4.1 Write Linear Equations in Slope-Intercept Form

Before You graphed equations of lines.

Now You will write equations of lines.

Why? So you can model distances in sports, as in Ex. 52.

Key Vocabulary
- *y*-intercept
- slope
- slope-intercept form

Recall that the graph of an equation in slope-intercept form, $y = mx + b$, is a line with a slope of m and a y-intercept of b. You can use this form to write an equation of a line if you know its slope and y-intercept.

EXAMPLE 1 Use slope and *y*-intercept to write an equation

Write an equation of the line with a slope of −2 and a *y*-intercept of 5.

$y = mx + b$ Write slope-intercept form.

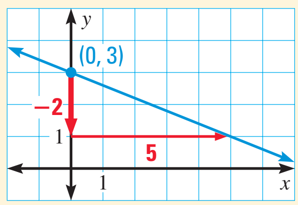
$y = -2x + 5$ Substitute −2 for *m* and 5 for *b*.

★ EXAMPLE 2 Standardized Test Practice

Which equation represents the line shown?

Ⓐ $y = -\frac{2}{5}x + 3$ Ⓑ $y = -\frac{5}{2}x + 3$

Ⓒ $y = -\frac{2}{5}x + 1$ Ⓓ $y = 3x + \frac{2}{5}$



ELIMINATE CHOICES
In Example 2, you can eliminate choices C and D because the *y*-intercepts of the graphs of these equations are not 3.

The slope of the line is $\frac{\text{rise}}{\text{run}} = \frac{-2}{5} = -\frac{2}{5}$.

The line crosses the y-axis at (0, 3). So, the y-intercept is 3.

$y = mx + b$ Write slope-intercept form.

$y = -\frac{2}{5}x + 3$ Substitute $-\frac{2}{5}$ for *m* and 3 for *b*.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

Animated Algebra at my.hrw.com

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

✓ GUIDED PRACTICE for Examples 1 and 2

Write an equation of the line with the given slope and *y*-intercept.

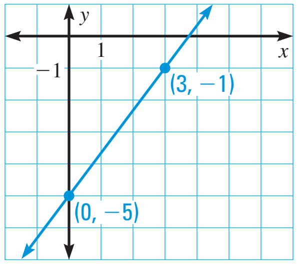
1. Slope is 8; y-intercept is −7.

2. Slope is $\frac{3}{4}$; y-intercept is −3.

USING TWO POINTS If you know the point where a line crosses the y-axis and any other point on the line, you can write an equation of the line.

EXAMPLE 3 Write an equation of a line given two points

Write an equation of the line shown.



Solution

STEP 1 **Calculate** the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-5)}{3 - 0} = \frac{4}{3}$$

STEP 2 **Write** an equation of the line. The line crosses the y-axis at $(0, -5)$. So, the y-intercept is -5.

$y = mx + b$ **Write slope-intercept form.**

$y = \frac{4}{3}x - 5$ **Substitute $\frac{4}{3}$ for m and -5 for b.**

WRITING FUNCTIONS Recall that the graphs of linear functions are lines. You can use slope-intercept form to write a linear function.

EXAMPLE 4 Write a linear function

REVIEW FUNCTIONS
You may want to review function notation before writing an equation for a function.

Write an equation for the linear function f with the values $f(0) = 5$ and $f(4) = 17$.

Solution

STEP 1 **Write** $f(0) = 5$ as $(0, 5)$ and $f(4) = 17$ as $(4, 17)$.

STEP 2 **Calculate** the slope of the line that passes through $(0, 5)$ and $(4, 17)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{17 - 5}{4 - 0} = \frac{12}{4} = 3$$

STEP 3 **Write** an equation of the line. The line crosses the y-axis at $(0, 5)$. So, the y-intercept is 5.

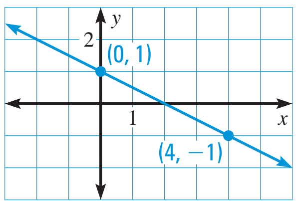
$y = mx + b$ **Write slope-intercept form.**

$y = 3x + 5$ **Substitute 3 for m and 5 for b.**

▶ The function is $f(x) = 3x + 5$.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Write an equation of the line shown.



Write an equation for the linear function f with the given values.

4. $f(0) = -2, f(8) = 4$

5. $f(-3) = 6, f(0) = 5$

READING
The value b is a starting value in a real-world situation modeled by $y = mx + b$, because when $x = 0$, the value of y is b.

MODELING REAL-WORLD SITUATIONS When a quantity y changes at a constant rate with respect to a quantity x, you can use the equation $y = mx + b$ to model the relationship. The value of m is the constant rate of change, and the value of b is an initial, or starting, value for y.

EXAMPLE 5 Solve a multi-step problem

RECORDING STUDIO A recording studio charges musicians an initial fee of \$50 to record an album. Studio time costs an additional \$35 per hour.

a. Write an equation that gives the total cost of an album as a function of studio time (in hours).

b. Find the total cost of recording an album that takes 10 hours of studio time.

Solution

a. The cost changes at a constant rate, so you can write an equation in slope-intercept form to model the total cost.

STEP 1 **Identify** the rate of change and the starting value.

Rate of change, *m*: cost per hour
Starting value, *b*: initial fee

STEP 2 **Write** a verbal model. Then write the equation.

Total cost (dollars)	=	Cost per hour (dollars per hour)	•	Studio time (hours)	+	Initial fee (dollars)
C	=	35	•	t	+	50

CHECK Use unit analysis to check the equation.

$$\text{dollars} = \frac{\text{dollars}}{\cancel{\text{hour}}} \cdot \cancel{\text{hours}} + \text{dollars} \checkmark$$

▶ The total cost C is given by the function $C = 35t + 50$ where t is the studio time (in hours).

b. Evaluate the function for $t = 10$.

$C = 35(10) + 50 = 400$ **Substitute 10 for *t* and simplify.**

▶ The total cost for 10 hours of studio time is \$400.

✓ GUIDED PRACTICE for Example 5

6. WHAT IF? In Example 5, suppose the recording studio raises its initial fee to \$75 and charges \$40 per hour for studio time.

a. Write an equation that gives the total cost of an album as a function of studio time (in hours).

b. Find the total cost of recording an album that takes 10 hours of studio time.

4.1 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 11, 19, and 47

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 40, 43, 48, and 50

◆ = **MULTIPLE REPRESENTATIONS** Ex. 49

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The ratio of the rise to the run between any two points on a nonvertical line is called the _?_.

2. ★ **WRITING** *Explain* how you can use slope-intercept form to write an equation of a line given its slope and y-intercept.

EXAMPLE 1 for Exs. 3–9, 16

WRITING EQUATIONS **Write an equation of the line with the given slope and y-intercept.**

3. slope: 2
   y-intercept: 9

4. slope: 1
   y-intercept: 5

5. slope: -3
   y-intercept: 0

6. slope: -7
   y-intercept: 1

7. slope: $\frac{2}{3}$
   y-intercept: -9

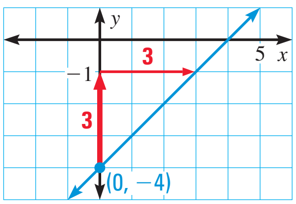
8. slope: $\frac{3}{4}$
   y-intercept: -6

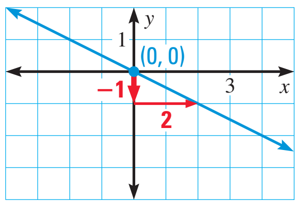
9. ★ **MULTIPLE CHOICE** Which equation represents the line with a slope of -1 and a y-intercept of 2?

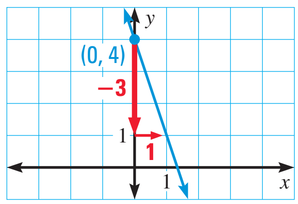
   Ⓐ $y = -x + 2$ Ⓑ $y = 2x - 1$ Ⓒ $y = x - 2$ Ⓓ $y = 2x + 1$

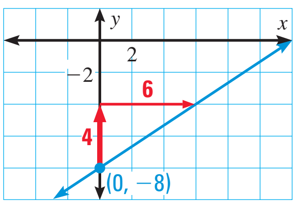
EXAMPLE 2 for Exs. 10–15

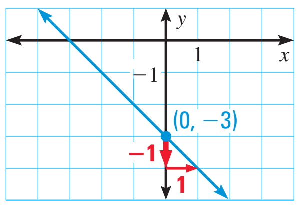
WRITING EQUATIONS **Write an equation of the line shown.**

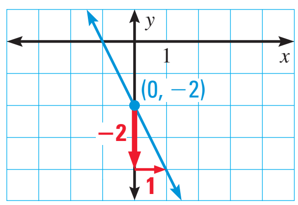
10. 

11. 

12. 

13. 

14. 

15. 

16. **ERROR ANALYSIS** *Describe* and correct the error in writing an equation of the line with a slope of 2 and a y-intercept of 7.

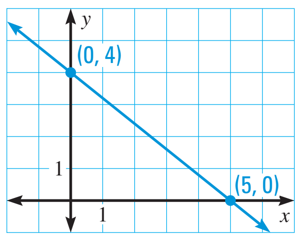
$y = 7x + 2$

EXAMPLE 3 for Exs. 17–29

17. **ERROR ANALYSIS** *Describe* and correct the error in writing an equation of the line shown.

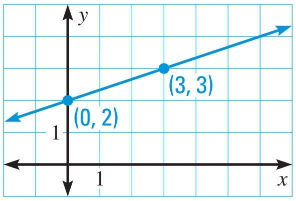
$$\text{slope} = \frac{0-4}{0-5} = \frac{-4}{-5} = \frac{4}{5}$$

$$y = \frac{4}{5}x + 4$$



USING A GRAPH **Write an equation of the line shown.**

18.



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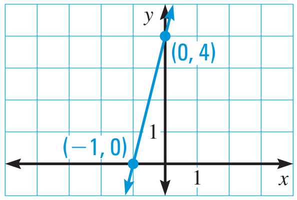

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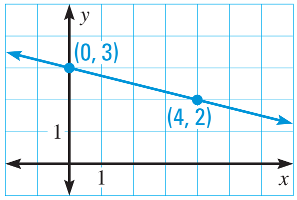

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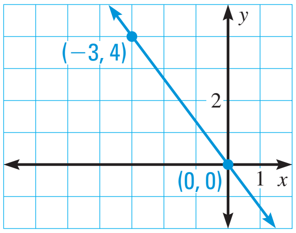

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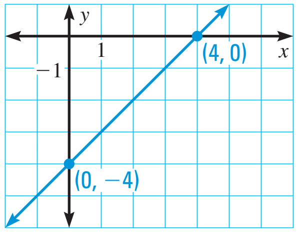

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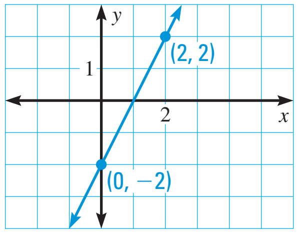


USING TWO POINTS **Write an equation of the line that passes through the given points.**

24. (−3, 1), (0, −8) **25.** (2, −7), (0, −5) **26.** (2, −4), (0, −4)

27. (0, 4), (8, 3.5) **28.** (0, 5), (1.5, 1) **29.** (−6, 0), (0, −24)

EXAMPLE 4
for Exs. 30–38

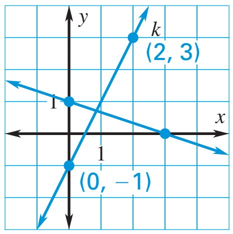
WRITING FUNCTIONS **Write an equation for the linear function *f* with the given values.**

30. $f(0) = 2, f(2) = 4$ **31.** $f(0) = 7, f(3) = 1$ **32.** $f(0) = -2, f(4) = -3$

33. $f(0) = -1, f(5) = -5$ **34.** $f(-2) = 6, f(0) = -4$ **35.** $f(-6) = -1, f(0) = 3$

36. $f(4) = 13, f(0) = 21$ **37.** $f(0) = 9, f(3) = 0$ **38.** $f(0.2) = 1, f(0) = 0.6$

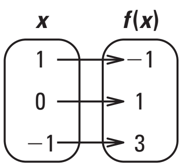
39. **VISUAL THINKING** Line ℓ passes through the points (0, 1) and (3, 0). What change(s) in the parameters m and b in the slope-intercept equation of k occurred to produce the slope-intercept equation of ℓ?



40. ★ **OPEN-ENDED** *Describe* a real-world situation that can be modeled by the function $y = 4x + 9$.

USING A DIAGRAM OR TABLE **Write an equation that represents the linear function shown in the mapping diagram or table.**

41.



42.

x	$f(x)$
−4	−2
−2	−1
0	0

43. ★ **WRITING** A line passes through the points (3, 5) and (3, −7). Is it possible to write an equation of the line in slope-intercept form? *Justify* your answer.

44. **CHALLENGE** Show that the equation of the line that passes through the points $(0, b)$ and $(1, b + m)$ is $y = mx + b$. *Explain* how you can be sure that the point $(-1, b - m)$ also lies on the line.

PROBLEM SOLVING

EXAMPLE 5
for Exs. 45–49

45. WEB SERVER The initial fee to have a website set up using a server is \$48. It costs \$44 per month to maintain the website.

a. Write an equation that gives the total cost of setting up and maintaining a website as a function of the number of months it is maintained.

b. Find the total cost of setting up and maintaining the website for 6 months.

46. PHOTOGRAPHS A camera shop charges \$3.99 for an enlargement of a photograph. Enlargements can be delivered for a charge of \$1.49 per order. Write an equation that gives the total cost of an order with delivery as a function of the number of enlargements. Find the total cost of ordering 8 photograph enlargements with delivery.

47. AQUARIUM Your family spends \$30 for tickets to an aquarium and \$3 per hour for parking. Write an equation that gives the total cost of your family's visit to the aquarium as a function of the number of hours that you are there. Find the total cost of 4 hours at the aquarium.

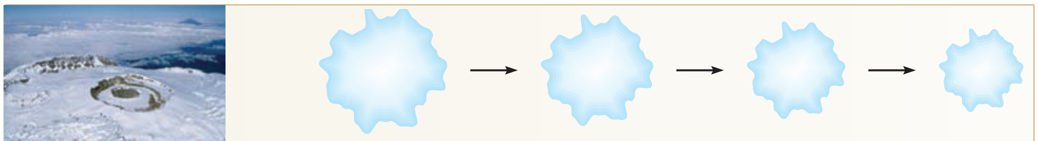
48. ★ SHORT RESPONSE Scientists found that the number of ant species in Clark Canyon, Nevada, increases at a rate of 0.0037 species per meter of elevation. There are approximately 3 ant species at sea level.

a. Write an equation that gives the number of ant species as a function of the elevation (in meters).

b. Identify the dependent and independent variables in this situation.

c. *Explain* how you can use the equation from part (a) to approximate the number of ant species at an elevation of 2 meters.

49. MULTIPLE REPRESENTATIONS The timeline shows the approximate total area of glaciers on Mount Kilimanjaro from 1970 to 2000.



Year	1970	1980	1990	2000
Area	5.2 km^2	4.1 km^2	3.0 km^2	1.9 km^2

a. Making a Table Make a table that shows the number of years x since 1970 and the area of the glaciers y (in square kilometers).

b. Drawing a Graph Graph the data in the table. *Explain* how you know the area of glaciers changed at a constant rate.

c. Writing an Equation Write an equation that models the area of glaciers as a function of the number of years since 1970. By how much did the area of the glaciers decrease each year from 1970 to 2000?

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**   = **MULTIPLE REPRESENTATIONS**

50. ★ **EXTENDED RESPONSE** The Harris Dam in Maine releases water into the Kennebec River. From 10:00 A.M. to 1:00 P.M. during each day of whitewater rafting season, water is released at a greater rate than usual.

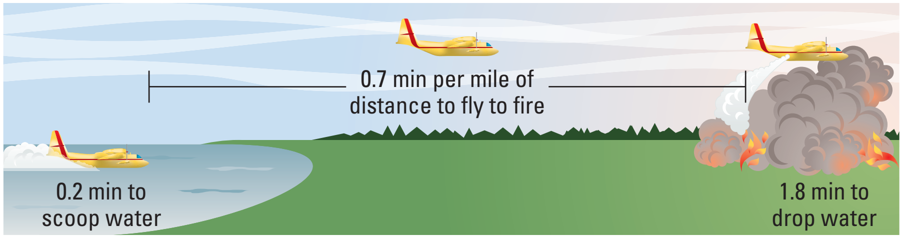
Time interval	Release rate (gallons per hour)
12:00 A.M. to 10:00 A.M.	8.1 million
10:00 A.M. to 1:00 P.M.	130 million

a. On a day during rafting season, how much water is released by 10:00 A.M.?

b. Write an equation that gives, for a day during rafting season, the total amount of water (in gallons) released as a function of the number of hours since 10:00 A.M.

c. What is the domain of the function from part (b)? *Explain.*

51. **FIREFIGHTING** The diagram shows the time a firefighting aircraft takes to scoop water from a lake, fly to a fire, and drop the water on the fire.



a. **Model** Write an equation that gives the total time (in minutes) that the aircraft takes to scoop, fly, and drop as a function of the distance (in miles) flown from the lake to the fire.

b. **Predict** Find the time the aircraft takes to scoop, fly, and drop if it travels 20 miles from the lake to the fire.

52. **CHALLENGE** The elevation at which a baseball game is played affects the distance a ball travels when hit. For every increase of 1000 feet in elevation, the ball travels about 7 feet farther. Suppose a baseball travels 400 feet when hit in a ball park at sea level.

a. **Model** Write an equation that gives the distance (in feet) the baseball travels as a function of the elevation of the ball park in which it is hit.

b. **Justify** *Justify* the equation from part (a) using unit analysis.

c. **Predict** If the ball were hit in exactly the same way at a park with an elevation of 3500 feet, how far would it travel?

Investigate Families of Lines

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

QUESTION How can you use a graphing calculator to find equations of lines using slopes and *y*-intercepts?

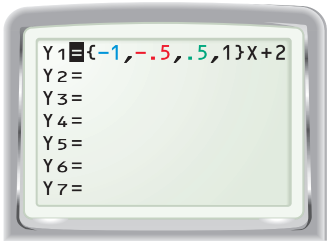
Recall that you can create families of lines by varying the value of either m or b in $y = mx + b$. The constants m and b are called *parameters*. Given the value of one parameter, you can determine the value of the other parameter if you also have information that uniquely identifies one member of the family of lines.

EXAMPLE 1 Find the slope of a line and write an equation

In the same viewing window, display the four lines that have slopes of −1, −0.5, 0.5, and 1 and a *y*-intercept of 2. Then use the graphs to determine which line passes through the point (12, 8). Write an equation of the line.

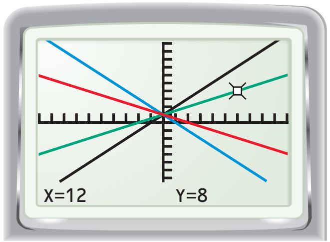
***STEP 1* Enter equations**

Press Y= and enter the four equations. Because the lines all have the same *y*-intercept, they constitute a family of lines and can be entered as shown.



***STEP 2* Display graphs**

Graph the equations in an appropriate viewing window. Press TRACE and use the left and right arrow keys to move along one of the lines until $x = 12$. Use the up and down arrow keys to see which line passes through (12, 8).



***STEP 3* Find the line**

The line that passes through (12, 8) is the line with a slope of 0.5. So, an equation of the line is $y = 0.5x + 2$.

PRACTICE

Display the lines that have the same *y*-intercept but different slopes, as given, in the same viewing window. Determine which line passes through the given point. Write an equation of the line.

1. Slopes: −3, −2, 2, 3; *y*-intercept: 5; point: (−3, 11)
2. Slopes: 4, −2.5, 2.5, 4; *y*-intercept: −1; point: (4, −11)
3. Slopes: −2, −1, 1, 2; *y*-intercept: 1.5; point: (1, 3.5)