use the table below, which gives the saturation specific humidity *y* (in grams per kilogram) for Earth's atmosphere at various temperatures *x* (in degrees Celsius).

Temperature, *x* (°C)	−40	−30	−20	−10	0	10	20	30	40
Saturation specific humidity, *y* (g/kg)	0.1	0.3	0.75	2	3.5	7	14	26.5	47

11. Make a scatter plot of the data.

12. Find an equation of the exponential function that models the data.

13. Use the quadratic model to predict the saturation specific humidity at a temperature of 25°C.

In Exercises 14–16, use the table below, which gives the population rank *x* and the estimated population *y* (in thousands) for the 9 most populated cities in Japan.

14. Make a scatter plot of the data.

Population rank, *x*	1	2	3	4	5	6	7	8	9
Estimated population, *y* (in thousands)	8240	3495	2597	2190	1848	1530	1474	1471	1369

15. Find an equation of the power function that models the data.

16. Use the power model to predict the population of the city in Japan that has a population rank of 13.

In Exercises 17 and 18, use the following table.

Planet	**Mean distance from Sun, *x* (in millions of miles)**	**Rotation time around Sun, *y* (in Earth years)**
Mercury	36	0.241
Venus	67	0.615
Earth	93	1.00
Mars	142	1.88
Jupiter	484	11.9
Saturn	891	29.4
Uranus	1785	83.8
Neptune	2793	164

17. Use a graphing calculator to find and graph a model for the set of data.

18. Pluto's mean distance from the Sun is 3,647,000,000 miles. Predict Pluto's rotation time around the Sun.

19. Use a newspaper, magazine, reference book, or Web site to find a set of nonlinear paired data. Use a graphing calculator to find a model for the data. Then use your model to make a prediction about the corresponding *y*-value for an *x*-value that is not contained in your data set.