4.2 Use Linear Equations in Slope-Intercept Form

Before You wrote an equation of a line using its slope and *y*-intercept.

Now You will write an equation of a line using points on the line.

Why So you can write a model for total cost, as in Example 5.

Key Vocabulary
- ***y*-intercept**
- **slope**
- **slope-intercept form**

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

KEY CONCEPT *For Your Notebook*

Writing an Equation of a Line in Slope-Intercept Form

STEP 1 **Identify** the slope m. You can use the slope formula to calculate the slope if you know two points on the line.

STEP 2 **Find** the y-intercept. You can substitute the slope and the coordinates of a point (x, y) on the line in $y = mx + b$. Then solve for b.

STEP 3 **Write** an equation using $y = mx + b$.

EXAMPLE 1 Write an equation given the slope and a point

Write an equation of the line that passes through the point $(-1, 3)$ and has a slope of -4.

Solution

STEP 1 **Identify** the slope. The slope is -4.

STEP 2 **Find** the y-intercept. Substitute the slope and the coordinates of the given point in $y = mx + b$. Solve for b.

$y = mx + b$ — Write slope-intercept form.

$3 = -4(-1) + b$ — Substitute -4 for m, -1 for x, and 3 for y.

$-1 = b$ — Solve for b.

AVOID ERRORS
When you substitute, be careful not to mix up the x- and y-values.

STEP 3 **Write** an equation of the line.

$y = mx + b$ — Write slope-intercept form.

$y = -4x - 1$ — Substitute -4 for m and -1 for b.

GUIDED PRACTICE for Example 1

1. Write an equation of the line that passes through the point (6, 3) and has a slope of 2.

EXAMPLE 2 Write an equation given two points

Write an equation of the line that passes through (−2, 5) and (2, −1).

Solution

STEP 1 **Calculate** the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{2 - (-2)} = \frac{-6}{4} = -\frac{3}{2}$$

ANOTHER WAY

You can also find the *y*-intercept using the coordinates of the other given point, (2, −1):

$y = mx + b$

$-1 = -\frac{3}{2}(2) + b$

$2 = b$

STEP 2 **Find** the *y*-intercept. Use the slope and the point (−2, 5).

$y = mx + b$ Write slope-intercept form.

$5 = -\frac{3}{2}(-2) + b$ Substitute $-\frac{3}{2}$ for *m*, −2 for *x*, and 5 for *y*.

$2 = b$ Solve for *b*.

STEP 3 **Write** an equation of the line.

$y = mx + b$ Write slope-intercept form.

$y = -\frac{3}{2}x + 2$ Substitute $-\frac{3}{2}$ for *m* and 2 for *b*.

EXAMPLE 3 Standardized Test Practice

Which function has the values $f(4) = 9$ and $f(-4) = -7$?

Ⓐ $f(x) = 2x + 10$ Ⓑ $f(x) = 2x + 1$

Ⓒ $f(x) = 2x - 13$ Ⓓ $f(x) = 2x - 14$

ELIMINATE CHOICES

You can also evaluate each function when $x = 4$ and $x = -4$. Eliminate any choices for which $f(4) \neq 9$ or $f(-4) \neq -7$.

STEP 1 **Calculate** the slope. Write $f(4) = 9$ as (4, 9) and $f(-4) = -7$ as (−4, −7).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 9}{-4 - 4} = \frac{-16}{-8} = 2$$

STEP 2 **Find** the *y*-intercept. Use the slope and the point (4, 9).

$y = mx + b$ Write slope-intercept form.

$9 = 2(4) + b$ Substitute 2 for *m*, 4 for *x*, and 9 for *y*.

$1 = b$ Solve for *b*.

STEP 3 **Write** an equation for the function. Use function notation.

$f(x) = 2x + 1$ Substitute 2 for *m* and 1 for *b*.

▶ The answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 2 and 3

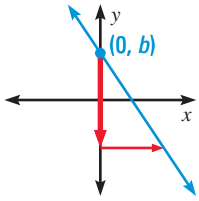
2. Write an equation of the line that passes through (1, −2) and (−5, 4).

3. Write an equation for the linear function with the values $f(-2) = 10$ and $f(4) = -2$.

CONCEPT SUMMARY *For Your Notebook*

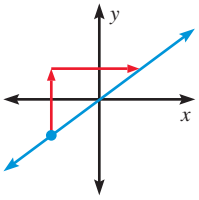
How to Write Equations in Slope-Intercept Form

Given slope m and y-intercept b



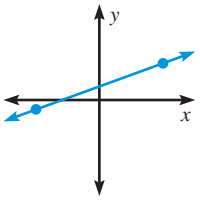
Substitute m and b in the equation $y = mx + b$.

Given slope m and one point



Substitute m and the coordinates of the point in $y = mx + b$. Solve for b. Write the equation.

Given two points



Use the points to find the slope m. Then follow the same steps described at the left.

MODELING REAL-WORLD SITUATIONS You can model a real-world situation that involves a constant rate of change with an equation in slope-intercept form.

EXAMPLE 4 Solve a multi-step problem

GYM MEMBERSHIP Your gym membership costs $33 per month after an initial membership fee. You paid a total of $228 after 6 months. Write an equation that gives the total cost as a function of the length of your gym membership (in months). Find the total cost after 9 months.

Solution

STEP 1 **Identify** the rate of change and starting value.

Rate of change, m: monthly cost, $33 per month
Starting value, b: initial membership fee

STEP 2 **Write** a verbal model. Then write an equation.

Total cost	=	Monthly cost	·	Number of months	+	Membership fee
C	=	33	·	t	+	b

STEP 3 **Find** the starting value. Membership for 6 months costs $228, so you can substitute 6 for t and 228 for C in the equation $C = 33t + b$.

$228 = 33(6) + b$ **Substitute 6 for t and 228 for C.**

$30 = b$ **Solve for b.**

STEP 4 **Write** an equation. Use the function from Step 2.

$C = 33t + 30$ **Substitute 30 for b.**

STEP 5 **Evaluate** the function when $t = 9$.

$C = 33(9) + 30 = 327$ **Substitute 9 for t. Simplify.**

▶ Your total cost after 9 months is $327.

EXAMPLE 5 Solve a multi-step problem

BMX RACING In Bicycle Moto Cross (BMX) racing, racers purchase a one year membership to a track. They also pay an entry fee for each race at that track. One racer paid a total of $125 after 5 races. A second racer paid a total of $170 after 8 races. How much does the track membership cost? What is the entry fee per race?

ANOTHER WAY
For alternative methods for solving the problem in Example 5, see the **Problem Solving Workshop**.

Solution

STEP 1 **Identify** the rate of change and starting value.

Rate of change, m: entry fee per race
Starting value, b: track membership cost

STEP 2 **Write** a verbal model. Then write an equation.

Total cost	=	Entry fee per race	·	Races entered	+	Membership cost
C	=	m	·	r	+	b

STEP 3 **Calculate** the rate of change. This is the entry fee per race. Use the slope formula. Racer 1 is represented by (5, 125). Racer 2 is represented by (8, 170).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{170 - 125}{8 - 5} = \frac{45}{3} = 15$$

STEP 4 **Find** the track membership cost b. Use the data pair (5, 125) for racer 1 and the entry fee per race from Step 3.

$C = mr + b$ Write the equation from Step 2.

$125 = 15(5) + b$ Substitute 15 for m, 5 for r, and 125 for C.

$50 = b$ Solve for b.

▶ The track membership cost is $50. The entry fee per race is $15.

GUIDED PRACTICE for Examples 4 and 5

4. **GYM MEMBERSHIP** A gym charges $35 per month after an initial membership fee. A member has paid a total of $250 after 6 months. Write an equation that gives the total cost of a gym membership as a function of the length of membership (in months). Find the total cost of membership after 10 months.

5. **BMX RACING** A BMX race track charges a membership fee and an entry fee per race. One racer paid a total of $76 after 3 races. Another racer paid a total of $124 after 7 races.

   a. How much does the track membership cost?

   b. What is the entry fee per race?

   c. Write an equation that gives the total cost as a function of the number of races entered.

4.2 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 5, 11, and 49
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 34–37, 41, and 49
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 53

SKILL PRACTICE

1. **VOCABULARY** What is the y-coordinate of a point where a graph crosses the y-axis called?

2. ★ **WRITING** If the equation $y = mx + b$ is used to model a quantity y as a function of the quantity x, why is b considered to be the starting value?

EXAMPLE 1 for Exs. 3–9

WRITING EQUATIONS **Write an equation of the line that passes through the given point and has the given slope m.**

3. $(1, 1)$; $m = 3$
4. $(5, 1)$; $m = 2$
5. $(-4, 7)$; $m = -5$
6. $(5, -5)$; $m = -2$
7. $(8, -4)$; $m = -\frac{3}{4}$
8. $(-3, -11)$; $m = \frac{1}{2}$

9. **ERROR ANALYSIS** *Describe* and correct the error in finding the y-intercept of the line that passes through the point $(6, -3)$ and has a slope of -2.

$y = mx + b$
$6 = -2(-3) + b$
$6 = 6 + b$
$0 = b$

EXAMPLE 4 for Ex. 10

10. **ERROR ANALYSIS** An Internet service provider charges \$18 per month plus an initial set-up fee. One customer paid a total of \$81 after 2 months of service. *Describe* and correct the error in finding the set-up fee.

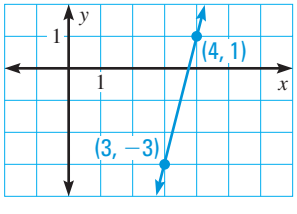
$C = mt + b$
$81 = m(2) + 18$
$63 = m(2)$
$31.50 = m$

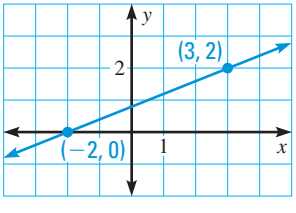
EXAMPLE 2 for Exs. 11–22

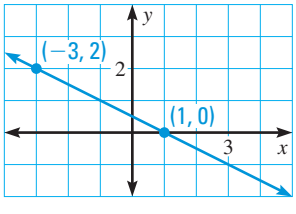
USING TWO POINTS **Write an equation of the line that passes through the given points.**

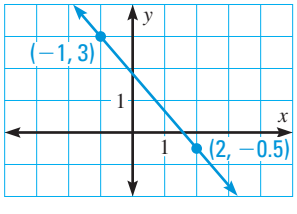
11. $(1, 4)$, $(2, 7)$
12. $(3, 2)$, $(4, 9)$
13. $(10, -5)$, $(-5, 1)$
14. $(-2, 8)$, $(-6, 0)$
15. $\left(\frac{9}{2}, 1\right)$, $\left(-\frac{7}{2}, 7\right)$
16. $\left(-5, \frac{3}{4}\right)$, $\left(-2, -\frac{3}{4}\right)$

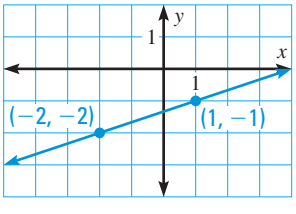
USING A GRAPH **Write an equation of the line shown.**

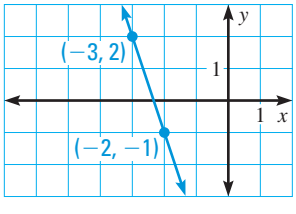
17. 

18. 

19. 

20. 

21. 

22. 

EXAMPLE 3
for Exs. 23–33

WRITING LINEAR FUNCTIONS **Write an equation for a linear function f that has the given values.**

23. $f(-2) = 15, f(1) = 9$

24. $f(-2) = -2, f(4) = -8$

25. $f(2) = 7, f(4) = 6$

26. $f(-4) = -8, f(-8) = -11$

27. $f(3) = 1, f(6) = 4$

28. $f(-5) = 9, f(11) = -39$

29. ★ **MULTIPLE CHOICE** Which function has the values $f(4) = -15$ and $f(7) = 57$?

Ⓐ $f(x) = 14x - 71$

Ⓑ $f(x) = 24x - 1361$

Ⓒ $f(x) = 24x + 360$

Ⓓ $f(x) = 24x - 111$

USING A TABLE OR DIAGRAM **Write an equation that represents the linear function shown in the table or mapping diagram.**

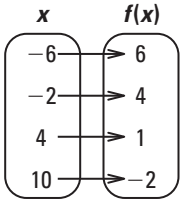
30.

x	$f(x)$
−4	6
4	4
8	3
12	2

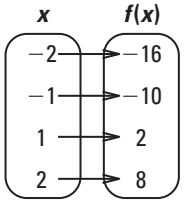
31.

x	$f(x)$
−3	8
3	4
6	2
9	0

32.



33.



★ **SHORT RESPONSE** **Tell whether the given information is enough to write an equation of a line. *Justify* your answer.**

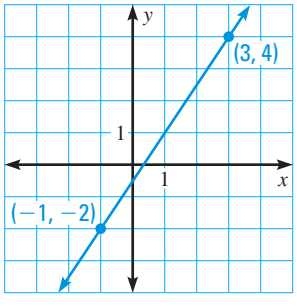
34. Two points on the line

35. The slope and a point on the line

36. The slope of the line

37. Both intercepts of the line

USING A GRAPH **In Exercises 38–41, use the graph at the right.**



38. Write an equation of the line shown.

39. Write an equation of a line that has the same y-intercept as the line shown but has a slope that is 3 times the slope of the line shown.

40. Write an equation of a line that has the same slope as the line shown but has a y-intercept that is 6 more than the y-intercept of the line shown.

41. ★ **WRITING** Which of the lines from Exercises 38–40 intersect? Which of the lines never intersect? *Justify* your answers.

REASONING **Decide whether the three points lie on the same line. *Explain* how you know. If the points do lie on the same line, write an equation of the line that passes through all three points.**

42. (−4, −2), (2, 2.5), (8, 7)

43. (2, 2), (−4, 5), (6, 1)

44. (−10, 4), (−3, 2.8), (−17, 6.8)

45. (−5.5, 3), (−7.5, 4), (−4, 5)

46. **CHALLENGE** A line passes through the points (−2, 3), (2, 5), and (6, k). Find the value of k. *Explain* your steps.

PROBLEM SOLVING

EXAMPLES 4 and 5 for Exs. 47–50

47. BIOLOGY Four years after a hedge maple tree was planted, its height was 9 feet. Eight years after it was planted, the hedge maple tree's height was 12 feet. What is the growth rate of the hedge maple? What was its height when it was planted?

48. TECHNOLOGY You have a subscription to an online magazine that allows you to view 25 articles from the magazine's archives. You are charged an additional fee for each article after the first 25 articles viewed. After viewing 28 archived articles, you paid a total of $34.80. After viewing 30 archived articles, you paid a total of $40.70.

a. What is the cost per archived article after the first 25 articles viewed?

b. What is cost of the magazine subscription?

49. ★ **SHORT RESPONSE** You are cooking a roast beef until it is well-done. You must allow 30 minutes of cooking time for every pound of beef, plus some extra time. The last time you cooked a 2 pound roast, it was well-done after 1 hour and 25 minutes. How much time will it take to cook a 3 pound roast? *Explain* how you found your answer.

HINT In part (b), let t represent the number of years since 1981.

50. TELEPHONE SERVICE The annual household cost of telephone service in the United States increased at a relatively constant rate of $27.80 per year from 1981 to 2001. In 2001 the annual household cost of telephone service was $914.

a. What was the annual household cost of telephone service in 1981?

b. Write an equation that gives the annual household cost of telephone service as a function of the number of years since 1981.

c. Find the household cost of telephone service in 2000.

51. NEWSPAPERS Use the information in the article about the circulation of Sunday newspapers.

a. About how many Sunday newspapers were in circulation in 1970?

b. Write an equation that gives the number of Sunday newspapers in circulation as a function of the number of years since 1970.

c. About how many Sunday newspapers were in circulation in 2000?

Sunday Edition C9

SUNDAY PAPERS INCREASE

From 1970 to 2000, the number of Sunday newspapers in circulation increased at a relatively constant rate of 11.8 newspapers per year. In 1997 there were 903 Sunday newspapers in circulation.

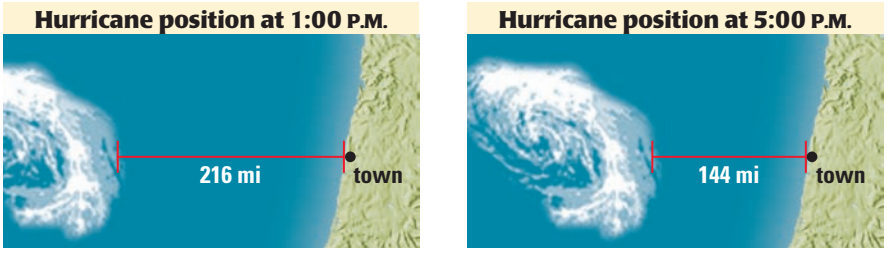
52. AIRPORTS From 1990 to 2001, the number of airports in the United States increased at a relatively constant rate of 175 airports per year. There were 19,306 airports in the United States in 2001.

a. How many U.S. airports were there in 1990?

b. Write an equation that gives the number of U.S. airports as a function of the number of years since 1990.

c. Find the year in which the number of U.S. airports reached 19,200.

53. **MULTIPLE REPRESENTATIONS** A hurricane is traveling at a constant speed on a straight path toward a coastal town, as shown below.



a. Writing an Equation Write an equation that gives the distance (in miles) of the hurricane from the town as a function of the number of hours since 12:00 P.M.

b. Drawing a Graph Graph the equation from part (a). *Explain* what the slope and the y-intercept of the graph mean in this situation.

c. Describing in Words Predict the time at which the hurricane will reach the town. Your answer should include the following information:

- an explanation of how you used your equation
- a description of the steps you followed to obtain your prediction

54. **CHALLENGE** An in-line skater practices at a race track. In two trials, the skater travels the same distance going from a standstill to his top racing speed. He then travels at his top racing speed for different distances.

Trial number	Time at top racing speed (seconds)	Total distance traveled (meters)
1	24	300
2	29	350

a. Model Write an equation that gives the total distance traveled (in meters) as a function of the time (in seconds) at top racing speed.

b. Justify What do the rate of change and initial value in your equation represent? *Explain* your answer using unit analysis.

c. Predict One lap around the race track is 200 meters. The skater starts at a standstill and completes 3 laps. Predict the number of seconds the skater travels at his top racing speed. *Explain* your method.

PROBLEM SOLVING WORKSHOP
LESSON 4.2

Using ALTERNATIVE METHODS

Another Way to Solve Example 5

MULTIPLE REPRESENTATIONS In Example 5, you saw how to solve a problem about BMX racing using an equation. You can also solve this problem using a graph or a table.

PROBLEM

BMX RACING In Bicycle Moto Cross (BMX) racing, racers purchase a one year membership to a track. They also pay an entry fee for each race at that track. One racer paid a total of $125 after 5 races. A second racer paid a total of $170 after 8 races. How much does the track membership cost? What is the entry fee per race?

METHOD 1

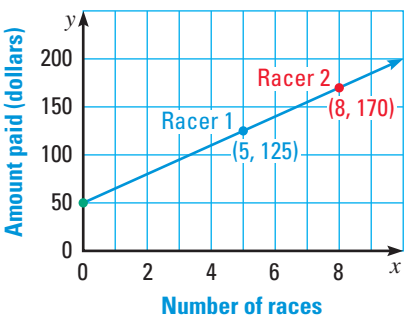
Using a Graph One alternative approach is to use a graph.

STEP 1 **Read** the problem. It tells you the number of races and amount paid for each racer. Write this information as ordered pairs.

Racer 1: (5, 125)
Racer 2: (8, 170)

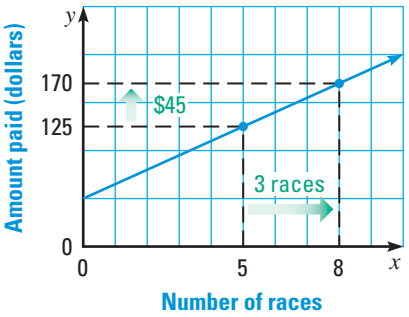
STEP 2 **Graph** the ordered pairs. Draw a line through the points.

The *y*-intercept is 50.
So, the track membership is $50.



STEP 3 **Find** the slope of the line. This is the entry fee per race.

$$\text{Fee} = \frac{\text{45 dollars}}{\text{3 races}} = \$15 \text{ per race}$$



METHOD 2

Using a Table Another approach is to use a table showing the amount paid for various numbers of races.

STEP 1 **Calculate** the race entry fee.

Number of races	Amount paid
5	$125
6	?
7	?
8	$170

+ 3 (5 → 8) + $45 ($125 → $170)

The number of races increased by 3, and the amount paid increased by $45, so the race entry fee is $45 ÷ 3 = $15.

STEP 2 **Find** the membership cost.

Number of races	Amount paid	
0	$50	
1	$65	− $15
2	$80	− $15
3	$95	− $15
4	$110	− $15
5	$125	− $15

The membership cost is the cost with no races. Use the race entry fee and work backwards to fill in the table. The membership cost is $50.

PRACTICE

1. **CALENDARS** A company makes calendars from personal photos. You pay a delivery fee for each order plus a cost per calendar. The cost of 2 calendars plus delivery is $43. The cost of 4 calendars plus delivery is $81. What is the delivery fee? What is the cost per calendar? Solve this problem using two different methods.

2. **BOOKSHELVES** A furniture maker offers bookshelves that have the same width and depth but that differ in height and price, as shown in the table. Find the cost of a bookshelf that is 72 inches high. Solve this problem using two different methods.

Height (inches)	Price (dollars)
36	56.54
48	77.42
60	98.30

3. **WHAT IF?** In Exercise 2, suppose the price of the 60 inch bookshelf was $99.30. Can you still solve the problem? *Explain*.

4. **CONCERT TICKETS** All tickets for a concert are the same price. The ticket agency adds a fixed fee to every order. A person who orders 5 tickets pays $93. A person who orders 3 tickets pays $57. How much will 4 tickets cost? Solve this problem using two different methods.

5. **ERROR ANALYSIS** A student solved the problem in Exercise 4 as shown below. *Describe* and correct the error.

Let p = price paid for 4 tickets

$$\frac{57}{3} = \frac{p}{4}$$

$$228 = 3p$$

$$76 = p$$