Name ______________________ Date ______________

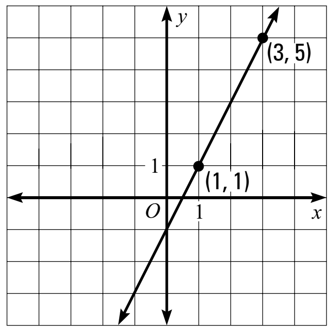
CHAPTER 4 Representations of Lines

Lines can be determined given any of the following: a graph, an equation, a slope and one point, or two points. Given one form of a line, the other forms can be derived from it.

Two points, (1, 1) and (3, 5), are plotted on a graph to form the line shown.

From the points, the slope of the line can be found.

Slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{3 - 1} = \frac{4}{2} = 2$



The slope with one of the given points can be used to find the equation of this line in point-slope form.

Point-slope form: $y - 1 = 2(x - 1)$

The point-slope form can be rewritten into slope-intercept form or general form.

Slope-intercept form: $y - 1 = 2x - 2 \rightarrow y = 2x - 1$

General form: $y - 1 = 2x - 2 \rightarrow 2x - y = 1$

Each of these forms represents the same line graphed through the two points (1, 1) and (3, 5).

EXAMPLE 1 Find the equation of a line from a graph

Find the point-slope form and slope-intercept form of the line shown in this graph.

Solution:

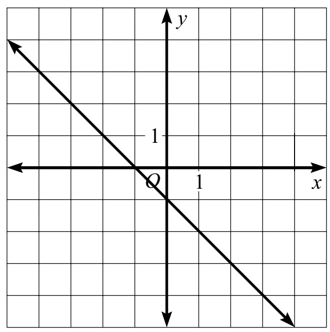
Two points located on this line are $(-3, 2)$ and $(1, -2)$.

The slope of the line is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{-3 - 1} = \frac{4}{-4} = -1$.

The y-intercept is at -1.

One point-slope form of this line is $y - 2 = -1(x + 3)$.

The slope-intercept form of this line is $y = -x - 1$. ■



Example 1 shows how equations of a line can be found given a graph or two points. Example 2 shows how the slope and one point can be found given the general form of a line.

EXAMPLE 2 Find the slope and a point on a line given its equation

Find the slope and a point on the line $3x + y = -4$.

Solution:

To find the slope, rewrite the equation in slope-intercept form as $y = -3x - 4$.

The slope is -3.

To find a point on the line, choose any x value and find the corresponding y value. When $x = -2$, $y = -3(-2) - 4 = 2$. A point on this line is $(-2, 2)$. ■

Name ______________________________ Date ______________

CHAPTER 4

Representations of Lines *continued*

Since lines have an infinite number of points, the number of possible x and corresponding y values that can be found is infinite. As a result, the number of equations that can be written for one line in point-slope form is also infinite.

EXAMPLE 3 Write multiple equations for one line in point-slope form

Find more than one equation in point-slope form for the line containing points (2, 1), (3, 3), and (4, 5).

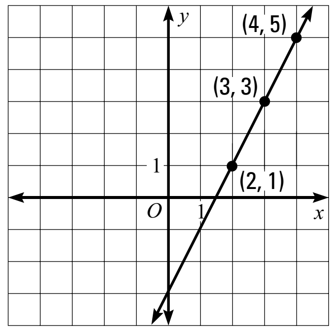
Solution:

Slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{3 - 2} = \frac{2}{1} = 2.$

One equation in point-slope form is $y - 1 = 2(x - 2)$.

Other equations are $y - 3 = 2(x - 3)$ and $y - 5 = 2(x - 4)$.

All equations represent the line $y = 2x - 3$. Since there are infinitely many points on a line, each point can be used to write an infinite number of equations in point-slope form. ■



Practice

Write the point-slope form of the line containing the given slope and point.

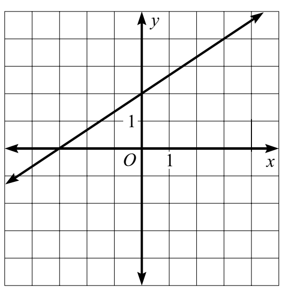
1. $m = 3$, point (0, 4)
2. $m = \frac{1}{3}$, point (3, 2)
3. $m = \frac{4}{3}$, point $(-1, -1)$
4. $m = -5$, point (6, 0)
5. $m = -\frac{5}{2}$, point $(-4, 2)$
6. $m = -1$, point $(4, -4)$

Write the slope-intercept form of the line containing the given two points.

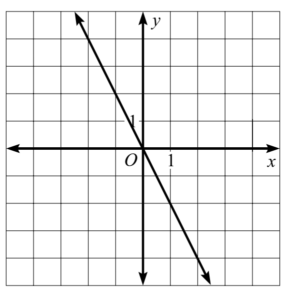
7. (1, 4) and (2, 5)
8. $(0, -3)$ and (4, 3)
9. (5, 0) and (0, 3)
10. $(-3, 1)$ and $(2, -3)$
11. $(3, -4)$ and $(-1, 2)$
12. $(-1, -2)$ and $(-3, -3)$

Write the slope-intercept form of the line graphed below.

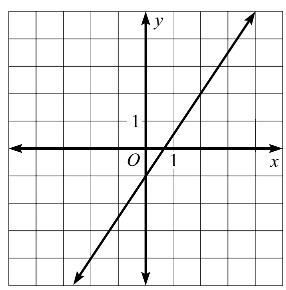
13.



14.



15.



Problem Solving

16. Mark bought a \$50 tennis racket and paid \$20 an hour for tennis lessons. Find Mark's total cost for the tennis racket and 1, 2, 3, and 4 hours of tennis lessons. Use this information to write four equations in point-slope form to model Mark's total cost for tennis racket and lessons.